TÜBİTAK

Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2015) 39: 178 – 186
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doi:10.3906/fiz-1412-3

Description of the yrast superdeformed bands in even-even nuclei in A $\sim$ 190 region using the nuclear softness model

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Received: 13.12.2014 • **Accepted/Published Online:** 09.04.2015 • **Printed:** 30.07.2015

Abstract: A two-parameter nuclear softness (NS2) model for the transition energies is proposed by treating the variation of the moment of inertia with spin. The model has been applied to the yrast superdeformed (SD) bands in even-even nuclei in the mass region A $\sim$ 190. The model parameters have been determined from the fitting procedure in order to minimize the relative root mean square deviation between the experimental E2 transition γ-ray energies and the calculated ones. The basic experimental data on SD bands are given in the form of a series of γ-ray transitional energies and the spins were determined in our previous works. The excellent agreement between the theory and the experiment give good support to the model. The presence of identical bands between the two isotopes ^{192}Hg and ^{194}Pb is investigated because their differences in γ-ray energies are small and constant up to rotational frequency $\hbar\omega \sim 0.25$ MeV. Our analysis also leads to the appearance of ΔI = 2 staggering effect in ^{194}Hg (SD1). A comment on equilibrium deformation for each nucleus is also given.

Key words: Softness model, moment of inertia, superdeformed nuclei, staggering

1. Introduction

Over the past decade, the study of superdeformation has been one of the most important and exciting topics in nuclear spectroscopy. Since the discovery of the first discrete superdeformed (SD) rotational band in the nucleus ^{152}Dy [1] in 1986, high-spin SD states were recognized in mass regions A = 40, 60, 80, 110, 130, 150, 190, and 240 [2,3]. Superdeformation in the mass 190 region was first observed in ^{191}Hg [4], and since then more 85 SD bands were reported in the mass A $\sim$ 190 region. Most SD bands of this region exhibit the same smooth increase of the dynamical moment of inertia with rotational frequency due to the gradual alignment of quasiparticles occupying specific high-N intruder orbitals (namely $j_{15/2}$ neutrons and $i_{13/2}$ protons) in the presence of pair correlations.

Because of the nonobservation of the discrete linking transitions between the SD states and the low-lying states of normal deformation, the spins, parity, and excitation energies have not been determined until now. Several approaches to assign the spins were proposed [5–9]. In this paper we will use the values of bandhead spins of our selected SD bands from our previous works [10–12]. Consequently, in the Hg-Pb mass region the lowest bandhead spins in SD bands are as low as about 10 $\hbar$. The observation of the SD band at low spins forced many researchers to study the structures of these well-deformed shape isomers.

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The ^{194}Pb nucleus is the first lead isotope where superdeformation was found experimentally [13,14]. The bandhead spin in this nucleus is $I_0 = 6\ \hbar$ [15] and its γ-ray energies are very close to the energies of the ^{192}Hg SD band [16]. Calculations using the cranked Nilsson–Strutinsky method [17] and the Hartree–Fock method [18] suggested that nuclei with N = 112 and Z = 80 or 82 should be particularly stable at these large deformations due to the presence of large shell gaps at deformation corresponding to a 1.6:1 axis ratio. As a result, ^{192}Hg and ^{194}Pb may be considered as doubly magic SD nuclei.

One of the unexpected features of the SD bands is the existence of identical bands (IBs) [19,20], that is, nearly identical γ-ray transition energies E_γ. It is found that several SD bands in the A $\sim$ 190 region have differences in E_γ of only 1–3 KeV. For the underlying physics of the IBs, some studies [8,9,21] showed that there is special physics on symmetry behind IBs, while others [22,23] suggested that the same E_γ and identical moments of inertia are due to the competition among the stretching effect, pairing interaction, blocking effect, rotation alignment, and Coriolis antipairing effect.

The ΔI = 2 staggering was also observed in some SD bands [24–28]. It manifests itself in systematic shifts of the energy levels, which are alternately pushed down and up with respect to a purely rotational sequence. It was suggested that their origin could be associated with the presence of C_4 symmetry [24,25]. To date, some models have been proposed to explain the experimental results [9,28]. The purpose of the present paper is to study some properties of the yrast SD bands in the A $\sim$ 190 mass region in terms of the nuclear softness model. The paper is organized as follows: in Section 2 we describe the formalism of the nuclear softness approach briefly. Sections 3, 4, and 5 deal the IBs and the staggering and electric quadrupole transition probabilities, respectively. In Section 6, the calculation results and some discussions are presented. Finally, a conclusion is given in Section 7.

