

Evidence that photons have extension in space

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Abstract: Satyendra Nath Bose realized that there was a logical inconsistency in the derivation of Planck’s radiation law since the derivation used to obtain the temperature-independent prefactor in the blackbody radiation law had been based in part on classical wave hypotheses, such as the number of degrees of freedom of the ether. Bose worked out a strictly quantum derivation by combining the quantum hypothesis with statistical mechanics to determine the number of states of each mode that would occupy a 6-dimensional phase space. Here I use the model of an extended photon to show that the temperature-independent prefactor of Planck’s radiation law can be derived in an alternative manner by calculating how many extended photons of each mode would fill a 3-dimensional Euclidean space. By combining my derivation of the temperature-independent prefactor with Planck’s temperature-dependent probability distribution, I show that all of the major equations that describe blackbody radiation can be derived from the assumption that the photon is an extended body in 3-dimensional Euclidean space. This derivation provides evidence for the suggestion that photons are neither mathematical points nor groups of infinite plane waves, but quantized and finite volume elements.

Key words: Blackbody radiation, Planck’s radiation law, photon

1. Introduction

Even though Planck believed that light itself was continuous and extended throughout space, his analysis of blackbody radiation led to a revolution in the conception of the nature of light [1]. At a time when the classical wave theory, which was based on the assumption that light was continuous and extended throughout space, was so productive in explaining optical phenomena, including diffraction, refraction, reflection, and dispersion, Einstein [2] thought that blackbody radiation and other phenomena, including the photoelectric effect “can be understood better if one assumes that the energy of light is discontinuously distributed in space”. Einstein went on to say “when a light ray is spreading from a point, the energy is not distributed continuously over ever-increasing spaces, but consists of a finite number of energy quanta that are localized in points in space, move without dividing, and can be absorbed or generated only as a whole”. According to Einstein’s [3] quantum hypothesis, the corpuscular properties of light, including energy ($\varepsilon = \frac{hc}{\lambda}$) and linear momentum ($p = \frac{h}{\lambda}$), were not only mechanical properties but also wave-like properties since they depended on the wavelength λ of light. However, the wave-like properties of the photon did not represent oscillation or extension in space but were encapsulated within a mathematical point. Einstein’s point-like photon was supported by Millikan’s [4] experiments on the photoelectric effect and Compton’s [5] scattering experiments that indicated that energy and momentum were conserved when photons scattered off electrons as if the photons were little billiard balls.

As quantum theory began to explain more and more phenomena, including the specific heat of solids

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and the spectral lines of atoms, Bose [6] realized that there was a logical inconsistency in the derivation of Planck's radiation law. The derivation used to obtain the temperature-independent prefactor in his blackbody radiation law had been based in part on classical wave hypotheses, such as the number of degrees of freedom of the ether. Bose then worked out a strictly quantum derivation by combining the quantum hypothesis with statistical mechanics.

Bose assumed that the quantum of light had a moment of magnitude equal to $\frac{h}{\lambda}$. That, is the moment was inversely proportional to the wavelength of the quantum, the constant of proportionality being Planck's constant. Bose then represented the quantum of light as a coordinate point in a 6-dimensional phase space that satisfied the following relation:

$$p_x^2 + p_y^2 + p_z^2 = \left(\frac{h}{\lambda}\right)^2, \quad (1)$$

where p_x , p_y , and p_z are the components of the momentum in 3-dimensional Euclidean space. The wavelength domain between λ and $\lambda + d\lambda$ is associated with the phase space domain $dx dy dz dp_x dp_y dp_z$ according to the following relation:

$$\int dx dy dz dp_x dp_y dp_z = V 4\pi \left(\frac{h}{\lambda}\right)^2 \frac{h}{\lambda^2} d\lambda = V 4\pi \frac{h^3}{\lambda^4} d\lambda, \quad (2)$$

