

Magnetic wiggler-assisted third-harmonic generation of a Gaussian laser pulse in plasma

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Abstract: A Gaussian laser pulse propagating through plasma in the presence of a magnetic wiggler produces third-harmonic radiation. The wiggler's magnetic field provides additional momentum required for phase matching. The required wiggler wave number is sensitive to pulse duration and amplitude. The efficiency of the process is significant at the instant at which the phase matching condition is satisfied.

Key words: Harmonic generation, wiggler magnetic field, phase matching

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1. Introduction

Harmonic generation in plasma and semiconductors has been an area of significant interest for the past several decades [1–10]. Harmonic generation offers an alternative source for short wavelength generation and an important tool for diagnostics of nonlinear media. In plasmas the nonlinearity arises due to 3 effects: relativistic effects, ponderomotive force, and collisions between electrons and ions. With the development of high intensity ($\geq 10^{20} \text{W/cm}^2$) short-pulse ($\sim \text{fs}$) lasers, the electron motion becomes highly nonlinear, giving rise to nonlinearity rather than the anharmonicity of the bounded electron oscillation in atoms and molecules; thus, the relativistic nonlinearity plays a dominant role. The efficiency of the harmonic generation process is significantly affected due to the phase mismatch between the fundamental and generated harmonic radiation. Several schemes have been proposed to make the harmonic generation process a resonant one. Parashar and Pandey [11,12] proposed the employing of a density ripple or a magnetic wiggler to compensate for the momentum mismatch between the pump and second-harmonic wave in plasma and semiconductors, respectively. Their studies showed significant enhancement in the efficiency of the second-harmonic process. Shkolnikov et al. [13] demonstrated the feasibility of optimal quasiphase matching for higher-order harmonic generation in gases and plasmas with modulated density. Rax and Fisch [14] studied phase-matched relativistic third-harmonic generation employing a resonant density modulation in a plasma. Averchi et al. [15] proposed a different approach to obtain phase-matched generation of high-order harmonics based on the use of pulsed Bessel beams. Sheinfux et al. [16] demonstrated a scheme for creation of periodic plasma structures by ablating a lithographic pattern for quasiphase matched harmonic generation. Recently, Sapaev et al. [17] demonstrated a novel method of quasiphase matching for third-harmonic generation in noble gases employing ultrasound. Sodha

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et al. [18] derived the phase matching conditions for generation of third-harmonic and combination frequency in collisional magnetoplasma. Kuo et al. [19] reported enhancement of relativistic third-harmonic generation by using an optically preformed periodic plasma waveguide. Katzir et al. [20] also reported quasiphase matching and increase of laser interaction length by periodically modulated plasma guide formed by a discharge current ablating the inner capillary walls. Shibu and Tripathi [21] studied phase-matched third-harmonic generation of a laser beam guided through a plasma channel. Chen et al. [22] reported observation of phase-matched relativistic third-harmonic generation in plasmas employing a T: sapphire - Nd: glass laser system of ≤ 2 J and 400 fs pulse at $1.053 \mu\text{m}$.

In this paper we study the third-harmonic generation of a Gaussian laser pulse in a plasma in the presence of a wiggler magnetic field, including the relativistic effects. The wiggler provides the additional momentum required to make the process a resonant one. The physics of the process is as follows: the oscillatory velocity of the plasma electrons due to pump laser field at (ω, \vec{k}) beats with a laser magnetic field to produce a second-harmonic ponderomotive force at $(2\omega, 2\vec{k})$. The oscillatory velocity due to this ponderomotive force beats with wiggler magnetic field $(0, \vec{k}_w)$ to exert a ponderomotive force at $(2\omega, 2\vec{k} + \vec{k}_w)$. The oscillatory velocity due to this ponderomotive force at $(2\omega, 2\vec{k} + \vec{k}_w)$ couples with electron density oscillations at (ω, \vec{k}) to produce a nonlinear current at $(3\omega, 3\vec{k} + \vec{k}_w)$, which drives the third-harmonic radiation. In the following sections we give the analysis for nonlinear current density and third-harmonic field, and in the end, we discuss our results.

