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Level structure of ^{210}Po by means of surface delta interaction

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Abstract: Within the framework of the shell model, we consider the residual interaction to be a surface delta interaction. This interaction has been applied in the calculation of the energy levels of ^{210}Po, which belong to N = 126 magic closed shell nuclei. The pure and the configuration mixings are applied in the calculations of matrix elements. The predicted energy levels are compared with experimental data.

Key words: Shell model, surface delta interaction, magic numbers, energy levels

1. Introduction

The radioactive even-even heavy nucleus ^{210}Po has Z = 84 protons (2 protons) outside the magic closed shell at Z = 82, and N = 126 neutrons. This nucleus is part of over 25 radioactive Po-isotopes with atomic masses ranging from A = 192 to A = 218.

The level structure of ^{210}Po is a matter of many experimental works; see, for example, [1–4] and the references cited therein. Theorists as well try to investigate the structure of this nucleus using different models [5].

The experimental level structure of this nucleus should close to a collective vibrational nucleus, at least for the first 3 levels, due to the effect of a closed neutron shell. The (equation) ratio is $E_{4_1}/E_{2_1} = 2\hbar\omega/\hbar\omega = 2$ in a vibrational structure, while in this nucleus the experimental ratio is $E_{4_1}/E_{2_1} = 1426.7/1181.4 = 1.2$, which means that it is far from the common nature of vibrational nuclei [6]. The low energy levels of the ground band $(0_1, 2_1, 4_1, 6_1, 8_1)$ have been observed experimentally with many crowded levels, energies between 1.44 and 1.56 MeV, and spin of 4 to 8. The 0_2^+ does not exist; also, there is uncertainty in level spin $(1^+, 2^+)$ at energy 2.285 MeV (where the group of the 3-phonon triplet should exist if the nucleus is a vibrator). One can see also a large energy gap ($\approx$ 0.638 MeV) and then a high density of 7 levels with energy between 2.188 to 2.438 MeV. The problem of the energy spectrum of this nucleus is complicated and one should be careful in his conclusions about the structure of this nucleus.

A close look at the experimental energies of the ground band, in the Figure, shows the systematics of Po even-even isotopes from N = 120 to 132, where our interest lies in the middle of the chain at N = 126. From A = 216 to A = 212, there are neutron particles outside the closed shell, while from A = 204 to A = 208 there are 2 valance protons outside the closed shell. Those particles (holes) interact always with the 2 valance protons outside the closed shell at Z = 82 to create the energy spectrum of a certain nucleus. In the Figure,

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one can see the dominating effect of neutrons, close to the neutrons' magic shell, in the heavy nuclei despite the effect of the large number of charged protons. This leads to the assumption that in this region there is an increased number of mixed symmetry states [7].

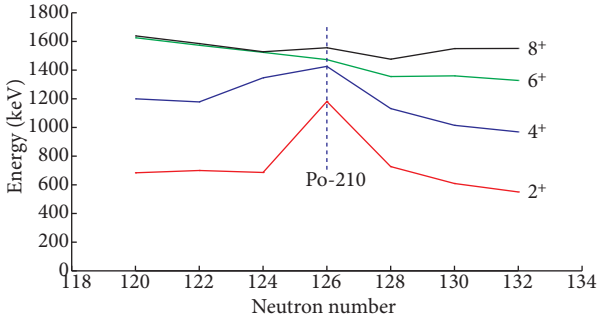

Figure. The experimental energy [6] of the ground band as a function of neutron number for Po isotopes, from A = 204 to A = 216.

The nucleus ^{210}Po, close to the mass region around the doubly magic nucleus ^{208}Pb, is one of the most important testing grounds for the nuclear shell model [8]. A few theoretical works have been made to calculate the spectrum of this nucleus and compare it with experimental results [9,10]

In this work, an attempt been made to calculate the energy spectrum of a ^{210}Po nucleus using surface delta interaction (SDI) using pure and mixing configurations [11,12].

