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Exact solution of FRW perfect fluid model interacting with zero-rest-mass scalar field and electromagnetic field in B-D theory

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Abstract: Considering a spherically symmetric nonstatic cosmological flat model of the Robertson–Walker universe, the problem of perfect fluid distribution interacting with the gravitational field in the presence of a zero-rest-mass scalar field and electromagnetic field in Brans–Dicke (B-D) theory has been investigated. Exact solutions are obtained by using a general approach of solving the partial differential equations and it has been observed that the electromagnetic field cannot survive the cosmological flat model due to the action caused by the presence of the zero-rest-mass scalar field, and the isotropic pressure p turns out to be negative. The pressure and density are found to be independent of time.

Key words: FRW model, B-D theory, electromagnetic field, zero-rest-mass

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1. Introduction

The study of the coupled electromagnetic and scalar field has drawn the attention of many researchers as a result of the discovery of meson particles interacting with charged electrons and masses of the order of magnitude of 200 electron masses present in the cosmic rays. Various problems relating to the charged particles in presence of a scalar field have been investigated by various authors. By the method of approximation, Stephenson [1] obtained exact solutions by taking a source-free electromagnetic and scalar field for a static spherically symmetric space-time. Exact solutions for the static coupled electromagnetic and zero-rest-mass meson fields were obtained by Janis et al. [2], and Penny [3] generalized the Reissner–Nordström solutions in the presence of a point charge by considering coupled gravitational and zero-rest-mass scalar meson fields. Ibotombi et al. [4] investigated the same problem by using a different method of finding exact solutions where electromagnetic fields survive. Many studies, such as [5–12], involved charging the well-known uncharged perfect fluid solutions.

In view of the above investigations and with respect to the scalar fields, the present authors have developed a different approach of solving the problem and exact solutions have been obtained for a cosmological model where electromagnetic fields cannot survive. This implies that charged perfect fluid cannot have interaction with the zero-rest-mass scalar fields in Brans–Dicke (B-D) theory for a cosmological model. In Section 2, we present the field equations and their solutions. In Section 3, physical interpretations of the solutions are presented.

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2. Field equations and their solutions

The B-D field equations in Dicke's unit transformation are given by:

$$G_j^i \equiv R_j^i - \frac{1}{2}\delta_j^i R = -8\pi\left(T_j^i + E_j^i + S_j^i\right) - \frac{2\omega+3}{2}\frac{1}{\varphi^2}\left[\varphi^{,i}\varphi_{,j} - \frac{1}{2}\delta_j^i\varphi^{,\mu}_{,\mu}\right], \tag{1}$$

and the wave equation for the scalar field is

$$\Box\left(\ell n\varphi\right) \equiv \left(\ell n\varphi\right)^{;\mu}_{;\mu} = \frac{8\pi}{2\omega+3}T, \tag{2}$$

where T_j^i, E_j^i, and S_j^i are the energy-momentum tensors for the perfect fluid, electromagnetic field, and zero-rest-mass scalar field, respectively, and are given by the following equations.

$$T_j^i = (p+\rho)u^i u_j - p\delta_j^i, \tag{3}$$

where p is the isotropic pressure, ρ is the fluid density, and u^i is the 4-velocity vector of the flow satisfying

$$u^i u_i = 1. \tag{4}$$

$$E_j^i = \frac{1}{4\pi}\left[-F^{i\alpha}F_{j\alpha} + \tfrac{1}{4}\delta_j^i F_{\alpha l}F^{\alpha l}\right], \tag{5}$$

where F_{ij} are the electromagnetic field tensors, and

$$S_j^i = \frac{1}{4\pi}\left[V^{,i}V_{,j} - \tfrac{1}{2}\delta_j^i V^{,k}V_{,k}\right], \tag{6}$$

where V is the scalar potential satisfying the wave equation

$$g^{ij}V_{;ij} = \sigma(r,t), \tag{7}$$

where σ is the source density of the scalar potential.

Using the co-moving coordinate system, we get

$$u^1 = u^2 = u^3 = 0 \text{ and } u^4 = 1. \tag{8}$$

The electromagnetic field equations are given by

$$F^{ij}_{;j} = -J^i \tag{9}$$

and

$$F_{[ij,k]} = 0, \tag{10}$$

where J^i is the current 4-vector and, in general, is the sum of the convention current and conduction current, i.e.

$$J^i = \in_0 u^i - \sigma_0 u^\gamma F^i_\gamma, \tag{11}$$

where $\in_0$ is the rest charge density and σ_0 is the conductivity.