where $4\pi \left(\frac{h}{\lambda}\right)^2$ is the surface area of the phase space for wavelength λ , $\frac{h}{\lambda^2} d\lambda$ is the thickness of the shell in phase space that represents the wavelength interval $d\lambda$, and V is the total volume of the radiation. The phase space is equal to the product of the volume of radiation and the momentum of radiation and has dimensions of $\text{J}^3 \text{s}^3$ or $\text{kg}^3 \text{m}^6 \text{s}^{-3}$. Bose subdivided the phase space into unit cells with magnitude h^3 , which represented the ultimate elementary region in phase space. The number of unit cells in phase space is equal to one-eighth the number of coordinate points in phase space or the number of coordinate points in an octant of phase space. The ratio of the phase space to h^3 is dimensionless and represents the number $Nd\lambda$ of fundamental unit cells that belong to a wavelength domain from λ to $\lambda + d\lambda$. $Nd\lambda$ is given by:

$$Nd\lambda = \frac{1}{h^3} \int dx dy dz dp_x dp_y dp_z = V \frac{4\pi}{\lambda^4} d\lambda. \quad (3)$$

The number $Nd\lambda$ of fundamental unit cells or states that belong to a wavelength domain from λ to $\lambda + d\lambda$ is related to a ratio of volumes. Specifically, it is related to the ratio of the volume of the radiation to the third power of the wavelength of radiation.

Bose interpreted the number density $d\varphi$ to represent the number $Nd\lambda$ of possible arrangements or states in a given volume V that one quantum of light in the wavelength domain from λ to $\lambda + d\lambda$ could assume. The relation between $d\varphi$ and wavelength λ is given by:

$$d\varphi = \frac{Nd\lambda}{V} = \frac{4\pi}{\lambda^4} d\lambda. \quad (4)$$

In order to account for the 2 orthogonal polarizations or spin states of light, Bose multiplied the above value of possible states by 2:

$$d\varphi = \frac{Nd\lambda}{V} = \frac{8\pi}{\lambda^4} d\lambda. \quad (5)$$

Bose derived the temperature-independent prefactor for Planck's radiation law by considering the number of states a photon with a given wavelength could occupy. While Bose's derivation is better known, de Broglie [7] published a similar momentum-based quantum-based derivation 2 years before Bose.

While treating the quantum of light as a mathematical point in the positive octant of phase space allowed Bose to derive Planck's blackbody radiation law from quantum principles, treating the quantum of light as a geometrical point in 3-dimensional Euclidean space bothered many of Einstein's contemporaries, including Planck [1], Bohr [8], Lorentz [9], and Millikan [4], who believed that a theory of light should also be able to explain wave phenomena that required an extended photon. Ludwik Silberstein [10] introduced the cross-sectional area of Einstein's light quantum or light dart as a useful working hypothesis to quantitatively describe the relationship between the size of silver halide grains and the light-induced darkening of film that takes place during in the photographic process. Silberstein calculated the cross-sectional area of a 470-nm photon to be between $8.1 \times 10^{-15} \text{ m}^2$ and $97.3 \times 10^{-15} \text{ m}^2$. Given that the number $Nd\lambda$ of fundamental unit cells that belong to a wavelength domain from λ to $\lambda + d\lambda$ is related to the ratio of the volume of the radiation to the third power of the wavelength of radiation, is it possible that that number $Nd\lambda$ of fundamental unit cells that belong to a wavelength domain from λ to $\lambda + d\lambda$ is also related to the ratio of the volume of the radiation to the volume of the photons of a given mode?

2. Results and discussion

I have previously modeled the photon as a composite particle that oscillates in 3 dimensions [11]. I have been able to use this model of a quantized and extended photon to explain phenomena that are usually considered to be a result of the relativity of space and time. For example, I have explained why charged particles cannot exceed the vacuum speed of light [12], the arrow of time [13], and the observed gravitational deflection of starlight [14]. Here I make use of this model of the extended photon to derive the temperature-independent prefactor of Planck's blackbody radiation law. Instead of making use of 6-dimensional phase space, I use 3-dimensional Euclidean space to determine how many photons, whose volume depends on their wavelength, can fit in a unit volume. I show that Planck's blackbody radiation law as well as the major equations that describe blackbody radiation can be logically and simply derived from the assumption that photons have extension in space.