2. Nonlinear current density

Consider the propagation of a Gaussian laser pulse through a plasma of electron density n_0^0 . The electric and magnetic fields of laser pulse are given by:

$$\begin{aligned}\vec{E} &= \hat{x} A e^{-i(\omega t - kz)}, \\ \vec{B} &= \hat{y} \frac{ck}{\omega} A e^{-i(\omega t - kz)}, \\ A^2 &= A_0^2 e^{-(t-z/v_g)^2/\tau^2},\end{aligned}\quad (1)$$

where $k = (\omega/c)\eta$, η is the refractive index of the plasma, and $v_g = c\eta \approx c$ is the group velocity. The oscillatory velocity of electrons due to the laser on solving the equation of motion $m(d\vec{v}/dt) = -e\vec{E} - (e/c)\vec{v} \times \vec{B}$ is

$$\vec{v} = \hat{x} \frac{eA}{m\omega\gamma_0} e^{-i(\omega t - kz)}, \quad (2)$$

where $-e$ and m are electronic charge and mass, respectively, and $\gamma_0 \approx (1 + a^2/2)^{1/2}$, $a = eA/m\omega c$, and $a < 1$. In terms of γ_0 and the plasma frequency $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, the refractive index η (in the limit $\omega_p^2/\omega^2 \ll 1$) can be written as

$$\eta(\omega) = 1 - \omega_p^2/2\omega^2\gamma_0. \quad (3)$$

There also exists a wiggler magnetic field given by

$$\vec{B}_w = \hat{y} B_0 e^{ik_w z}. \quad (4)$$

For third-harmonic generation, the third-harmonic wave vector is $k_3 > 3k_1$. For the process to be a resonant one, the phase matching conditions demand

$$\omega_3 = 3\omega_1,$$

and

$$\hbar k_3 = 3\hbar k_1 + \hbar k_w. \quad (5)$$

To satisfy the phase matching conditions, the required wiggler wave number k_w is

$$k_w \approx \frac{4}{3} \frac{\omega}{c} \frac{\omega_p^2}{\omega^2 \gamma_0}. \quad (6)$$

In Figures 1 and 2, we show the variation of normalized wiggler wave number ck_w/ω_p with t'/τ ($t' = t - z/c$) at different values of $a_0 = (eA_0/m\omega c)$ and ω_p/ω . The required wiggler wave number is smaller for higher values of a_0 and larger for higher plasma density and pulse duration.

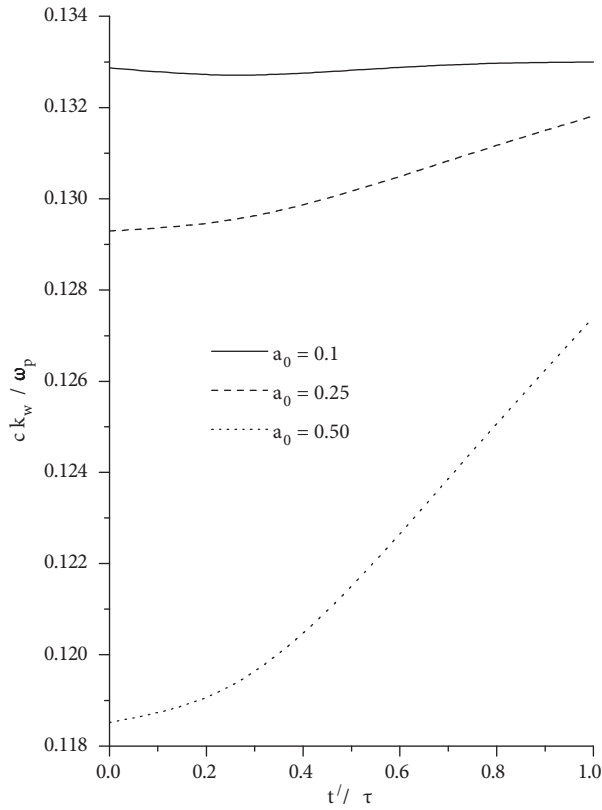


Figure 1. Variation of normalized wiggler wave number ck_w/ω_p with t'/τ for $a_0 = 0.1, 0.25,$ and $0.50,$ respectively, at $\omega_p/\omega = 0.1$.