2. The theoretical model

In the traditional shell model calculations, it is customary to calculate the energy levels relative to a closed shell rather than the total energy of the system, and for a single nucleon outside the doubly magic core. Energy in this case is taken to be an eigenvalue of the Hamiltonian H_0, when there is more than one nucleon outside the core and the total Hamiltonian is written as:

$$H = \sum_{k=l}(H_o)_k + \sum_{k\leq l} V_{kl} \tag{1}$$

where V_{kl} is the residual 2-body interaction, which exists in addition to the average shell-model potential, and we can express this as:

$$\sum_{k<l} V_{kl} = \sum_{JM} \sum_{j_a \geq j_b} \sum_{j_c \geq j_d} \langle j_a j_b | V_{12} | j_c j_d \rangle_J \; a^+_{JM}(j_a j_b) \; a_{JM}(j_c j_d) \tag{2}$$

where $<\quad>$ is the matrix element of the residual 2-body interaction.

$a^+_{JM}(j_a j_b)$ is an operator that creates a pair of nucleons in the single-particle states j_a and j_b with total angular momentum JM and $a_{JM}(j_c j_d)$ is the one that destroys a pair of particles in states j_c and j_d; it is the Hermitian adjoin operator to $a^+_{JM}(j_a j_b)$.

If the 2 particles occupy the same level, the energy relative to the closed shell is:

$$\langle H \rangle = 2\varepsilon_j + \rangle jj/V_{12}|jj\rangle_j, \tag{3}$$

where ε_j is the single-particle energy.

In order to describe the low-lying states, one should consider a situation where several single-particle levels are the foundation; for example, if there are 2 states denoted by $|\ \mathrm{j}_1\ \mathrm{j}_2\ \mathrm{J\ M}>$ and $|\ \mathrm{j}_3\ \mathrm{j}_4\ \mathrm{J\ M}>$, then their energies with respect to the core are given by

$$\begin{array}{l}\langle H\rangle_{11} = \varepsilon_{j_1} + \varepsilon_{j_2} + \langle j_1 j_2|V_{12}|j_1 j_2\rangle_J, \\ \langle H\rangle_{22} = \varepsilon_{j_3} + \varepsilon_{j_4} + \langle j_3 j_4|V_{12}|j_3 j_4\rangle_J,\end{array} \tag{4}$$

The above equations calculate the energies that consider only pure configuration states, neglecting the mixing between orbits. However, the nucleons may be scattered from one state, $|\ \mathrm{j}_1\ \mathrm{j}_2\ \mathrm{J\ M}>$, into other, $|\ \mathrm{j}_3\ \mathrm{j}_4\ \mathrm{J\ M}>$. Thus, a mixture of states must give the actual state. This means that a term like $<\mathrm{H}>_{12}$ should be added to Eq. (3), which can be written as [13]:

$$\langle H\rangle_{12} = \langle j_1 j_2|V_{12}|j_3 j_4\rangle_J. \tag{5}$$

Thus, energy separation between the states changes when configuration mixing between 2 states is taken into account.

The residual nucleon–nucleon interaction is assumed to be the SDI; in order to calculate the matrix element, this interaction can be considered as a δ-interaction for which one makes a further assumption that all the radial integrals are equal. This interaction has strength V_0, and it can written as a delta function in the angular coordinates of the interacting particles [10,12,13]:

$$V_{ik}^{SDI} = -4\pi V_o \delta(\Omega_{ik}), \tag{6}$$

where $\delta(\Omega_{ik}) = \delta(r(i) - r(k))\delta(r(i) - R_0)$, $r(i)$ $r(k)$ are the position vector of the interacting particles and R_0 is the nuclear radius. The 2-body matrix elements are given by:

$$\langle j_a j_b|V_{12}^{SDI}|j_c j_d\rangle_{JT} = \langle j_a j_b| - 4\pi V_o \delta(\Omega_{12}|j_c j_d\rangle_{JT}, \tag{7}$$

where j_a stands for the complete set of single-particle quantum numbers n_a (radial nodes), l_a (orbital angular momentum), and j_a (total angular momentum). The 2-particle states are each coupled to total J and T (isospin), which can be used to label the strength parameter.

