

Survey of muon stripping in the muon-catalyzed $d - d$ fusion cycle in pure deuterium

Seyed Mohammad MOTEVALLI*, Maedeh MOHAMMADPOURFARD
Department of Physics, Faculty of Sciences, University of Mazandaran, Babolsar, Iran

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Abstract: Muon-catalyzed fusion is a very efficient nuclear fusion method to which a wide variety of experimental and theoretical studies have been devoted in recent years. The mixtures of hydrogen isotopes are the most appropriate particles to do this kind of fusion. The efficiency of muon-catalyzed fusion in mixtures of pure deuterium is due to the abundance of this isotope as compared to other hydrogen isotopes. The most important issue of reducing the efficiency of the muon-catalyzed fusion chain in pure deuterium mixtures is the probability of muons sticking to the helium-3 particle produced after the $d - d$ nuclear fusion reaction. In this investigation, the density dependence of the slowing down, muon stripping, and effective sticking in pure deuterium fuel are presented by considering all possible effective processes and solving a set of coupled differential equations. The results show that our calculated values are in agreement with available experimental and theoretical data.

Key words: Fusion, muon stripping, effective sticking, pure deuterium, slowing down

1. Introduction

The catalysis of nuclear fusion reactions by negative muons in a mixture of hydrogen isotopes is a considerable physical phenomenon known as muon-catalyzed fusion (μ CF) [1–6]. The injection of negative muons into a mixture of hydrogen isotopes creates repetitive cycles of processes that are nuclear fusion and the release of energy. So far, many studies have been carried out on μ CF [7–16]. The μ CF allows nuclear fusion to take place at temperatures significantly lower than the temperatures required for thermonuclear fusion. Hydrogen isotopes are the most appropriate particles for these processes, because they have only 1 charge and require a lower thermal energy to get closer to one another; in other words, the fusion of hydrogen isotopes occurs at low temperatures. The existence of negative muons plays a key role in this cycle.

Muons are unstable subatomic particles with lifetime $\tau_\mu = 2.197 \times 10^{-6} s$ created by pion scattering, $\pi^- \rightarrow \mu^- + \bar{\nu}$. The muon mass is about 206.77 times larger than the mass of electron, Therefore, the size of a muonic hydrogen atom is smaller than that of an electronic hydrogen atom by the same rate approximately.

After the injection of a muon into surroundings containing pure deuterium, the muon is captured by particles in the environment and $d\mu$ muonic atoms are formed [7]:



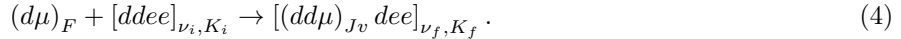
In this process, the muon leaves its high-energy initial state with principle quantum number $n \approx \sqrt{m_\mu/m_e} \cong 14$. The subsequent deexcitation of the $d\mu$ muonic atom occurs via Stark, Auger, scattering, radiative, and transfer

*Correspondence: motavali@umz.ac.ir

processes, which occur on the 100 ps time scale [17,18]. The energy difference between the two hyperfine states of the $d\mu$ muonic atom ($F = 3/2, 1/2$) is 48.5 MeV [19]. After fast thermalization, hyperfine transitions take place by inelastic scattering processes with rates $\lambda_{d\mu}^{FF'}$ [19,20]. In general, λ is given by

$$\lambda = \rho \sigma \nu \text{ (s}^{-1}\text{)}, \quad (2)$$

where $\rho = \varphi \rho_0 \text{ (cm}^{-3}\text{)}$ is the density of the surrounding environment, $\sigma \text{ (cm}^{-2}\text{)}$ is the cross-section describing the process, $\lambda = \rho \sigma \nu \text{ (sec}^{-1}\text{)}$ is the relative velocity, and ϕ is the deuterium density in units of liquid hydrogen density ($\rho_0 = LHD = 4.25 \times 10^{22} \text{cm}^{-3}$). The formation of muonic molecules of hydrogen isotopes and their nuclear reactions have been the subject of many experimental and theoretical studies [7–12]. One of these important studies involves the reaction of $d\mu$ muonic atoms with D_2 molecules. Tritium is of particular interest because of the abundance of deuterium relative to other hydrogen isotopes. Furthermore, nuclear fusion in channel $d-d$, unlike fusion in channel $d-t$, does not cause problems of provision of tritium in the primary fuel. The $dd\mu$ muonic molecules can be formed either via the nonresonant Auger process (rate $\lambda_{dd\mu}^{non-res}$), where the energy released under $dd\mu$ -formation is transferred to the conversion electron, or via the Vesman resonance mechanism (rates $\lambda_{dd\mu}^{FS}$, F , and S being the total spin of the initial muonic atom and the final muonic molecule, respectively) [17,21]:

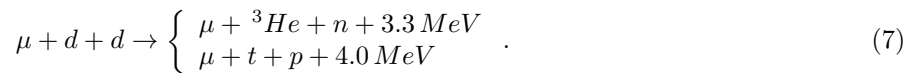


Here, J and ν are the orbital angular momentum and vibrational quantum numbers, respectively, while, $\nu_i K_i$ and $\nu_f K_f$ are the vibrational and rotational quantum numbers of the D_2 molecule and the mesomolecular complex, respectively. The $(J, \nu) = (1, 1)$ muonic molecule produced in the $[(dd\mu)de]^*$ complex by resonant formation has an extraordinarily long lifetime. For this state, back-decay rates (for the $(J, \nu) = (1, 0)$ state) are faster than the fusion rate (for the $(J, \nu) = (1, 1)$ state) and the very slow Auger rate $\lambda_e \approx 0.03 \times 10^9 \text{ s}^{-1}$ [22], and only about one-fourth of the $dd\mu$ muonic molecules formed actually undergo fusion. The effective $dd\mu$ formation rate $\tilde{\lambda}_{dd\mu}^F$ and the hyperfine transition rate $\tilde{\lambda}_{d\mu}^{FF'}$ are expressed as the sum of nonresonant and resonant terms:

$$\tilde{\lambda}_{dd\mu}^F = \lambda_{dd\mu}^{non-res} + \left(\tilde{\lambda}_{dd\mu}^F\right)^{res} = \lambda_{dd\mu}^{non-res} + \sum_S \lambda_{dd\mu}^{FS} \frac{\tilde{\lambda}_f}{\tilde{\lambda}_f + \sum_{F''} \Gamma_{SF''}}, \quad (5)$$

$$\tilde{\lambda}_{d\mu}^{FF'} = \lambda_{d\mu}^{FF'} + \sum_S \lambda_{dd\mu}^{FS} \frac{\Gamma_{SF'}}{\tilde{\lambda}_f + \sum_{F''} \Gamma_{SF''}} \quad F = 3/2, \quad F' = 1/2, \quad (6)$$

where $\tilde{\lambda}_f$ is the sum of the fusion rate (λ_f) and the rate for Auger transitions (λ_e), and Γ_{SF} are the back-decay rates for $(dd\mu)_S \rightarrow (d\mu)_F + d$ [23]. The $d-d$ nuclear fusion reaction takes place in about 10^{-9} s [24], which is much shorter than the muon lifetime. There are two nuclear fusion channels:

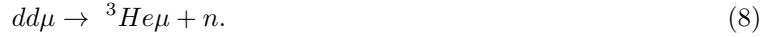


This chain reaction is repeated until the muon gets out of the cycle by absorbing or scattering. The $d-d$ nuclear fusion rate λ_f depends on the quantum numbers of the $dd\mu$ state in which fusion occurs. In particular,

theory predicts $\lambda_f \approx 0.44 \times 10^9 s^{-1}$ [25] for the ($J = 1, \nu = 1$) state and $\lambda_f \approx 1.5 \times 10^9 s^{-1}$ [23] for the ($J = 1, \nu = 0$) state.

2. Theoretical formalism

One of the important factors in limiting the muon in the μCF cycling is the muon sticking on the helium-3 particles produced in the reaction $dd\mu \rightarrow {}^3He + n + \mu$. This process is the main loss mechanism in muon-catalyzed fusion. The formation of muonic ions (${}^3He\mu$) upon the muon's sticking to 3He is called initial sticking:

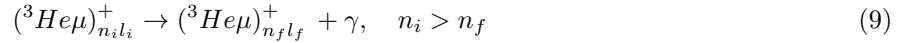


The initial sticking probability depends only on intramolecular dynamics, ($\omega_d^0 = 13.5\%$). The initial kinetic energy of the muonic helium-3 ion is 0.79 MeV ($v = 3.20 a.u.$, $1 a.u = 2.1877 \times 10^8 cm/s$) [26–28].