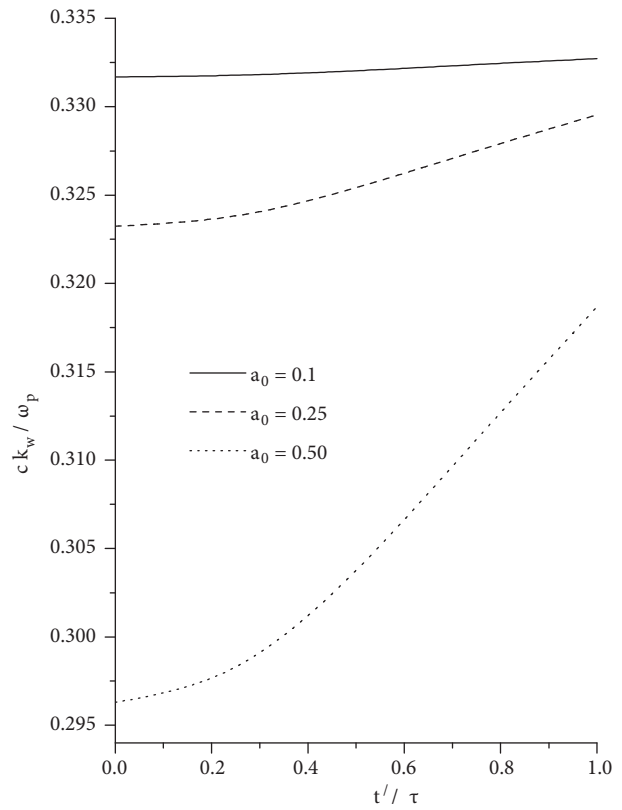


Figure 2. Variation of normalized wiggler wave number ck_w/ω_p with t'/τ for $a_0 = 0.1, 0.25,$ and $0.50,$ respectively, at $\omega_p/\omega = 0.25$.

Using Eq. (2) in the equation of continuity $\partial n/\partial t + \nabla \cdot (n_0 \vec{v}_1) = 0$, the electron density perturbation n_1 at (ω, \vec{k}) is obtained as

$$n_1 = \frac{k}{\omega} \frac{n_0 e A}{m \omega i \gamma_0} e^{-i(\omega t - kz)}. \quad (7)$$

The electron velocity \vec{v} beats with laser magnetic field \vec{B} to produce a ponderomotive force,

$$\vec{F}_{p2} = -\frac{e}{2c}\vec{v} \times \vec{B} = -\hat{z}\frac{e^2 A^2 k}{2mi\omega^2 \gamma_0} e^{-2i(\omega t - kz)}. \quad (8)$$

The electron velocity \vec{v} at $(2\omega, 2\vec{k})$ due to \vec{F}_p is

$$\vec{v}_2 = -\hat{z}\frac{e^2 A^2 k}{4m^2 \omega^3 \gamma_0} e^{-2i(\omega t - kz)}. \quad (9)$$

\vec{v}_2 beats with \vec{B}_w to exert a ponderomotive force \vec{F}'_{p2} at $(2\omega_1, 2\vec{k} + \vec{k}_w)$,

$$\vec{F}'_{p2} = -\frac{e}{2c}\vec{v}_2 \times \vec{B}_w = -\hat{x}\frac{e}{2c}\frac{e^2 A^2 k B_0}{4m^2 \omega^3 \gamma_0} e^{-i[2\omega t - (2k+k_w)z]}. \quad (10)$$

Electron oscillatory $\vec{v}_2'^{at}(2\omega_1, 2\vec{k} + \vec{k}_w)$ due to \vec{F}'_{p2} is

$$\vec{v}_2' = -\hat{x}\frac{e}{2c}\frac{e^2 A^2 B_0 k}{4m^2 \omega^3 \gamma_0 2mi\omega} e^{-i[2\omega t - (2k+k_w)z]}. \quad (11)$$

The electron velocity \vec{v}_2' beats with electron density perturbation n_1 to produce a nonlinear current density \vec{J}_3^{NL} at $(3\omega, 3\vec{k} + \vec{k}_w)$ as

$$\vec{J}_3^{NL} = -\frac{1}{2}n_1 e \vec{v}_2' = -\hat{x}\frac{1}{16}\frac{n_0 e^5 A^3 B_0 k^2}{\omega^6 m^4 \gamma_0^2 c}. \quad (12)$$

There also exists a self-consistent third-harmonic field $\vec{E}_3 = \hat{x}A_3 e^{-i[3\omega t - (3k+k_w)z]}$. The linear current density \vec{J}_3^L due to \vec{E}_3 is

$$\vec{J}_3^L = -\frac{n_0 e^2 \vec{E}_3}{3im\omega}. \quad (13)$$

3. Third-harmonic field

The wave equation governing the third-harmonic field is

$$\frac{\partial^2 \vec{E}_3}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}_3}{\partial t^2} - \frac{1}{c^2} \frac{\partial \vec{J}_3^L}{\partial t} = -\frac{4\pi}{c^2} 3i\omega \vec{J}_3^{NL} = \hat{x}\alpha e^{-i[3\omega t - (3k+k_w)z]}, \quad (14)$$

where $\alpha = \frac{3i}{16} \frac{\omega_p^2}{\omega^2} \frac{a^2}{\gamma_0^2} \frac{\omega_c}{\omega} k^2 A$, $\omega_c = \frac{eB_0}{mc}$ and $a = \frac{eA}{m\omega c}$.