In this work, we restrict ourselves to the assumption that the residual interaction is obtained between the identical particles (protons), i.e. $\mathrm{T} = 1$, and so the SDI matrix elements in Eq. (3) take the following form [9]: $\delta(\Omega_{ik}) = \delta(r(i) - r(k))\delta(r(i) - R_0)$, $r(i)$ $r(k)$ are the position vector of the interacting particles and R_0 is the nuclear radius. The 2-body matrix elements are given by:

$$\begin{aligned}\langle j_a j_b|V_{12}^{SDI}|j_c j_d\rangle_J \;=\; & (-1)^Q\{1+(-1)^{l_a+l_b+l_c+l_d}\}\{1+(-1)^{l_c+l_d+J}\} \\ & \{V_0 R_0/4(2J+1)\}\{(2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)/(1+\delta_{ab}), \\ & (1+\delta_{cd})\}^{1/2}\Big\langle j_a \frac{1}{2} j_b - \frac{1}{2}|J0\Big\rangle\Big\langle j_c \frac{1}{2} j_d - \frac{1}{2}|J0\Big\rangle\end{aligned} \tag{8}$$

where $Q = j_a + j_c + l_b + l_d + n_a + n_b + n_c + n_d$, R_o is a positive number, and $\langle|\rangle$ is the Clebsch–Gordan coefficient [14].

3. Calculations and results

The SDI interaction discussed in the previous section has been used extensively as the effective 2-body interaction in many regions in the periodic table [15,16]. A number of nuclear properties of many nuclei are compared with the calculations of other models and experimental data [17,18]. In this work, the energy levels of ^{210}Po are calculated according to the following details.

3.1. ^{210}Po in a complete shell

In this case, there are 2 protons outside the closed shell Z = 82. The inert core used in this calculation is $^{208}Pb_{82}$, and the 2 valance protons are distributed over the model space that is restricted by the orbits from $2f87/2$ to $1i_{13/2}$. The 2 particles' possible configurations for this nucleus are shown in Table 1. The wave function of the excited states contains mostly proton contribution. In this case, the original SDI Hamiltonian is adjusted to fit exactly the ground state energy. It is found that the agreement with the experimental data is not very good; this may be due to the effect of reducing collectivity in the wave function of these states in this nucleus. In order to include the neutron's contribution, configuration mixing between the orbit is applied.

Table 1. The possible 2-particle configurations of ^{210}Po.