The metric taken for the present problem is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right], \tag{12}$$

where t is the cosmic time, R(t) is the radius of the Universe, and K is the curvature index, which takes the values +1, 0, and –1. Here a comma or semicolon followed by a subscript denotes partial or covariant differentiation, respectively. A dot and a dash over a letter denote partial differentiation with respect to time t and radial distance r.

The surviving field equations for the metric (12) are:

$$G_1^1 \equiv \frac{2R\ddot{R} + \dot{R}^2 + K}{R^2} = -8\pi p + 8\pi E_1^1 - \frac{1-kr^2}{R^2}V'^2 - \dot{V}^2 - \frac{2\omega+3}{4}\left[\frac{1-Kr^2}{R^2}\left(\frac{\varphi'}{\varphi}\right)^2\left(\frac{\dot{\varphi}}{\varphi}\right)^2\right], \tag{13}$$

$$G_2^2 \equiv \frac{2R\ddot{R} + \dot{R}^2 + K}{R^2} = -8\pi p + 8\pi E_2^2 + \frac{1-Kr^2}{R^2}V'^2 - \dot{V}^2 + \frac{2\omega+3}{4}\left[\frac{1-Kr^2}{R^2}\left(\frac{\varphi'}{\varphi}\right)^2 - \left(\frac{\dot{\varphi}}{\varphi}\right)^2\right], \tag{14}$$

$$G_3^3 \equiv G_2^2, \tag{15}$$

$$G_4^4 \equiv 3\left(\frac{\dot{R}^2 + K}{R^2}\right) = 8\pi\rho + 8\pi E_4^4 + \frac{1-Kr^2}{R^2}V'^2 + \dot{V}^2 + \frac{2\omega+3}{4}\left[\frac{1-Kr^2}{R^2}\left(\frac{\varphi'}{\varphi}\right)^2 + \left(\frac{\dot{\varphi}}{\varphi}\right)^2\right], \tag{16}$$

$$G_4^1 \equiv 0 = 4\pi E_4^1 + V'\dot{V} + \frac{2\omega+3}{4}\left(\frac{\varphi'}{\varphi}\right)\left(\frac{\dot{\varphi}}{\varphi}\right), \tag{17}$$

$$G_2^1 \equiv 0 = -\frac{1-Kr^2}{R^4 r^2\sin^2\theta}F_{13}F_{23} + \frac{1-Kr^2}{R^2}F_{14}F_{24}, \tag{18}$$

$$G_3^1 \equiv 0 = \frac{1}{R^2 r^2}F_{12}F_{23} + F_{14}F_{34}, \tag{19}$$

$$G_3^2 \equiv 0 = -\frac{1-kr^2}{R^2}F_{12}F_{13} + F_{24}F_{34}, \tag{20}$$

$$G_4^2 \equiv 0 = -(1-kr^2)F_{12}F_{14} + \frac{1}{r^2\sin^2\theta}F_{23}F_{34}, \tag{21}$$

and

$$G_4^3 \equiv 0 = (1-kr^2)F_{13}F_{14} + \frac{1}{r^2}F_{23}F_{24}. \tag{22}$$

The wave equation (2) reduces to

$$-\frac{1-kr^3}{R^2}(\ln\varphi)'' - \frac{2-3kr^3}{R^2 r}(\ln\varphi)' + \frac{3\dot{R}}{R}(\dot{\ln\varphi}) + (\ddot{\ln\varphi}) = \frac{8\pi}{2\omega+3}(\rho - 3p) + \frac{2}{2\omega+3}\left[\frac{1-kr^2}{R^2}V'^2 - \dot{V}^2\right]. \tag{23}$$

Also, from equation (7), we get:

$$-\frac{1-kr^2}{R^2}V'' - \frac{2-3kr^2}{R^2 r}V' + 3\frac{\dot{R}}{R}\dot{V} + \ddot{V} = \sigma(r,t). \tag{24}$$

