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Higher-dimensional Bianchi type-III universe with strange quark matter attached to string cloud in general relativity

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Abstract: In this paper, our intention is to construct 5-dimensional Bianchi type-III cosmological models for quark matter attached to a string cloud in general relativity. Different cases for the metric potentials are considered and studied. The physical and kinematical behaviors of all the models are discussed. It is observed that most of the models admit initial singularity.

Key words: Bianchi type-III, quark matter, string cloud

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1. Introduction

The exact cosmological solutions of Einstein field equations with different equations of state and different symmetries have been found by many authors in 5 dimensions [1–3] and other dimensions [4–7]. The physics of the universe in higher-dimensional space-time have been studied by many authors [8–10].

The possibility that the world may have more than 4 dimensions is due to Kaluza [11] and Klein [12], who used 1 extra dimension to unify gravity and electromagnetism in a theory that was essentially 5-dimensional in general relativity. Sabbatta [13], Lee [14], Appelquist and Chodos [15], and Collins et al. [16] accepted this idea and constructed cosmological models in higher dimensions by using various phenomena of particle physics and cosmology. Overduin and Wesson [17] presented an excellent review of Kaluza–Klein theory and higher-dimensional unified theories, in which the cosmological and astrophysical implications of extra dimensions were studied. Many authors also studied Kaluza–Klein cosmological models with different matters [18–22]. Aliev et al. [23] derived a plane symmetric solution that describes a 3-brane world embedded in a 5-dimensional bulk space-time. Mohanty and Mahanta [24] constructed a 5-dimensional homogeneous anisotropic cosmological model in Barber's second self-creation theory in the presence of perfect fluid. Mohanty and Samanta [25] studied 5-dimensional string cosmological model in the presence and absence of bulk viscosity. Moreover, Mohanty and Mahanta [26] constructed a stiff fluid model in 5-dimensional space-time based on Lyra geometry. There is now extensive literature dealing with different aspects of higher-dimensional cosmology.

In this study, we will examine quark matter attached with cosmic string in the higher-dimensional Bianchi type-III space-time. The possibility of the existence of quark matter dates back to the early 1970s. Itoh [27], Bodmer [28], and Witten [29] proposed 2 ways of formation of quark matter: the quark-hadron phase transition

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in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. In the theories of strong interaction, quark bag models suppose that the breaking of physical vacuum takes place inside hadrons. As a result, vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system.

Typically, strange quark matter is modeled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought of as degenerate Fermi gases, which exist only in a region of space endowed with vacuum energy density B_C (called the bag model). Additionally, in the framework of this model, the quark matter is composed of massless u, d quarks; massive s quarks; and electrons. In the simplified version of this model, on which our study is based, quarks are massless and noninteracting. We then have quark pressure $p_q = \frac{\rho_q}{3}$ (ρ_q is the quark energy density); the total energy density is

$$\rho = \rho_q + B_C, \tag{1.1}$$

while total pressure is

$$p = p_q - B_C. \tag{1.2}$$

Yilmaz et al. [30] studied strange quark matter for the Robertson–Walker model in the context of the general theory of relativity. Yilmaz and Yavuz [31] obtained higher-dimensional Robertson–Walker cosmological models in the presence of quark-gluon plasma in the general theory of relativity. Khadekar et al. [32] confined their work to the quark matter attached to topological defects in general relativity. Adhav et al. [33] obtained a Bianchi type-III cosmological model with strange quark matter attached to string cloud in general relativity. Mahanta et al. [34] investigated Bianchi type-III cosmological models with strange quark matter attached to string cloud in self-creation theory. Khadekar and Wanjari [35] studied the geometry of quark and strange quark matter in higher-dimensional general relativity. Katore and Shaikh [36] investigated cosmological models with strange quark matter attached to the string cloud in the general theory of relativity for axially symmetric space-time. Mahanta and Biswal [37] studied string cloud and domain walls with quark matter in Lyra geometry. Rao and Neelima [38] discussed axially symmetric space-time with strange quark matter attached to the string cloud in self-creation theory and general relativity. Rao and Sireesha [39] discussed axially symmetric space-time with strange quark matter attached to the string cloud in the Brans–Dicke theory of gravitation. Recently, Sahoo and Mishra [40,41] studied axially symmetric and plane symmetric cosmological solutions for quark matter coupled with string cloud and domain walls in bimetric theory.

In this paper, we have studied 5-dimensional Bianchi type-III space-time with strange quark matter coupled with string cloud. In Section 2, the metric and energy momentum tensor are described. In Section 3, the field equations and their solutions are derived. The concluding remarks are given in Section 4.