The effective sticking probability is defined as $\omega_d^{eff} = \omega_d^0(1 - R)$, where R is the reactivation coefficient of the muon for the ${}^3He\mu$ ion and ω_d^0 is the initial sticking probability. The muon reactivation probability of the ${}^3He\mu$ ion depends on stopping power and several important cross-sections. The kinetics of muon stripping is described by the various rates in one set of coupled differential equations.

Different types of interactions take place between muonic ions and the electrons apart from the dominant electron contribution processes such as radiation, ionization, charge transfer, Auger deexcitation, Stark mixing, Coulomb excitation, and Coulomb deexcitation. The probabilities of all these effects are used in writing the stripping dynamical equation. In this paper, the following basic processes have been studied:

Radiative:



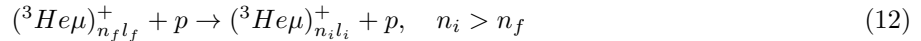
Ionization:



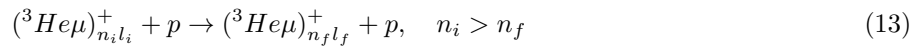
Charge transfer:



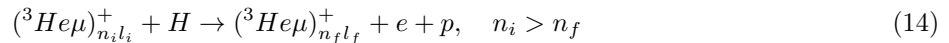
Coulomb excitation:



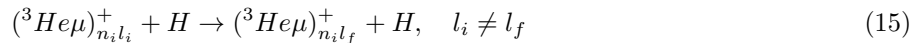
Coulomb deexcitation:



Auger deexcitation:



Stark mixing:



Experiments on muonic systems are very difficult to perform due to the muon's short lifetime. Therefore, one possibility is to do experiments on electronic systems and then scale them to muonic systems. If the rate for

any of the above-mentioned processes is $\Lambda(v(t)) = \rho v \sigma(v)$, the muon stripping probabilities of the ${}^3\text{He}\mu$ ion in terms of population probabilities, which is a time-dependent quantity, can be written as

$$\frac{dP_{strip}(t)}{dt} = \sum_i \Lambda_{strip}^i(v(t)) P_i(t), \quad (16)$$

$$\Lambda_{strip}^i(v(t)) = \Lambda_{charge}^i + \Lambda_{ion}^i, \quad (17)$$

where $\Lambda_{strip}^i(v(t))$ are velocity-dependent stripping rates in the individual energy levels and Λ_{charge}^i and Λ_{ion}^i are charge transfer and ionization rates in the i states, respectively [7]. $P_i(t)$ are the time-dependent population probabilities for the state i of the muonic helium-3 ion and are determined by

$$\frac{dP_i(t)}{dt} = \Lambda_{pop}^i P_{i'}(t) - \Lambda_{de-pop}^i P_i(t), \quad (18)$$

where

$$\Lambda_{pop}^i = \sum_{i'(n_{i'}=n_i)} \Lambda_{St}^{i' \rightarrow i} + \sum_{i'(n_{i'} < n_i)} \Lambda_{ex}^{i' \rightarrow i} + \sum_{i'(n_{i'} > n_i)} \left\{ \Lambda_{ra}^{i' \rightarrow i} + \Lambda_{Au}^{i' \rightarrow i} + \Lambda_{de-ex}^{i' \rightarrow i} \right\} \quad (19)$$

is the rate of populating of state i , and

$$\Lambda_{de-pop}^i = \sum_{i'(n_{i'}=n_i)} \Lambda_{St}^{i \rightarrow i'} + \sum_{i'(n_{i'} > n_i)} \Lambda_{ex}^{i \rightarrow i'} + \sum_{i'(n_{i'} < n_i)} \left\{ \Lambda_{ra}^{i \rightarrow i'} + \Lambda_{Au}^{i \rightarrow i'} + \Lambda_{de-ex}^{i \rightarrow i'} \right\} + \Lambda_{strip}^i \quad (20)$$