On further simplification of Eq. (14) considering the group velocity of the third harmonic as c and $k_3 = (3\omega/c)(1 - \omega_p^2/18\omega^2\gamma_0)$, we obtain

$$\frac{\partial A_3}{\partial z} + \frac{1}{c} \frac{\partial A_3}{\partial t} = \frac{\alpha}{2ik_3}. \quad (15)$$

Introducing a new set of variables $z' = z$, $t' = t - z/c$, Eq. (15) reduces to

$$\frac{\partial A_3}{\partial z'} = \frac{\alpha}{2ik_3} e^{-i\Delta z'}. \quad (16)$$

Here, $\Delta = k_3 - 3k - k_w$. For the Gaussian pulse $a^2 = a_0^2 \exp(-t'^2/\tau^2)$, $a_0 = eA_0/m\omega c$, $\gamma_0 = [1 + (a_0^2/2) \exp(-t'^2/\tau^2)]^{1/2}$ is a function of time. For a given k_w , one cannot have phase matching ($\Delta = 0$) for harmonic generation at all times. If one matches the wiggler wave number at the peak of the laser pulse ($t' = 0$), $k_w \approx \frac{4}{3} \frac{\omega}{c} \frac{\omega_p^2}{\omega^2 \gamma_{00}}$, where $\gamma_{00} = (1 + a_0^2/2)^{1/2}$. At all other times we have $\Delta = k_w \left(\frac{\gamma_{00}}{\gamma_0} - 1 \right)$, and Eq. (16) gives

$$A_3 = \frac{\alpha [e^{-i\Delta(\gamma_{00}/\gamma_0 - 1)z'} - 1]}{2k_3 k_w (\gamma_{00}/\gamma_0 - 1)}. \tag{17}$$

At a distance $z = L$ we obtain the following expression for a normalized third-harmonic wave amplitude from Eq. (17):

$$\left| \frac{A_3}{A_0} \right| = \left[\frac{1}{16} \frac{\omega_p^2}{\omega^2} \frac{\omega_c}{\omega} \frac{k}{k_3} \frac{k}{k_w} a_0^2 \right] \frac{e^{-3t'^2/\tau^2} \text{sinc} k_w L (\gamma_{00}/\gamma_0 - 1)}{\gamma_0^2 (\gamma_{00}/\gamma_0 - 1)}. \tag{18}$$

In Figures 3 and 4 we show the variation of $|A_3/A_0|$ with t'/τ at $a_0 = 0.1$ and 0.25 , respectively. The other parameters are $\omega_p/\omega = 0.1$ & 0.25 , $\omega_c/\omega = 0.01$, and $L\omega_p/c = 6 \times 10^3$. The efficiency of the process is sensitive