Again from equation (9), we obtain the following 4 equations:

$$\frac{\partial F^{14}}{\partial t} + 3\frac{\dot{R}}{R}F^{14} = \sigma_0 F_4^1, \tag{25}$$

$$\frac{\partial F^{14}}{\partial r} + \left\{\frac{kr}{1-kr^2} + \frac{2}{r}\right\} F^{14} = \varepsilon_0, \tag{26}$$

$$\frac{\partial F^{23}}{\partial \theta} + F^{23}\cot\theta = 0, \tag{27}$$

and

$$\frac{\partial F^{23}}{\partial \varphi} = 0. \tag{28}$$

Also, from equation (10), we obtain

$$\frac{\partial F_{14}}{\partial \theta} = \frac{\partial F_{14}}{\partial \varphi} = 0 \tag{29}$$

and

$$\frac{\partial F_{23}}{\partial r} = \frac{\partial F_{23}}{\partial t} = 0. \tag{30}$$

From equations (18) through (22), we have the following 3 possible cases:

i) $F_{12} = F_{13} = F_{34} = F_{24} = 0$, at least one of F_{14}, F_{23} being nonzero.

ii) $F_{12} = F_{14} = F_{34} = F_{23} = 0$, at least one of F_{24}, F_{13} being nonzero.

iii) $F_{14} = F_{24} = F_{13} = F_{23} = 0$, at least one of F_{12}, F_{34} being nonzero.

Hence, the electromagnetic field is nonnull and consists of an electric and/or magnetic field, both of which are in the direction of the same space axis. Without loss of generality we may consider case (i), in which also the component $F_{14} \neq 0$, $F_{23} = 0$, which is directed in the direction of the x-axis.

Using the above assumption in equation (5), we obtain

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = \frac{1}{8\pi}.\frac{1-kr^2}{R^2}(F_{14}), \tag{31}$$

and

$$E_{ij} = 0, (i \neq j; i, j = 1, 2, 3, 4). \tag{32}$$

Using equation (31), the field equations (13), (14), and (16) respectively become

$$\frac{2R\ddot{R} + \dot{R}^2 + k}{R^2} = -8\pi p + \frac{1-kr^2}{R^2}(F_{14})^2 - \frac{1-kr^2}{R^2}V'^2 - \dot{V}^2 - \frac{2\omega+3}{4}\left[\frac{1-kr^2}{R^2}\left(\frac{\varphi'}{\varphi}\right)^2 + \left(\frac{\dot{\varphi}}{\varphi}\right)^2\right], \tag{33}$$

$$\frac{2R\ddot{R} + \dot{R}^2 + k}{R^2} = -8\pi p - \frac{1-kr^2}{R^2}(F_{14})^2 + \frac{1-kr^2}{R^2}V'^2 - \dot{V}^2 + \frac{2\omega+3}{4}\left[\frac{1-kr^2}{R^2}\left(\frac{\varphi'}{\varphi}\right)^2 - \left(\frac{\dot{\varphi}}{\varphi}\right)^2\right], \tag{34}$$

and

$$\frac{3\dot{R}^2+3k}{R^2}=8\pi\rho+\frac{1-kr^2}{R^2}\left(F_{14}\right)^2+\frac{1-kr^2}{R^2}V'^2+\dot{V}^2+\frac{2\omega+3}{4}\left[\frac{1-kr^2}{R^2}\left(\frac{\varphi'}{\varphi}\right)^2+\left(\frac{\dot{\varphi}}{\varphi}\right)^2\right]. \tag{35}$$

By using equation (32) in equation (17), we get

$$0=V'\dot{V}+\frac{2\omega+3}{4}\left(\frac{\varphi'}{\varphi}\right)\left(\frac{\dot{\varphi}}{\varphi}\right). \tag{36}$$

Solving equation (36), we obtain

$$V=\frac{\sqrt{-(2\omega+3)}}{2}\log\varphi+a_1, \tag{37}$$

where a_1 is an arbitrary constant.