is the depopulating probability of state i , where Λ_{Au} , Λ_{ra} , Λ_{de-ex} , Λ_{ex} , Λ_{St} , and Λ_{strip} are the Auger deexcitation, radiative process, Coulomb deexcitation, Coulomb excitation, Stark mixing, and stripping rates, respectively, and n_i refers to the principal quantum number of state i . The energy dependences of cross-sections were obtained in previous studies [4,6,29–31]. The dependence of velocity on time is calculated using

$$\frac{dE_{3\text{He}\mu}}{dt} = -\rho v_{3\text{He}\mu} S(v_{3\text{He}\mu}), \quad (21)$$

where $S(v_{3\text{He}\mu})$ is stopping power of the target. Therefore, the slowing down time is:

$$t_{stop} = \int_{E_{in}}^0 \rho v_{3\text{He}\mu} S(v_{3\text{He}\mu}) dE_{3\text{He}\mu}. \quad (22)$$

Clearly, slowing down will depend on the deuterium density of the target [6]. The stopping power depends only on the charge and velocity (not on the mass) of the muonic helium-3 ion. The stopping power is the same and does not depend on the type of mixture [32]. The stopping power is given by:

$$S = \frac{dE}{dx} = 4\pi m \phi Z (r_0 z \frac{c^2}{v})^2 \left\{ \ln\left(\frac{2mc^2 v^2}{I}\right) + \ln\left(\frac{1}{c^2 - v^2}\right) - \left(\frac{v}{c}\right)^2 \right\}, \quad (23)$$

where r_0 is the classical electron radius, v is the ion velocity in atomic units, c is the speed of light in vacuum, z is the charge of the incident particle, ϕ is the density of target, Z is the atomic number of the material, and

I is the mean average excitation potential of the media. This formula predicts the energy loss accurately at energies larger than 1 MeV per atomic mass unit (a.m.u.). The initial conditions are:

$$E_{3He\mu}(0) = E_{3He\mu}^{in} = 0.79 \text{ MeV}, \quad (24)$$

$$P_{strip}(0) = 0. \quad (25)$$

The initial values of populated levels are determined by the initial sticking, $P_i(0) = \omega_d^0(i)/\omega_d^0$. The reactivation probability is equivalent to stripping fraction at $t \rightarrow \infty$.

The velocity dependence of the stopping power of the ${}^3He\mu$ ion is shown in different densities of pure deuterium environment in Figure 1. As shown in this figure, the stopping power of muonic helium-3 ion increases slowly with decreasing velocity to reach a maximum and then decreases rapidly at a specific density. The stopping power increases with increasing density. The time dependences of population probabilities, $P_i(t)$ for $1s$, $2s$, $2p$, $3s$, $3p$, and $3d$ states, are shown in Figure 2 for pure deuterium mixture at density 1.2 LHD.

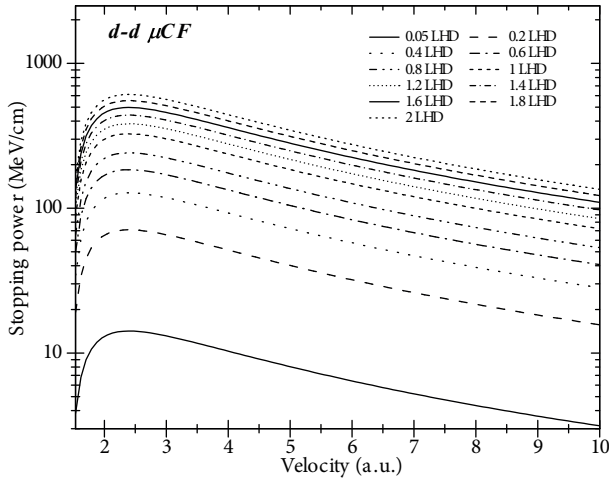


Figure 1. The velocity dependence of stopping power of the ${}^3He\mu$ ion at different deuterium densities presented in the inset for the $d-d$ μCF .