2. Soft rotor formula

The energy expression for a rigid rotator is given by:

$$E\left(I\right) = \frac{\hbar^2}{2J} I(I+1), \tag{1}$$

where J is the variable moment of inertia. We can write J in terms of the softness parameters σ to the first order as:

$$J = \theta\left(1 + \sigma I\right), \tag{2}$$

where θ is a proportional constant and the bandhead moment of inertia is J_0 when I = I_0, the bandhead spin.

Substituting from Eq. (2) into the energy expression of Eq. (1) yields:

$$E\left(I\right) = A\frac{I\left(I+1\right)}{1+\sigma I}, \tag{3}$$

with

$$A = \frac{\hbar^2}{2\theta}. \tag{4}$$

Eq. (3) contains only two parameters, A and σ. Eq. (3) is denoted by nuclear softness NS2 or the soft rotor formula [29]. For excited states, Eq. (3) is written as:

$$E\left(I\right) = A\frac{I\left(I+1\right)}{1+\sigma I} + E_{bh}, \tag{5}$$

where E_{bh} denotes the bandhead energy.

The transition energies take the following formulae:

$$E_\gamma\left(I_0+4\rightarrow I_0\right)=E\left(I_0+4\right)-E\left(I_0\right)=A\left[\frac{\left(I_0+4\right)\left(I_0+5\right)}{1+\sigma(I_0+4)}-\frac{I_0(I_0+1)}{1+\sigma I_0}\right] \tag{6}$$

$$E_\gamma\left(I_0+2\rightarrow I_0\right)=E\left(I_0+2\right)-E\left(I_0\right)=A\left[\frac{\left(I_0+2\right)\left(I_0+3\right)}{1+\sigma(I_0+2)}-\frac{I_0(I_0+1)}{1+\sigma I_0}\right] \tag{7}$$

Now we define the transitional energy ratio λ as

$$\lambda=\frac{E_\gamma(I_0+4\rightarrow I_0)}{E_\gamma(I_0+2\rightarrow I_0)}. \tag{8}$$

This leads to a quadratic equation in σ:

$$\begin{aligned}&\left[I_0\left(I_0+2\right)\left(I_0+4\right)\lambda-2I_0\left(I_0+2\right)\left(I_0+4\right)\right]\sigma^2+\left[\left(3I_0^2+13I_0+12\right)\lambda-2\left(3I_0^2+13I_0+10\right)\right]\sigma\\&+\left[\left(2I_0+3\right)\lambda-2(2I_0+5)\right]=0.\end{aligned} \tag{9}$$

The positive root of Eq. (9) represents the softness σ, and, when substituting in Eq. (7), we can determine the other parameter, A. Therefore, the two parameters σ and A of the NS2 model energies are then calculated.

3. Identical transition energies

The discovery of the phenomenon of IBs [19] awoke considerable interest. It was found that several SD bands were identical to other bands. That is, the γ-transition energies of the two bands are identical to within ± 3 KeV. Their IBs have identical supershell structures, but generally different alignment configurations. The observed range of frequencies of the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are completely determined by the supershell structure and the high –j configuration. Hence, the moments of inertia are nearly identical. There are more examples of IBs in the A $\sim$ 190 region occurring in pairs separated by two mass units and assuming ^{192}Hg as a doubly magic core. Usually, the difference between transition energies ΔE_γ for the identical pair of SD bands is plotted versus the transition energy E_γ, which shows the degree of similarity between transition energies in the pair. Another way of relating the energies of different bands to those of reference ^{192}Hg is to use incremental alignment.