J	Configurations
0	$(h_{9/2})^2$, $(f_{7/2})^2$, $(i_{13/2})^2$, $(f_{5/2})^2$, $(p_{3/2})^2$, $(p_{1/2})^2$
1	$h_{9/2}\ f_{7/2}$, $f_{7/2}\ f_{5/2}$, $f_{5/2}\ p_{3/2}$, $p_{3/2}\ p_{1/2}$
2	$(h_{9/2})^2$, $h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $h_{9/2}\ f_{5/2}$, $(f_{7/2})^2$, $f_{7/2}\ f_{5/2}$, $f_{7/2}\ p_{3/2}$, $(i_{13/2})^2$, $(f_{5/2})^2$, $f_{5/2}\ p_{3/2}$, $f_{5/2}\ p_{1/2}$, $(p_{3/2})^2$, $p_{3/2}\ p_{1/2}$
3	$h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $h_{9/2}\ f_{5/2}$, $h_{9/2}\ p_{3/2}$, $f_{7/2}\ i_{13/2}$, $f_{7/2}\ f_{5/2}\ f_{7/2}\ p_{3/2}$, $f_{7/2}\ p_{1/2}$, $f_{5/2}\ p_{3/2}$, $f_{5/2}\ p_{1/2}$
4	$(h_{9/2})^2$, $h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $h_{9/2}\ f_{5/2}$, $h_{9/2}\ p_{3/2}$, $h_{9/2}\ p_{1/2}$ $(f_{7/2})^2$, $f_{7/2}\ i_{13/2}$, $f_{7/2}\ f_{5/2}$, $f_{7/2}\ p_{3/2}$, $f_{7/2}\ p_{1/2}$, $(i_{13/2})^2$, $i_{13/2}\ f_{5/2}$, $f_{5/2}\ p_{3/2}$, $(f_{5/2})^2$
5	$h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $h_{9/2}\ f_{5/2}$, $h_{9/2}\ p_{3/2}$, $h_{9/2}\ p_{1/2}$, $f_{7/2}\ i_{13/2}$ $f_{7/2}\ f_{5/2}$, $f_{7/2}\ p_{3/2}$, $i_{13/2}\ f_{5/2}$, $i_{13/2}\ p_{3/2}$
6	$(h_{9/2})^2$, $h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $h_{9/2}\ f_{5/2}$, $h_{9/2}\ p_{3/2}$, $(f_{7/2})^2$ $f_{7/2}\ i_{13/2}$, $f_{7/2}\ f_{5/2}$, $(i_{13/2})^2$, $i_{13/2}\ f_{5/2}$, $i_{13/2}\ p_{3/2}$, $i_{13/2}\ p_{1/2}$
7	$h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $h_{9/2}\ f_{5/2}$, $f_{7/2}\ i_{13/2}$, $i_{13/2}\ f_{5/2}$, $i_{13/2}\ p_{3/2}$, $i_{13/2}\ p_{1/2}$
8	$(h_{9/2})^2$, $h_{9/2}\ f_{7/2}$, $h_{9/2}\ i_{13/2}$, $f_{7/2}\ i_{13/2}$, $i_{13/2}\ f_{5/2}$, $i_{13/2}\ p_{3/2}$, $i_{13/2}\ p_{1/2}$
9	$h_{9/2}\ i_{13/2}$, $f_{7/2}\ i_{13/2}$, $i_{13/2}\ f_{5/2}$
10	$h_{9/2}\ i_{13/2}$, $f_{7/2}\ i_{13/2}$, $(i_{13/2})^2$
11	$h_{9/2}\ i_{13/2}$
12	$(i_{13/2})^2$

The calculated energy levels are summarized in Tables 2 and 3, including the comparisons with available experimental data.

From the details of Table 2, one can make the following remarks.

1. In the table, one can recognize that there is fair agreement between theory and experimental data for the mixing configuration; indeed, in many cases, the calculated levels are too low in comparison with the experimental value (remember that, in our model, there is one fitting parameter). Concerning the pure configuration, shown in Table 3, the levels are pushed down by a factor of 2. Despite the fact that there is no clear band structure in this isotope, we were able to produce the same systematic structure of levels. By using a mixed configuration, much better agreement with experimental data was produced. However,

Table 2. The calculated energy levels with their spin and parity, using configuration mixing, compared with experimental data.