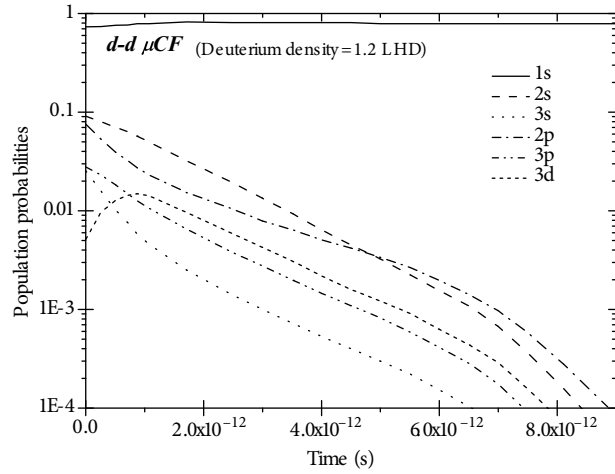


Figure 2. Time-dependent fractional populations of the ${}^3He\mu$ ion at deuterium density $\phi = 1.2$ LHD for the $d-d$ μCF .

Calculations of reactivation probability, effective sticking coefficient, and initial sticking coefficient of the ${}^3He\mu$ ion as a function of density for a pure deuterium environment are shown in Figure 3. As is clear from Figure 3, when density increases reactivation probability increases, and then effective sticking decreases slowly.

The muon reactivation probability and effective sticking probability in pure deuterium environment at density 1.2 LHD and 0.05 LHD are summarized in the Table, where we compare our calculated results with available theoretical and experimental data.

3. Conclusions

The sticking of muons to helium-3 particles after $d-d$ fusion in a pure deuterium mixture is an unwanted process and eliminates muons from the chain of nuclear fusion reactions. The stopping power of the ${}^3He\mu$ ion as a function of velocity has been calculated in different densities of a pure deuterium environment. In this paper, we consider all possible effective processes that separate the muon from the ${}^3He\mu$ ion, namely Auger deexcitation, Stark mixing, radiative process, Coulomb excitation, Coulomb deexcitation, charge transfer, and ionization.

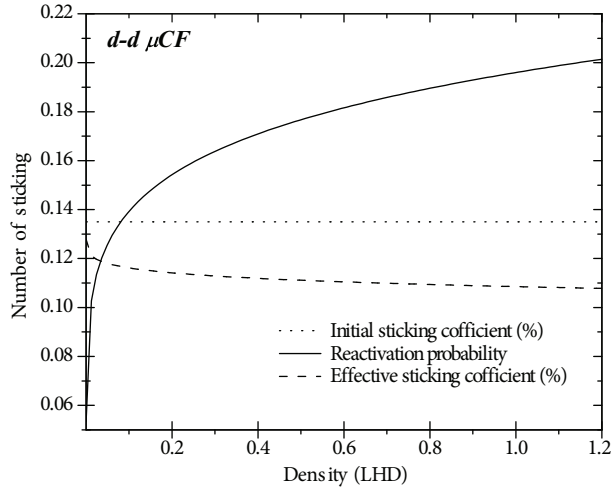


Figure 3. Reactivation probability, effective sticking coefficient (%), and initial sticking coefficient (%) as a function of density in pure deuterium for the $d-d \mu CF$.

Table. Theoretical and experimental values of the reactivation probability and the effective sticking coefficient for the ${}^3He\mu$ ion, at $\phi = 1.2$ LHD and $\phi = 0.05$ LHD in pure deuterium.

Experiment Ref. [24]	Theory				Present theory	Density (LHD)	
	Ref. [36]	Ref. [35]	Ref. [34]	Ref. [33]			
—	0.17	0.207	—	0.14	0.203	1.2	Reactivation probability
—	—	0.127	—	—	0.124	0.05	
—	0.112	0.107	0.100	0.113	0.1075	1.2	Effective sticking coefficient
0.122 ± 0.003	—	0.118	—	—	0.1182	0.05	

Our results show that the stopping power increases slowly with decreasing velocity to reach a maximum and then decreases rapidly at a specific density. The density dependence of muon reactivation probability, effective sticking coefficient, and initial sticking coefficient in a pure deuterium environment show that our calculations are in good agreement with the available experimental and theoretical data.

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