4. Test of a $\Delta I=2$ staggering

Another surprising feature in SD nuclear bands is the ΔI = 2 staggering. Sequences of states differing by four units of angular momentum are displaced relative to each other. A few theoretical proposals for the possible explanation of this ΔI = 4 bifurcation were made [9,24,25,28].The deviation of the γ-ray transition energies from the rigid rotor behavior can be measured by the staggering quantity [26]:

$$\Delta^4 E_\gamma\left(I\right)=\frac{1}{16}\left[6E_\gamma\left(I\right)-4E_\gamma\left(I-2\right)-4E_\gamma\left(I+2\right)+E_\gamma\left(I-4\right)+E_\gamma(I+4)\right]. \tag{10}$$

It represents the finite difference approximation to the fourth derivative of the transition energies with respect to the spin in a ΔI = 2 band. The staggering quantity $\Delta^4 E_\gamma$ (I) contains five consecutive E_γ values and is called the 5-point formula. We define the staggering $S^{(4)}$ as the difference between the experimental transition energies and an axillary reference as

$$S^{(4)} = 2^4 \left[\Delta^4 E_\gamma^{exp}(I) - \Delta^4 E_\gamma^{cal}(I)\right]. \tag{11}$$

5. Quadrupole moment and deformation

A number of high-precision measurements are now available for charge quadrupole moments of yrast SD bands [30], which provide opportunities to further challenge our predictions. In the axial rotor model [31], where the configuration is described by the Nilsson diagram, the transition electric quadrupole moment Q_t is derived from the reduced transition probability B (E2) using the following formula:

$$B(E2, \Delta I = 2) = \frac{5}{16\pi} Q_t^2 \left|\langle IK20 \mid (I-2)K\rangle\right|^2 e^2 fm^2. \tag{12}$$

Q_t is related to the deformation parameter β_2 by the following relation:

$$\begin{aligned} Q_t &= \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 \left(1 + \frac{2}{7}\sqrt{\frac{5}{\pi}}\beta_2\right) 10^{-2} eb \\ &= 0.757 Z R^2 \beta_2 (1 + 0.36\beta_2) 10^{-2} eb \\ &= 1.09 Z A^{2/3} \beta_2 (1 + 0.36\beta_2) 10^{-2} eb, \end{aligned} \tag{13}$$

where R = 1.2 $A^{1/3}$ fm, A is the nucleon number, and Z is the proton number.

The bandhead moment of inertia J_0 is related to the quadrupole deformation β_2 by the Grodzins formula [32]:

$$J_0 = C(Z) A^{5/3} \beta_2^2, \tag{14}$$

where C (Z) describes the calibration of the relationship between J_o and β_2, which remains constant for each isotopic chain and varies smoothly with Z.

Table 1. The calculated values of the model parameters λ, σ, and A employed in the calculations for eight yrast SD bands in even-even nuclei in the A $\sim$ 190 region. I_0 is the bandhead spin value obtained from our previous works [10–12]. For each band transition from $I_0 + 2$ to I_0 is included. J_0 and β_2 denote the bandhead moment of inertia and the quadrupole deformation, respectively.

SD Band	I_0 ($\hbar$)	E_γ ($I_0 + 2 \rightarrow I_0$) (KeV)	λ	σ	A (KeV)	Θ ($\hbar^2$ MeV^{-1})	J_0 ($\hbar^2$ MeV^{-1})	C(Z)	B_2
^{190}Hg (SD1)	12	316.9	2.1360	3.7710×10^3	6.3098	79.2418	82.8276	0.050	0.5136
^{192}Hg (SD1)	8	214.4	2.2024	2.3117×10^3	5.8226	85.8722	87.4602	0.050	0.5232
^{194}Hg (SD1)	8	211.7	2.1994	3.2016×10^3	5.8182	85.9372	88.1382	0.050	0.5207
^{192}Pb (SD1)	8	214.8	2.2100	1.4554×10^3	5.6639	88.2783	88.3810	0.040	0.5880
^{194}Pb (SD1)	4	124.9	2.3572	2.2618×10^3	5.7778	86.5381	87.3210	0.040	0.5794
^{196}Pb (SD1)	6	171.4	2.2578	2.4032×10^3	5.8620	85.2951	86.5249	0.040	0.5719
^{198}Pb (SD1)	12	304.4	2.1422	1.7844×10^3	5.8367	85.6648	87.4991	0.040	0.5702
^{198}Po (SD1)	6	175.9	2.2528	3.8152×10^3	6.1060	81.8866	83.7610	0.045	0.5260

6. Calculations and discussion

We have applied the transition energy E_γ(I) formula described in Section 2 to eight yrast SD bands in even-even nuclei in the A $\sim$ 190 region, namely $^{190-194}$Hg, $^{192-198}$Pb, and ^{198}Po. In our nuclear softness (NS2) model, the two parameters σ and A occurring in Eq. (3) are determined after calculating the transitional energy ratio λ. In our calculations, the spin assignments of these SD bands are taken from our previous works [10–12]. In Table 1 we list the parameters used in the calculations and some useful calculating quantities. Using these set of parameters, the E_γ(I) transitions are calculated and compared with experimental ones. The results are shown in Figure 1 and are listed in Table 2.