In order to derive the blackbody radiation formula, I start with the assumptions that 2 factors set the limits on the distribution of energy within a cavity. The first assumption is due to Planck. It is that the temperature of the walls of the cavity sets the limit for the short wavelength radiation. While the derivation of Planck's blackbody radiation law is based on the assumption that the volume of the cavity is large compared with the wavelength of light [15], the long wavelength radiation may still be limited if each photon is quantized into a real and finite volume element such that the ratio of the geometrical volume of the cavity to the geometrical volume of photons limits the number of long wavelength photons. The importance of geometry in quantum optics is illustrated by the characteristics of quantum dots where the structure of the optical spectrum produced by an electron-hole pair in the semiconductor of which the quantum dot is made is a function of the size of the quantum dot [16]. Each volume element or mode in a blackbody cavity is characteristic of a photon with a given wavelength. Each volume element or mode is related to the linear momentum ($\frac{h}{\lambda}$) and total energy ($\frac{hc}{\lambda}$) of a photon [11]. According to my model [11], the average length (L_λ) of a photon of wavelength λ oscillating between approximately $L = 0$ and $L = \lambda$ is given by:

$$L_\lambda = \frac{\lambda}{2}. \quad (6)$$

The cross sectional area (A_λ) of a photon, which is calculated based on its quantized angular momentum [11] and its rotational kinetic energy [14], is also a function of wavelength λ and is given by:

$$A_\lambda = \pi r^2 = \pi \left(\frac{\lambda\sqrt{2}}{2\pi} \right)^2 = \frac{\lambda^2}{2\pi}. \quad (7)$$

The average volume (V_λ) of a photon [14] with wavelength λ approximates a cylinder and is given by:

$$V_\lambda = L_\lambda A_\lambda = \frac{\lambda}{2} \frac{\lambda^2}{2\pi} = \frac{\lambda^3}{4\pi}. \quad (8)$$

The volume occupied by a given photon is proportional to the cube of its wavelength—the longer the wavelength, the greater the volume occupied by the photon and the more “ethereal” is the photon (Figure). The shorter the wavelength, the smaller the volume occupied by the photon and the more “particle-like” is the photon. As an illustration, the volume of a radio wave photon with a wavelength of 1 km is about $8 \times 10^7 \text{ m}^3$ and the volume of a gamma ray photon with a wavelength of 10^{-12} m is about $8 \times 10^{-38} \text{ m}^3$. Because the photon oscillates, the average volume of a photon with a given wavelength is quantized, but its instantaneous volume varies in time. Indeed, this time variation in volume may be the basis of the uncertainty principle.

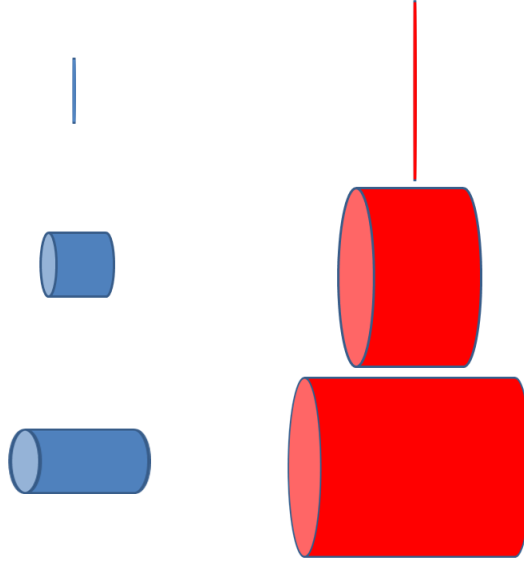


Figure. The average volume of a photon oscillating in space is a function of its wavelength. The shorter the wavelength (blue), the smaller is the average volume and the more particle-like is the photon. The longer the wavelength (red), the larger is the average volume and the more “ethereal” is the photon. The wavelength-independent parameters characteristic of all photons include angular momentum ($\frac{h}{2\pi}$) and vacuum velocity (c).