2. Metric and energy momentum tensor

We consider the 5-dimensional Bianchi type-III metric in the form of

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2ax}dy^2 - C^2dz^2 - D^2du^2, \tag{2.1}$$

where A, B, C, and D are the functions of time t only.

The energy momentum tensor for string cloud [42] is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j. \tag{2.2}$$

Here, ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s, \tag{2.3}$$

where ρ_p is the particle energy density.

We know that string is free to vibrate. The vibration models of the string represent different types of particles because these models are seen as different masses or spins. Therefore, here we consider quarks instead of particles in the string cloud. Moreover, we consider here quark matter energy density instead of particle energy density in the string cloud.

In this case, from Eq. (2.3), we get

$$\rho = \rho_q + \rho_s + B_C. \tag{2.4}$$

From Eqs. (2.3) and (2.4), we have the energy momentum tensor for strange quark matter attached to the string cloud [43] as follows:

$$T_{ij} = (\rho_q + \rho_s + B_C)\, u_i u_j - \rho_s x_i x_j, \tag{2.5}$$

where u_i is the 5 velocity of the particles and x_i is the unit space-like vector representing the direction of string.

We also u_i and x_i with

$$u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0. \tag{2.6}$$

We have taken the direction of the string along the z-axis. The components of the energy momentum tensor are then

$$T_1^1 = T_2^2 = T_4^4 = 0\,,\ T_3^3 = \rho_s\,,\ T_5^5 = \rho, \tag{2.7}$$

where ρ and ρ_s are functions of t only.

3. Field equations and their solutions

Einstein's field equations read as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij}. \tag{3.1}$$

The field equations of Eq. (3.1) for the metric of Eq. (2.1) can be written as follows:

$$\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{D_{55}}{D} + \frac{B_5 C_5}{BC} + \frac{B_5 D_5}{BD} + \frac{C_5 D_5}{CD} = 0, \tag{3.2}$$

$$\frac{A_{55}}{A} + \frac{C_{55}}{C} + \frac{D_{55}}{D} + \frac{A_5 C_5}{AC} + \frac{A_5 D_5}{AD} + \frac{C_5 D_5}{CD} = 0, \tag{3.3}$$

$$\frac{A_{55}}{A} + \frac{B_{55}}{B} + \frac{D_{55}}{D} + \frac{A_5 B_5}{AB} + \frac{A_5 D_5}{AD} + \frac{B_5 D_5}{BD} - \frac{a^2}{A^2} = 8\pi\rho_s, \tag{3.4}$$

$$\frac{A_{55}}{A} + \frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{A_5 B_5}{AB} + \frac{A_5 C_5}{AC} + \frac{B_5 C_5}{BC} - \frac{a^2}{A^2} = 0, \tag{3.5}$$

$$\frac{A_5 B_5}{AB} + \frac{A_5 C_5}{AC} + \frac{B_5 C_5}{BC} + \frac{A_5 D_5}{AD} + \frac{B_5 D_5}{BD} + \frac{C_5 D_5}{CD} - \frac{a^2}{A^2} = 8\pi\rho, \tag{3.6}$$

$$\frac{A_5}{A} - \frac{B_5}{B} = 0, \tag{3.7}$$

where the subscript 5 after A, B, C, and D denotes ordinary differentiation with respect to t.

From Eq. (3.7), we get

$$A = \alpha B.$$

Without loss of generality we take the arbitrary constant $\alpha = 1$, such that we have

$$A = B. \tag{3.8}$$

Using Eq. (3.8), the field equations of Eqs. (3.2) through (3.6) reduce to the following:

$$\frac{B_{55}}{B} + \frac{C_{55}}{C} + \frac{D_{55}}{D} + \frac{B_5 C_5}{BC} + \frac{B_5 D_5}{BD} + \frac{C_5 D_5}{CD} = 0, \tag{3.9}$$

$$2\left(\frac{B_{55}}{B}\right) + \frac{D_{55}}{D} + \left(\frac{B_5}{B}\right)^2 + 2\left(\frac{B_5 D_5}{BD}\right) - \frac{a^2}{B^2} = 8\pi\rho_s, \tag{3.10}$$

$$2\left(\frac{B_{55}}{B}\right) + \frac{C_{55}}{C} + \left(\frac{B_5}{B}\right)^2 + 2\left(\frac{B_5 C_5}{BC}\right) - \frac{a^2}{B^2} = 0, \tag{3.11}$$

$$\left(\frac{B_5}{B}\right)^2 + 2\left(\frac{B_5 C_5}{BC}\right) + 2\left(\frac{B_5 D_5}{BD}\right) + \frac{C_5 D_5}{CD} - \frac{a^2}{B^2} = 8\pi\rho. \tag{3.12}$$

Thus, we have 4 equations with 5 unknowns, B, C, D, ρ, and ρ_s. Since these equations are highly nonlinear in nature, in order to get a deterministic solution we need one assumption. We shall explore physically meaningful solutions of the field equations of Eqs. (3.9) through (3.12) by considering a simplifying assumption of the field variables B, C, and D.