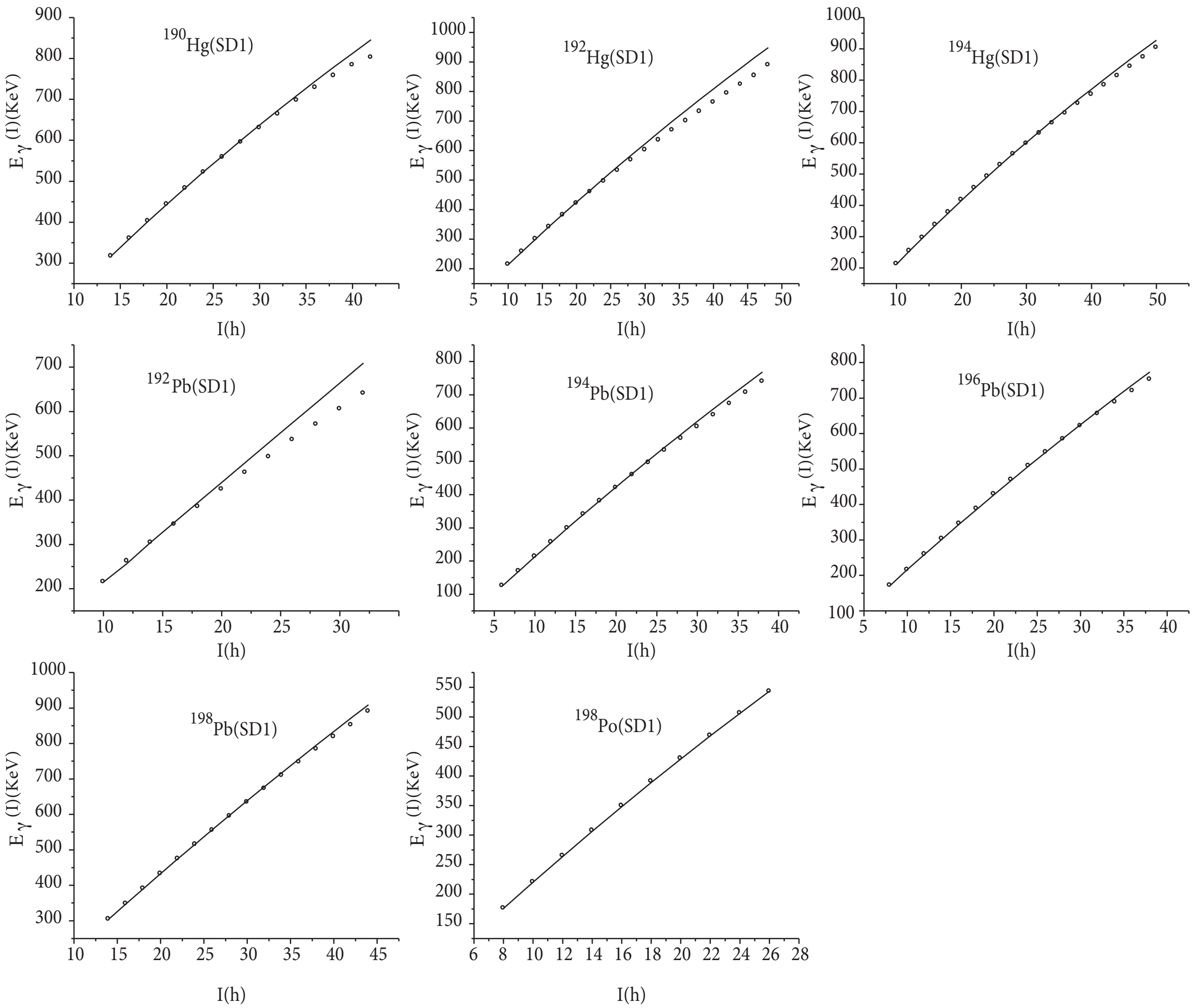

Figure 1. Calculated γ-ray transition energies E_γ(I) versus spin I for the eight SD bands are compared with experimental values [2,3]. Solid curves indicate theoretical calculations and filled circles indicate experimental values.