Here I assume that the unit cell (in m^3) in a black body cavity is equal to L^3 , which contrasts with the unit cell in Bose’s derivation that is equal to h^3 . The largest single photon of a given wavelength that can occupy a unit cell (in m^3) is:

$$L^3 = \frac{\lambda_1^3}{4\pi} n_1, \quad (9)$$

where n_1 indicates that a photon with wavelength λ_1 represents the first or fundamental mode. Since a cylinder-like photon can be oriented arbitrarily in 3 directions in space, for spatial symmetry, L^3 is the volume of the unit cell that can contain a photon propagating in any direction in space. This ensures that the radiation is isotropic. Eq. (9) can be written in terms of the symmetrical volume that can contain 1 photon propagating in any direction in 3-dimensional space:

$$L^3 = \frac{3\lambda_1^3}{4\pi} n_1. \quad (10)$$

To ensure that the radiation is unpolarized when it is emitted by a blackbody, the number of unit cells containing a photon of a given mode must be doubled to account for the necessity of equal numbers of photons in unpolarized radiation having 2 orthogonal polarizations:

$$2L^3 = \frac{3\lambda_1^3}{4\pi} n_1. \quad (11)$$

Eq. (11) can be written in terms of the symmetrical volume that contains 1 polarized photon propagating in 1 of the 3 orthogonal spatial directions:

$$L^3 = \frac{3\lambda_1^3}{8\pi} n_1. \quad (12)$$

One unit cell containing 1 photon in my derivation is equivalent to 1 coordinate point in the positive octant in Bose's derivation.

When the energy of the system is increased, L^3 can be populated with additional modes, where $n_i > n_1$ and $\lambda_i < \lambda_1$. The greater the mode number, the smaller the volume of each photon that populates that mode $\left(\frac{\lambda_i^3}{4\pi} < \frac{\lambda_1^3}{4\pi}\right)$ and the greater the number of photons that can populate that mode. For the infinite series of photons that can populate each volume, for each mode, Eq. (12) becomes:

$$L^3 = \frac{3\lambda_i^3}{8\pi} n_i. \quad (13)$$

The number density (φ) of photons of a given mode (i) is given by the following equation:

$$\varphi = \frac{n}{L^3} = \frac{8\pi}{3\lambda^3}. \quad (14)$$

Assuming that φ is very large and $d\varphi$ is very small, the number of photons in the wavelength range from λ to $\lambda + d\lambda$ that can fit in the unit volume can be obtained by differentiating Eq. (14) with respect to wavelength:

$$\frac{d\varphi}{d\lambda} = -8\pi\lambda^{-4}. \quad (15)$$

After rearranging, we get:

$$d\varphi = -8\pi\lambda^{-4}d\lambda, \quad (16)$$

where $d\varphi$ represents the number of photons in the wavelength range from λ to $\lambda + d\lambda$ that can fit in the 3-dimensional unit volume. Reversing the limits of integration ensures that the number of photons is greater than zero. The number $d\varphi$ of photons in the wavelength range from $\lambda + d\lambda$ to λ that can fit in the 3-dimensional unit volume is:

$$d\varphi = 8\pi\lambda^{-4}d\lambda. \quad (17)$$

While Bose interpreted $d\varphi$ as the number $Nd\lambda$ of possible states in the given volume V that 1 quantum of light in the wavelength domain from $\lambda + d\lambda$ to λ could assume, I interpret $d\varphi$ as the number of photons in the wavelength range from $\lambda + d\lambda$ to λ that can fit in the 3-dimensional unit volume.

Because the energy of a photon is a function of its wavelength, the distribution of energy among photons is not equipartitioned among modes, as postulated by classical theory, but is weighted. If it were equipartitioned and the average energy per mode $\langle\varepsilon\rangle$ were equal to kT , then the energy density $ud\lambda$ in the wavelength range between $\lambda + d\lambda$ and λ would be:

$$ud\lambda = \langle\varepsilon\rangle d\varphi = 8\pi\lambda^{-4}kTd\lambda, \quad (18)$$

which is the Rayleigh-Jeans equation based on the classical theory of equipartition. This formula does not fit the observed data for short wavelengths. In fact, it also incorrectly predicts that the energy density would be infinite for short wavelengths. This suggests that the energy in the cavity is not equipartitioned and that the number of short wavelength photons must be limited by another effect. Planck [1] postulated that the number of short-wavelength photons is limited by the fact that the energy of a photon is inversely proportional to its wavelength. This makes it less probable for short-wavelength photons to be created compared with long-wavelength photons. The probability of photon creation is expressed by a weighting factor that is a function of the temperature of the cavity and the wavelength of the photon. By combining my derivation of the temperature-independent prefactor with Planck's temperature-dependent probability distribution, I will show that the major equations [17–20] that describe blackbody radiation can be derived from the assumption that the photon is an extended body in 3-dimensional Euclidean space.