Present work		Exp.		|ΔE|	Present work		Exp.		|ΔE|
$J\pi$ E(MeV)		$J\pi$	E	MeV	$J\pi$	E (MeV)	J	E (MeV)	MeV
0^+	0.0	0^+	0.0	0.0	9^-	3.609			
2^+	0.911	2^+	1.181	0.270	$3^-, 4^-, 6^-, 8^-, 10^-$		6^-	3.525	0.132
4^+	1.041	4^+	1.426	0.385		3.657			
6^+	1.092	6^+	1.473	0.381	4^+	3.665		3.694	0.029
8^+	1.120	8^+	1.557	0.437	0^+	3.821	(2^+)	3.792	0.074
8^+	1.853	8^+	2.188	0.335	6^+	3.911		4.043	0.132
6^+	1.951	6^+	2.326	0.375	$3^+, 5, 7^+$	3.978	$(7, 8, 9^-)$	4.026	0.048
4^+	1.986	4^+	2.383	0.397	6^+	4.088	(6^+)	4.141	0.052
2^+	2.014	2^+	2.290	0.276	4^+	4.210	(4^+)	4.029	0.181
1^+	2.048	1^+	2.393	0.345	2^+	4.259		4.237	0.022
3^+	2.048	5^+	2.425	0.377	$3^+, 5^+$	4.271	(3^+)	4.320	0.049
0^+	2.399	0^+	2.609	0.210	4^+	4.318	$(11)^-$	4.324	
9^-	2.650		2.658	0.001	8^+	4.322		4.330	
7^-	2.688				6^+	4.344	(6^+)	4.470	0.126
5^-	2.710	5^-	2.910		$10^+, 12^+$	4.369	(12^-)	4.503	0.134
2^+	2.721				4^+	4.684	(4^+)	4.542	0.142
3^-	2.741	$(3)^-$	2.846		6^+	4.767	(6^+)	4.645	0.122
2^-	2.761	11^-	2.849		5^+	4.785	(5^+)	4.624	0.161
4^-	2.761				2^+	4.846		4.948	
6^-	2.761				4^+	4.855			
4^+	2.823	2.872			$1^+, 3^+, 5^+$				
6^+	2.869				4.874			5.041	
5^-	3.487	5^-	3.428		2^+	5.028		5.186	
2^+	3.504				4^+	5.166			
7^-	3.562	7^-			3^+	5.167			
9^-	5.406	3.685							
7^-	5.489				2^+	6.722			
5^-	5.515				4^+	6.767			
4^-	5.587				4^+	7.004			
6^-	5.587				2^+	7.062			
8^-	5.587				$1^+, 3^+$				
4^+	5.620				7.097				
3^+	5.681				0^+	7.277			
5^+	5.681				2^+	7.340			
5^-	5.720				2^+	7.523			
7^-	5.820				3^+	7.729			
6^-	5.880				2^+	7.858			
8^-	5.880				1^+	7.904			
7^-	6.307				0^+	8.353			

for most of the excited state, the discrepancies are around 0.1 MeV. This is quite acceptable in the general model space used for this isotope.

2. Many calculated levels with their spin were in good agreement with the experimental data.
3. There is uncertainty in the spin of some high energy levels experimentally; the same energy is predicted with a single spin value. As examples of this: levels with energies of 3.6375, 3.6939, 4.1051, 4.1411, 4.2371, and 4.3461 MeV with a total angular momentum of 7, 3, 2, 5, 4, and 2, respectively, with positive parity.
4. Some energy levels with their spins are proposed in certain regions of the spectrum, including high energy levels up to 8.329 MeV (note that experimental data is lacking above 5.2701 MeV).

3.2. Calculations without $1i_{13/2}$

As was already mentioned before, there has been a mixture between close levels in the wave function. In this case, a mixed wave function between 2 orbits, $2f_{7/2}$ and $1i_{13/2}$, has been proposed. To check the ability of the model to calculate the energy levels, the calculation was repeated ignoring the $1i_{13/2}$ orbits. It was found that fewer levels in the energy range were produced and fewer levels in the allowed angular momentum range were produced (values 9, 10, 11, and 12 disappeared). This means that one should do the calculation with

Table 3. Calculated energy levels with their spin and parity using pure configuration, compared with experimental data.