Figure 2 shows the difference in γ-ray transition energies δE_γ between the yrast SD bands in ^{192}Hg and ^{194}Pb (closed circles) versus E_γ; they are very similar (the average deviation in energy is around 3 KeV). Therefore, these two bands have been considered as IBs. In the same figure δE_γ for the pair ^{192}Hg and ^{194}Hg (open circles) is also seen; the differences are too large to consider these two bands as identical ones.

Table 2. Calculated and experimental γ-ray transition energies $E_{\gamma(KeV)}$ and aligned spins for the eight SD bands in even-even nuclei in the A $\sim$ 190 region.

^{190}Hg (SD1)			^{192}Hg (SD1)			^{194}Hg (SD1)			^{192}Pb (SD1)		
Spin	E_γ^{cal}	$E_\gamma^{\exp[2]}$	Spin	E_γ^{cal}	$E_\gamma^{\exp[2]}$	Spin	E_γ^{cal}	$E_\gamma^{\exp[2]}$	Spin	E_γ^{cal}	$E_\gamma^{\exp[2]}$
14	316.896	316.9	10	214.399	214.4	10	211.69	211.7	10	214.797	214.8
16	359.994	360.0	12	257.793	257.8	12	253.911	253.93	12	259.9043	262.4
18	402.179	402.34	14	300.605	300.1	14	295.347	295.99	14	304.9724	303.7
20	443.476	442.98	16	342.844	341.4	16	336.027	337.18	16	350.0012	344.6
22	483.911	482.71	18	384.529	381.6	18	375.967	377.39	18	394.9908	384.6
24	523.506	521.3	20	425.636	421.1	20	415.187	416.6	20	439.9412	423.7
26	562.286	558.6	22	466.225	458.8	22	453.703	454.76	22	484.8525	461.5
28	600.272	594.9	24	506.273	496.0	24	491.531	491.86	24	529.7247	498.7
30	637.486	630.1	26	545.797	532.1	26	528.689	527.88	26	574.5579	535.3
32	673.948	664.1	28	584.806	567.4	28	565.192	562.92	28	619.352	570.3
34	709.68	696.9	30	623.309	601.7	30	601.055	596.87	30	664.1073	604.7
36	744.688	728.5	32	661.315	634.9	32	636.293	629.93	32	708.8236	640.0
38	779.025	757.4	34	698.832	668.1	34	670.920	662.07			
40	812.677	783.5	36	735.868	700.1	36	704.951	693.4			
42	845.671	801.8	38	772.433	731.5	38	738.400	723.91			
			40	808.533	762.3	40	771.279	753.92			
			42	844.176	792.7	42	803.601	783.67			
			44	879.371	822.9	44	835.378	813.12			
			46	914.125	853.1	46	866.624	842.55			
			48	948.445	888.7	48	897.35	872.41			
						50	927.566	903.1			

Table 2. Continued.

^{194}Pb (SD1)			^{196}Pb (SD1)			^{198}Pb (SD1)			^{198}Po (SD1)		
Spin	E_γ^{cal}	$E_\gamma^{exp[2]}$	Spin	E_γ^{cal}	$E_\gamma^{exp[2]}$	Spin	E_γ^{cal}	$E_\gamma^{exp[2]}$	Spin	E_γ^{cal}	$E_\gamma^{exp[2]}$
6	124.8985	124.9	8	171.398	171.4	14	304.396	304.4	8	175.898	175.8982
8	169.1895	169.52	10	216.893	215.6	16	347.681	347.7	10	220.365	220.3654
10	212.8928	213.26	12	259.151	259.5	18	390.517	390.3	12	263.859	263.8592
12	256.0186	256.22	14	302.111	303.0	20	432.910	432.4	14	306.407	306.4078
14	298.5771	298.49	16	344.474	345.8	22	474.867	473.8	16	348.038	348.0385
16	340.5782	339.9	18	386.251	387.6	24	516.393	514.6	18	388.777	388.7775
18	382.0317	380.2	20	427.454	428.5	26	557.494	554.8	20	428.65	428.65
20	422.9469	420.0	22	468.092	469.4	28	598.175	594.4	22	467.680	467.6804
22	463.3332	458.4	24	508.177	508.5	30	638.444	633.4	24	505.892	505.8924
24	503.1997	495.8	26	547.717	546.9	32	678.304	671.8	26	543.308	543.3086
26	542.5552	532.5	28	586.723	584.2	34	717.763	709.4			
28	581.4084	568.3	30	625.205	620.6	36	756.824	746.7			
30	619.7678	603.4	32	663.171	654.9	38	795.4939	782.7			
32	657.6418	638.1	34	700.631	688.8	40	833.7771	818.5			
34	695.0386	672.3	36	737.595	720.1	42	871.679	851.2			
36	731.966	706.2	38	774.069	752.1	44	909.2046	890.0			
38	768.432	739.5									