Planck's weighting factor $f(\varepsilon_n)$ is equal to $e^{-\frac{\varepsilon_n}{kT}}$. The average energy per mode is given by the sum of the various weighted energies divided by the sums of the weights:

$$\langle\varepsilon\rangle = \frac{\sum_n \varepsilon_n f(\varepsilon_n)}{\sum_n f(\varepsilon_n)}. \quad (19)$$

Since the energy $\langle\varepsilon\rangle$ of each mode is discrete and equal to $n\frac{hc}{\lambda}$, where n represents the mode number, I sum the discrete energies from $n = 0$ to $n = \infty$ to get:

$$\langle\varepsilon\rangle = \frac{\sum_{n=0}^{\infty} n\frac{hc}{\lambda} e^{-\frac{nhc}{\lambda kT}}}{\sum_{n=0}^{\infty} e^{-\frac{nhc}{\lambda kT}}} = kT \frac{\sum_{n=0}^{\infty} xne^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}}, \quad (20)$$

where $x \equiv \frac{hc}{\lambda kT}$. In order to solve for $\langle\varepsilon\rangle$, let the denominator in Eq. (20), which is the partition function, be denoted by $Z(x) \equiv \sum_{n=0}^{\infty} e^{-nx}$ and expand it as a geometric series:

$$Z(x) = \sum_{n=0}^{\infty} e^{-nx} = 1 + e^{-x} + e^{-2x} + \dots \quad (21)$$

The sum to infinity of the above geometric series, in which the first term is 1 and the common ratio of successive terms is e^{-x} , is:

$$Z(x) = 1 + e^{-x} + e^{-2x} + \dots = 1 \left[\frac{1 - e^{-\infty x}}{1 - e^{-x}} \right] = \frac{1}{1 - e^{-x}}. \quad (22)$$

The numerator of Eq. (20) can be presented as a function of the partition function $Z(x)$:

$$\sum_{n=0}^{\infty} xne^{-nx} = x \sum_{n=0}^{\infty} ne^{-nx} = -x \frac{d}{dx} \sum_{n=0}^{\infty} e^{-nx} = -x \frac{d}{dx} Z(x). \quad (23)$$

Thus, $\langle \varepsilon \rangle$ can be obtained by replacing $Z(x)$ with $\frac{1}{1-e^{-x}}$ and $\frac{d}{dx} Z(x)$ with $-\frac{e^x}{(e^x-1)^2}$:

$$\langle \varepsilon \rangle = kT \frac{\sum_{n=0}^{\infty} xne^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} = \frac{-xkT}{Z(x)} \frac{dZ(x)}{dx} = xkT (1 - e^{-x}) \frac{e^x}{(e^x - 1)^2} = xkT \frac{(e^x - 1)}{(e^x - 1)^2} = \frac{xkT}{e^x - 1} \quad (24)$$

After replacing x with $\frac{hc}{\lambda kT}$, we get:

$$\langle \varepsilon \rangle = \frac{\frac{hc}{\lambda kT} kT}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{\frac{hc}{\lambda}}{e^{\frac{hc}{\lambda kT}} - 1}. \quad (25)$$

By combining Eq. (17), which is based on the temperature-independent extended photon model, and Eq. (25), which is based on the temperature dependence of distributing energy, we can solve for the spectral energy density $ud\lambda$ in the wavelength range between $\lambda + d\lambda$ and λ :

$$ud\lambda = \langle \varepsilon \rangle d\varphi = 8\pi\lambda^{-4} d\lambda \frac{\frac{hc}{\lambda}}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1}, \quad (26)$$

which is Planck's blackbody radiation law. Here it was obtained by assuming that photons have extension in space. When λ is sufficiently short and $\frac{hc}{k} \gg \lambda T$, $e^{\frac{hc}{\lambda kT}}$ becomes large compared with unity and Eq. (26) becomes identical with Wien's approximation for the blackbody radiation law as long as $8\pi hc = c_1$ and $\frac{hc}{k} = c_2$, where c_1 and c_2 are constants in Wien's equation:

$$ud\lambda = \frac{c_1}{\lambda^5} e^{-\frac{c_2}{\lambda T}} d\lambda. \quad (27)$$