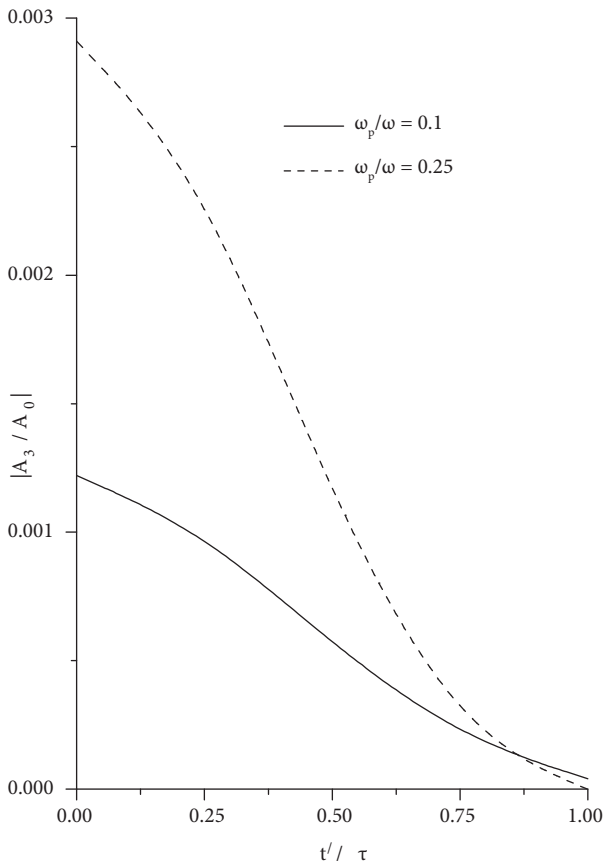


Figure 3. Variation of normalized third-harmonic field $|A_3/A_0|$ with t'/τ for $\omega_p/\omega = 0.1$ and 0.25 , respectively, at $a_0 = 0.1$, $\omega_c/\omega = 0.01$, and $L\omega_p/c = 6 \times 10^3$.

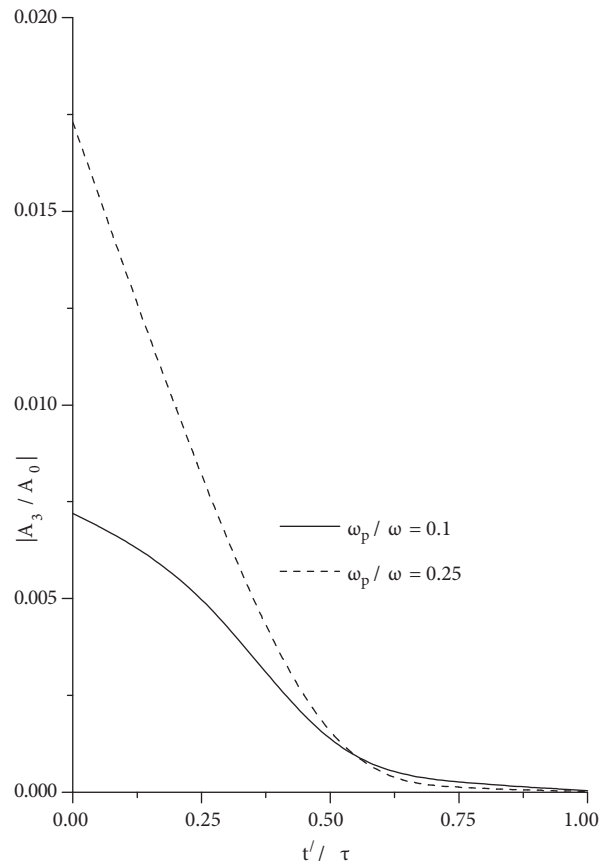


Figure 4. Variation of normalized third-harmonic field $|A_3/A_0|$ with t'/τ for $\omega_p/\omega = 0.1$ and 0.25 , respectively, at $a_0 = 0.25$, $\omega_c/\omega = 0.01$, and $L\omega_p/c = 6 \times 10^3$.

to laser pulse duration and is maximum at $t'/\tau = 0$, i.e. when the phase matching condition is perfectly satisfied.

4. Discussion

A magnetic wiggler can be employed to provide the additional momentum required for resonant third-harmonic generation. However, for a laser pulse of short duration, the phase matching can be achieved for an instant only at a particular wiggler wave number. At a later duration of laser pulse, one requires a larger wiggler wave number, k_w . The required wiggler wave number is smaller for higher values of a_0 and plasma frequency ω_p . The efficiency of the process is maximum at an instant when the phase matching is satisfied and thereafter decreases sharply at later durations of laser pulse. The maximum efficiency of the process is $|A_3/A_0| \sim 0.017$ for $\omega_p/\omega = 0.25$, $a_0 = 0.25$, $\omega_c/\omega = 0.01$, and $L\omega_p/c = 6 \times 10^3$. This set of parameters can be realized by using a CO₂ laser (10.6 μm , 10^{14} W/cm²) in a plasma of electron density of $\sim 10^{17}$ cm⁻³, wiggler with $\lambda_w = 2$ mm, $B_0 = 100$ kG, and plasma length $L = 10$ cm. The efficiency of the process can be increased for a larger duration by using a tapered wiggler magnetic field or plasma with a tapered density. We have ignored the other nonlinear effects arising due to the relativistic nonlinearity, namely self-phase modulation and filamentation of the laser beam. These effects can significantly alter the efficiency of the process. The filamentation of the laser beam can be checked by employing beam-smoothing techniques [23].

Acknowledgments

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