From equations (33), (34), and (37), we get

$$F_{14}=0. \tag{38}$$

Making use of equations (37) and (38) in (33) or (34) and (35), we obtain

$$8\pi p=-\frac{2R\ddot{R}+\dot{R}^2+k}{R^2} \tag{39}$$

and

$$8\pi\rho=\frac{3\dot{R}^2+3k}{R^2}. \tag{40}$$

When the curvature index k = 0, we obtain the following from equation (23):

$$-\frac{1}{R^2}(\ln\varphi)''-\frac{2}{R^2r}(\ln\varphi)'+\frac{3\dot{R}}{R}(\ln\varphi)^{\cdot}+(\ln\varphi)^{\cdot\cdot}=\frac{8\pi}{2\omega+3}(\rho-3p)+\frac{2}{2\omega+3}\left(\frac{1}{R^2}V'^2-\dot{V}^2\right). \tag{41}$$

Using the well-known Hubble's principle

$$\frac{\dot{R}}{R}=\alpha, \tag{42}$$

where α is the Hubble's constant, we obtain the following from (41):

$$\frac{3}{2}\left(\frac{\varphi'}{\varphi}\right)^2-\frac{\varphi''}{\varphi}-\frac{2}{r}\frac{\varphi'}{\varphi}=R^2\left[\frac{12\alpha^2}{2\omega+3}+\frac{3}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2-3\alpha\left(\frac{\dot{\varphi}}{\varphi}\right)-\frac{\ddot{\varphi}}{\varphi}\right]. \tag{43}$$

From equation (43), we obtain

$$\varphi=e^{nt}.\frac{r^2}{(A-Br)2}, \tag{44}$$

with the relation on constants

$$24\alpha^2+(n^2-6\alpha n)(2\omega+3)=0, \tag{45}$$

where n, A, and B are arbitrary constants.

From equations (37) and (44), we obtain

$$V=\frac{\sqrt{-(2\omega+3)}}{2}\log\left[\frac{e^{nt}r^2}{(A-Br)^2}\right]+a_1. \tag{46}$$

Using equations (42) and (44) in equation (24), we obtain

$$\sigma(r,t)=\sqrt{-(2\omega+3)}\left[\frac{3}{2}\alpha n-\frac{A^2e^{-2\alpha t}}{A_1^2r^2(A-Br)^2}\right], \tag{47}$$

where A_1 is an arbitrary constant. Equations (39) and (40) become

$$p=-\frac{3}{8\pi}\alpha^2 \tag{48}$$

and

$$\rho=\frac{3}{8\pi}\alpha^2. \tag{49}$$

From equation (26), we obtain

$$\varepsilon_0=0. \tag{50}$$

From equation (11), we obtain

$$J^i=0, i=1,2,3,4. \tag{51}$$

3. Physical interpretation of the solutions

It is clear from equation (38) that the electromagnetic field, when interacting with the scalar field and B-D field in the presence of perfect fluid, does not survive. From (44), it is observed that the B-D scalar field φ is an exponentially increasing function of time when we take A = 0, and it is a quadratically increasing function of radial coordinate r when B = 0. However, when t tends to infinity with r remaining constant, φ tends to infinity. From (46), we see that the scalar field, V, is physically realistic provided the coupling constant $\omega < -3/2$.

It is also observed that the scalar field V is a linear function of time t only when A = 0. When both r and t tend to infinity simultaneously, φ and V tend to infinity provided $B \neq 0$. The source density σ of the scalar potential reduces to a constant quantity when both r and t tend to infinity, but the magnitude of the source density is constrained to satisfy the relation $\frac{3}{2}\alpha n > \frac{A^2e^{-2\alpha t}}{A_1^2r^2(A-Br)^2}$, for all values of r and t. It is seen that the model of the Universe does not satisfy the fluid energy condition $\rho +$ p > 0. The pressure P is found to be negative because of the interaction caused by the presence of scalar field V and the fluid acquires a repulsive character in nature. At the same time, the electromagnetic field gives no contribution in yielding the pressure P to be negative. Since the charged density ε_0 and the current density J^0 become zero, it is thereby shown that the matter becomes electrically neutral.

The present work deals with a closed form of exact solutions of nonstatic static, spherically symmetric perfect fluid interacting with zero-rest-mass scalar field in B-D theory corresponding to the Friedmann–Robertson–Walker (FRW) model. The solutions will play a physically significant role in studies of the interaction of electromagnetic and zero-rest-mass scalar fields in a FRW model universe. Such solutions will also help in the study of B-D gravitational theory and the relationship between this theory and Einstein's theory. We hope that some physical insight can be gained from these solutions obtained in this paper.

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