In order to obtain a simple but physically realistic solution, let us choose a simple power-law form of the scale factor [44]:

$$B = t^n, \tag{3.13}$$

where n is an arbitrary constant.

Using Eq. (3.13) in Eq. (3.11), we get

$$\frac{C_{55}}{C} + 2\frac{2nC_5}{tC} + \frac{2n\,(n-1) + n^2}{t^2} - \frac{a^2}{t^{2n}} = 0. \tag{3.14}$$

Eq. (3.14) is solvable for $n = 1$. Hence, Eqs. (3.13) and (3.14) reduce to

$$B = t \tag{3.15}$$

and

$$t^2 C_{55} + 2tC_5 + \left(1 - a^2\right) C = 0. \tag{3.16}$$

Integrating Eq. (3.16) yields

$$C = t^{\frac{-1+\sqrt{4a^2-3}}{2}} \tag{3.17}$$

or

$$C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}. \tag{3.18}$$

Using Eq. (3.17) in Eq. (3.9), we get

$$t^2 D_{55} + t\left(1+\sqrt{4a^2-3}\right) D_5 + \frac{\left(-1+\sqrt{4a^2-3}\right)^2}{4} D = 0, \tag{3.19}$$

which on integration yields

$$D = t^{m_1} \tag{3.20}$$

or

$$D = t^{m_2}, \tag{3.21}$$

where

$$m_1 = \frac{-\sqrt{4a^2-3} + \sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}$$

and

$$m_2 = \frac{-\sqrt{4a^2-3} - \sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}.$$

Again using Eq. (3.18) in Eq. (3.9), we get

$$t^2 D_{55} + t\left(1-\sqrt{4a^2-3}\right) D_5 + \frac{\left(-1-\sqrt{4a^2-3}\right)^2}{4} D = 0, \tag{3.22}$$

which on integration yields

$$D = t^{k_1} \tag{3.23}$$

or

$$D = t^{k_2}, \tag{3.24}$$

where

$$k_1 = \frac{\sqrt{4a^2-3} + \sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}$$

and

$$k_2 = \frac{\sqrt{4a^2-3} - \sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2}}{2}.$$

Now the above solutions give 4 different models.

Case I: When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ and $D = t^{m_1}$

The 5-dimensional string cosmological model corresponding to Eqs. (3.15), (3.17), and (3.20) is written as

$$ds^2 = dt^2 - t^2 dx^2 - t^2 e^{-2ax} dy^2 - t^{-1+\sqrt{4a^2-3}} dz^2 - t^{2m_1} du^2. \tag{3.25}$$

From Eq.(3.10), we get the following rest energy density.

$$\rho = \frac{1}{32\pi t^2}\left[3\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} + \sqrt{4a^2-3} + \sqrt{4a^2-3}\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} - 8a^2 + 3\right] \tag{3.26}$$

From Eq. (3.10), we get the following:

$$\text{string tension density} = \rho_s = \frac{m_1^2 + m_1 + 1 - a^2}{8\pi t^2}, \tag{3.27}$$

$$\text{string particle density} = \rho_p = \rho - \rho_s = \frac{2n_1 + m_1 + n_1 m_1 - m_1^2}{8\pi t^2}, \tag{3.28}$$

where

$$n_1 = \frac{-1 + \sqrt{4a^2 - 3}}{2};$$

$$\begin{aligned} \text{quark energy density} \quad &= \quad \rho_q = \rho - B_C = \frac{1}{32\pi t^2}\Bigg[3\sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2-3}\right)^2} + \sqrt{4a^2-3} \\ &+ \sqrt{4a^2-3}\sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2-3}\right)^2} - 8a^2 + 3\Bigg] - B_C, \end{aligned} \tag{3.29}$$

$$\begin{aligned} \text{quark pressure} \quad &= \quad p_q = \frac{\rho_q}{3} = \frac{1}{96\pi t^2}\Bigg[3\sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2-3}\right)^2} + \sqrt{4a^2-3} \\ &+ \sqrt{4a^2-3}\sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2-3}\right)^2} - 8a^2 + 3\Bigg] - \frac{B_C}{3}. \end{aligned} \tag{3.30}$$