Present work		Exp.		$\|\Delta E\|$	Present work		Exp.		$\|\Delta E\|$
J^{π}	E(MeV)	J^{π}	E	MeV	J^{π}	E (MeV)	J	E (MeV)	MeV
0^+	0.0	0^+	0.0	0.0	7^-	3.028			
2^+	0.466	2^+	1.181	0.715	9^-	3.071	6^-	3.525	0.454
4^+	0.538	4^+	1.427	0.889	2^+	3.089			
6^+	0.569	6^+	1.473	0.904	3^-, 4^-	3.120		3.694	0.574
8^+	0.590	8^+	1.557	0.967	8^-, 10^-		(2^+)	3.792	
8^+	1.313	8^+	2188	0.875				4.043	
6^+	1.431	6^+	2.326	0.895			$(7, 8, 9^-)$	4.026	
4^+	1.467	4^+	2.383	0.916	4^+	3.313	(6^+)	4.141	0.828
2^+	1.490	2^+	2.290	0.800	6^+	3.381	(4^+)	4.029	0.641
1^+, 3^+	2.048	1^+,3	2.393	0.345	3^+,	5, 7		4.237	0.796
5^+, 7^+		5^+, 7	2.438		3.441		(3^+)	4.320	0.741
0^+	1.915	0^+	2.609	0.694	6^+	3.579	$(11)^-$	4.324	0.704
9^-	2.125		2.658	0.569	2^+	3.620		4.330	0.646
7^-	2.163				4^+	3.684	(6^+)	4.470	0.753
5^-	2.186	5^-	2.910	0.724	4^+	3.717	(12^-)	4.503	0.769
3^-	2.204				3^+, 5^+	3.734	(4^+)	4.542	0.784
2^-	2.224	$(3)^-$	2.840	0.616	6^+	3.758	(6^+)	4.645	0.864
4^-, 6^-		11^-	2.849		8^+	3.781	(5^+)	4.624	
8^-, 10^-, 11					10^+,	12^+		4.948	
2^+	2.290				3.832				
4^+	2.346	4^+	2.872	0.526	4^+	4.111			
6^+	2.379				6^+	4.165		5.041	
5^-	2.959	5^-	3.429	0.470	5^+	4.248		5.186	
0^+	2.971				4^+	4.273			
		7^-	3.686		2^+	4.309			
1^+,	3								
4.337					6^-				
					0^+				
2^+	4.377				2^+	6.1			
4^+	4.552				4^+				
3^+	4.630				4^+				
9^-	4.875				2^+				
7^-	4.974				1^+,	3^+			
5^-	5.013				6.560				
4^+	5.035				0^+	6.607			
4^-, 6^-	5.050				2^+	6.803			
5^-	5.126				2^+	6.926			
3^+, 5^+	5.144				3^+	7.074			
7^-	5.271				2^+	7.269			
6^-,	8				1^+	7.367			
5.343					0^+	7.758			
7^-	5.742								

the assumption that the wave function of the 2 levels is mixed in order to get a better agreement with the experimental data. Actually, this is true for most of the high energy levels.

3.3. Calculations without $3p_{1/2}$

The calculations of energy levels were repeated ignoring the effect of the $3p_{1/2}$ orbital, which means ignoring the effect of neutrons. It was found that there are minor effects on the values of the matrix elements, and this reduced the number of levels. All 3 methods discussed above were repeated using mixed configurations and the results were closer to the experimental data.

4. Conclusion

The present calculation of the energy levels using the SDI interaction reproduced the experimental energy levels well in nuclei that have a very small nucleon number outside the closed shell. The model used in the present work predicts most of the energy levels and their systematics with their total angular momentum. However, it was not possible to reproduce the position of some levels, for example the position of the 0^+ states. It was possible to calculate the level structure to high excitation energy and spins, which had not been reported earlier. Comparisons of the experimental results with the shell-model calculations showed that the level structures exhibit mainly single-particle character.

From the calculations, one can conclude that:

1. The mixing between shells should be included in order to get more realistic values compared with experimental data.
2. The effect of ignoring the $p_{1/2}$ orbit does not affect the results when configuration mixing is used, but it has an effect when neglecting orbit $i_{13/2}$.
3. The level structure of this isotope still needs experimental data for checking the model's predictions.

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