Likewise, when λ is sufficiently long and $\frac{hc}{k} \ll \lambda T$, we can expand $e^{\frac{hc}{\lambda kT}}$ in Eq. (26) using a Taylor series and drop terms that are of second order or higher so that $e^{\frac{hc}{\lambda kT}} \approx 1 + \frac{hc}{\lambda kT}$ and $\frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1}$ becomes $\frac{8\pi kT}{\lambda^4} d\lambda$. This gives the Rayleigh–Jeans approximation given in Eq. (18) for the blackbody radiation law.

The assumption that photons have extension in space also gives rise to the related equation for the spectral radiance $R(T)d\lambda$, which is the energy emitted per second per unit surface area of a blackbody in the wavelength range between $\lambda + d\lambda$ and λ . The spectral radiance is related to the energy density in the wavelength range between $\lambda + d\lambda$ and λ according to the following formula: $Rd\lambda = \frac{c}{4\pi} ud\lambda$, thus

$$R(T)d\lambda = \frac{2hc^2}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1}. \quad (28)$$

The assumption that photons have extension in space also gives rise to the related equation for the total energy density $U(T)$ for all wavelengths in a blackbody radiator. Integrating Eq. (26) from $\lambda = \infty$ to $\lambda = 0$ gives the

Stefan–Boltzmann law:

$$U(T) = \int_0^\infty \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda kT}} - 1} = \frac{8\pi k^4 T^4}{c^3 h^3} \int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{8\pi^5 k^4}{15c^3 h^3} T^4 = aT^4, \quad (29)$$

where $x \equiv \frac{hc}{\lambda kT}$, $\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$, and $a = \frac{8\pi^5 k^4}{15c^3 h^3}$. The total energy radiated $R(T)$ from a blackbody per second per unit surface area is related to the total energy density $U(T)$ by the relation: $R(T) = \frac{c}{4}U(T)$. Consequently,

$$R(T) = \frac{c}{4} \frac{8\pi^5 k^4}{15c^3 h^3} T^4 = \frac{2\pi^5 k^4}{15c^2 h^3} T^4 = \sigma T^4,$$

where σ is the Stefan–Boltzmann constant and is equal to $\frac{2\pi^5 k^4}{15c^2 h^3}$ as well as $\frac{ac}{4}$.

We can derive Wien's displacement law from Planck's blackbody radiation law (Eq. (26)) by taking the derivative with respect to wavelength and setting the derivative to zero:

$$\frac{du}{d\lambda} = 8\pi hc \left[\frac{hce^{\frac{hc}{\lambda kT}}}{kT\lambda^7 \left(e^{\frac{hc}{\lambda kT}} - 1\right)^2} - \frac{5}{\lambda^6 \left(e^{\frac{hc}{\lambda kT}} - 1\right)} \right] = 0, \quad (30)$$

which can be simplified to:

$$\left[\frac{hc}{\lambda kT} \frac{e^{\frac{hc}{\lambda kT}}}{e^{\frac{hc}{\lambda kT}} - 1} - 5 \right] = 0. \quad (31)$$

Again, letting $x \equiv \frac{hc}{\lambda kT}$, we get:

$$\left[x \frac{e^x}{e^x - 1} - 5 \right] = 0. \quad (32)$$

The solution to Eq. (32), which is transcendental and must be solved by trial and error, is $x = 4.965$. Thus,

$$4.965 = x = \frac{hc}{\lambda_{peak} kT} \quad (33)$$

and

$$\lambda_{peak} T = \frac{hc}{4.965k} = 2.897 \times 10^{-3} Km, \quad (34)$$

which is Wien's displacement law.

3. Conclusion

By combining my derivation of the temperature-independent prefactor with Planck's temperature-dependent probability distribution, I have shown that the major equations that describe blackbody radiation, also known as the specific heat of the vacuum [21], can be derived from the assumption that the photon is an extended body in 3-dimensional Euclidean space. This provides further support for the conjecture that the quantum of light is neither a geometrical point nor a single or group of infinite plane waves, but composed of particles [22] with energy and momentum whose extension in space depends upon wavelength.

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