The volume element of the model in Eq. (3.25) is given by

$$V = \sqrt{(-g)} = t^{n_1 + m_1 + 2} e^{-ax}. \tag{3.31}$$

The expression for the scalar expansion θ is given by

$$\theta = u^i_{;i} = \frac{n_1 + m_1 + 2}{t} \tag{3.32}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\left[2\left(\frac{1}{3} - \frac{1}{t}\right)^2 + \left(\frac{1}{3} - \frac{n_1}{t}\right)^2 + \left(\frac{1}{3} - \frac{m_1}{t}\right)^2\right]. \tag{3.33}$$

The deceleration parameter q is given by

$$q = \frac{-\left(1 + \sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2-3}\right)^2}\right)}{3 + \sqrt{4a^2 - 3 - \left(-1 + \sqrt{4a^2-3}\right)^2}}. \tag{3.34}$$

The rest energy density, string tension density, string particle density, quark energy density, quark pressure, expansion scalar, and shear become infinite for $t = 0$, which indicates that the universe starts at $t = 0$. Hence, the model of Eq. (3.25) admits initial singularity. The scalar expansion $\theta \to 0$ as $t \to \infty$. Since $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy for large values of t. The spatial volume V is zero when $t = 0$ and it becomes infinite when $t \to \infty$. The deceleration parameter is negative. Hence, the model of Eq. (3.35) is inflationary.

Case II: When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ and $D = t^{m_2}$

The 5-dimensional string cosmological model corresponding to Eqs. (3.15), (3.17), and (3.21) is written as

$$ds^2 = dt^2 - t^2 dx^2 - t^2 e^{-2ax} dy^2 - t^{-1+\sqrt{4a^2-3}} dz^2 - t^{2m_2} du^2. \tag{3.35}$$

From Eq. (3.12), we get the following rest energy density.

$$\rho = \frac{1}{32\pi t^2}\left[-3\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} + \sqrt{4a^2-3} - \sqrt{4a^2-3}\sqrt{4a^2-3-\left(-1+\sqrt{4a^2-3}\right)^2} - 8a^2 - 3\right] \tag{3.36}$$

From the above equation, it is seen that the rest energy density does not satisfy the reality conditions, i.e. $(\rho > 0)$. Hence. the model of Eq. (3.35) is physically unrealistic and not physically acceptable.

Case III: When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ and $D = t^{k_1}$

The metric of Eq. (2.1) corresponding to Eqs. (3.15), (3.18), and (3.23) is written as

$$ds^2 = dt^2 - t^2 dx^2 - t^2 e^{-2ax} dy^2 - t^{-1-\sqrt{4a^2-3}} dz^2 - t^{2k_1} du^2. \tag{3.37}$$

The physical and kinematical quantities for the model of Eq. (3.37) have the following expressions:

$$\begin{aligned}\text{rest energy density} \quad = \quad \rho = \frac{1}{32\pi t^2}\Bigg[&3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} \\ &-\sqrt{4a^2-3} - \sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - 8a^2 + 3\Bigg],\end{aligned} \tag{3.38}$$

$$\text{string tension density} = \rho_s = \frac{k_1^2 + k_1 + 1 - a^2}{8\pi t^2}, \tag{3.39}$$

$$\text{string particle density} = \rho_p = \frac{2n_2 + k_1 + n_2 k_1 - k_1^2}{8\pi t^2}, \tag{3.40}$$

where

$$n_2 = \frac{1+\sqrt{4a^2-3}}{2};$$

$$\begin{aligned}\text{quark energy density} \quad = \quad \rho_q = \frac{1}{32\pi t^2}\Bigg[&3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - \sqrt{4a^2-3} \\ &-\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2} - 8a^2 + 3\Bigg] - B_C,\end{aligned} \tag{3.41}$$

$$\text{quark pressure} = p_q \quad = \quad \frac{1}{96\pi t^2}\left[3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-\sqrt{4a^2-3}\right.$$
$$\left.-\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-8a^2+3\right]-\frac{B_C}{3}, \qquad (3.42)$$

$$\text{volume} = V = t^{n_2+k_1+2}e^{-ax}, \qquad (3.43)$$

$$\text{scalar expansion} = \theta = \frac{n_2+k_1+2}{t}, \qquad (3.44)$$

$$\text{shear} = \sigma^2 = \frac{1}{2}\left[2\left(\frac{1}{3}-\frac{1}{t}\right)^2+\left(\frac{1}{3}-\frac{n_2}{t}\right)^2+\left(\frac{1}{3}-\frac{k_1}{t}\right)^2\right], \qquad (3.45)$$

$$\text{deceleration parameter} = q = \frac{-\left(1+\sqrt{4a^2-3-(1+\sqrt{4a^2-3})^2}\right)}{3+\sqrt{4a^2-3-(1+\sqrt{4a^2-3})^2}}. \qquad (3.46)$$

The rest energy density, string tension density, string particle density, quark energy density, quark pressure, expansion scalar, and shear become infinite for $t = 0$, which indicates that the universe starts at $t = 0$. Hence, the model of Eq. (3.37) admits initial singularity. The scalar expansion $\theta \to 0$ as $t \to \infty$. Since $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, the model does not approach isotropy for large values of t. The spatial volume V is zero when $t = 0$ and it becomes infinite when $t \to \infty$. The deceleration parameter is negative. Hence, the model of Eq. (3.37) is inflationary.

Case IV: When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ and $D = t^{k_2}$

The metric of Eq. (2.1) corresponding to Eqs. (3.15), (3.18), and (3.24) is written as

$$ds^2 = dt^2 - t^2dx^2 - t^2e^{-2ax}dy^2 - t^{-1-\sqrt{4a^2-3}}dz^2 - t^{2k_2}du^2. \qquad (3.47)$$

The physical and kinematical parameters for the model of Eq. (3.47) are obtained as follows:

$$\text{rest energy density} \quad = \quad \rho = \frac{1}{32\pi t^2}\left[-3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-\sqrt{4a^2-3}\right.$$
$$\left.+\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-8a^2+3\right], \qquad (3.48)$$

$$\text{string tension density} = \rho_s = \frac{k_2^2+k_2+1-a^2}{8\pi t^2}, \qquad (3.49)$$

$$\text{string particle density} = \rho_p = \frac{2n_2+k_2+n_2k_2-k_2^2}{8\pi t^2}, \qquad (3.50)$$

$$\text{quark density} \quad = \quad \rho_q = \frac{1}{32\pi t^2}\left[-3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-\sqrt{4a^2-3}\right.$$
$$\left.+\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-8a^2+3\right]-B_C, \qquad (3.51)$$

$$\text{quark pressure} \quad = \quad p_q = \frac{1}{96\pi t^2}\left[-3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-\sqrt{4a^2-3}\right.$$
$$\left.+\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}-8a^2+3\right]-\frac{B_C}{3}, \tag{3.52}$$

$$\text{volume} = V = t^{n_2+k_2+2}e^{-ax}, \tag{3.53}$$

$$\text{scalar expansion} = \theta = \frac{n_2+k_2+2}{t}, \tag{3.54}$$

$$\text{shear} = \sigma^2 = \frac{1}{2}\left[2\left(\frac{1}{3}-\frac{1}{t}\right)^2+\left(\frac{1}{3}-\frac{n_2}{t}\right)^2+\left(\frac{1}{3}-\frac{k_2}{t}\right)^2\right], \tag{3.55}$$

$$\text{deceleration parameter} = q = \frac{-\left(1+\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}\right)}{3+\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}}. \tag{3.56}$$

The reality condition $(\rho > 0)$ is satisfied for

$$3\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}+\sqrt{4a^2-3}+8a^2<\sqrt{4a^2-3}\sqrt{4a^2-3-\left(1+\sqrt{4a^2-3}\right)^2}+3.$$

The rest energy density, string tension density, string particle density, quark energy density, quark pressure, expansion scalar, and shear become infinite for $t = 0$. It is observed that the model of Eq. (3.47) admits initial singularity.

4. Conclusion

We have constructed 5-dimensional Bianchi type-III cosmological models with strange quark matter attached to the string cloud in general relativity. The following results were obtained for different cases:

(i) When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ and $D = t^{m_1}$, the model of Eq. (3.25) admits initial singularity and does not approach isotropy for large values of t. The deceleration parameter is negative and hence the model is inflationary.

(ii) When $C = t^{\frac{-1+\sqrt{4a^2-3}}{2}}$ and $D = t^{m_2}$, the rest energy density does not satisfy the reality conditions and hence the model of Eq. (3.35) leads to unphysical situations.

(iii) When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ and $D = t^{k_1}$, the universe starts at $t = 0$ and the model does not approach isotropy for large values of t.

(iv) When $C = t^{\frac{-1-\sqrt{4a^2-3}}{2}}$ and $D = t^{k_2}$, all the physical parameters become infinite for $t = 0$. Hence, it is concluded that the model of Eq. (3.47) admits initial singularity.

In summary, it is observed that of the 4 models discussed here, 3 are inflationary and possess initial singularity while 1 is physically unrealistic.

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