Another result in the present work is the observation of a $\Delta I = 2$ staggering effect in the γ-ray transition energies in ^{194}Hg (SD1). We calculated the staggering quantity $S^{(4)}$ and plotted it as a function of rotational frequency $\hbar\omega$ in Figure 3, and the values are listed in Table 3. Significant anomalous staggering has been observed. The calculated bandhead moments of inertia J_0 in terms of our NS2 model for ^{190}Hg, ^{192}Hg, and ^{194}Hg are listed in Table 1. Assuming that C (Z) in Eq. (14) is constant for the three bands C (Z) = 0.05, the extracted quadrupole deformation β_2 for ^{190}Hg and ^{194}Hg would be 1.8% and 0.47% smaller than that for ^{192}Hg. This is consistent with the extracted β_2 in [17], where the β_2 deformations for ^{190,192,194}Hg are 0.463,

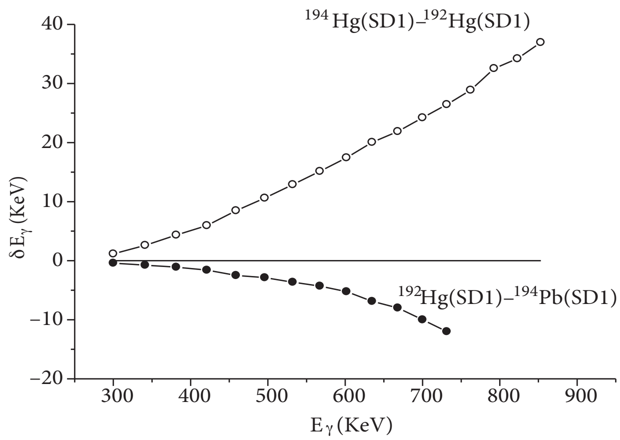

Figure 2. Differences in γ-ray transition energies $\delta E\gamma$ versus $E\gamma$ between the yrast bands in ^{192}Hg and ^{194}Pb (closed circles) and ^{192}Hg and ^{194}Hg (open circles).

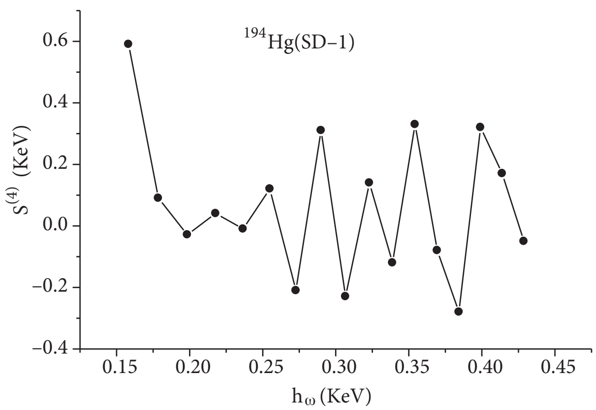

Figure 3. $S^{(4)}$ staggering pattern for the yrast SD band in ^{194}Hg.

Table 3. The staggering parameter $S^{(4)}$ as a function of rotational frequency $\hbar\omega$ for ^{194}Hg (SD1).

$\hbar\omega$ (KeV)	$S^{(4)}$ (KeV)
0.1582	0.5906
0.1786	0.0907
0.1984	–0.0284
0.2178	0.0406
0.2366	–0.0096
0.2549	0.1206
0.2727	–0.2096
0.2899	0.3103
0.3067	–0.2293
0.3230	0.1402
0.3388	–0.1194
0.3543	0.3301
0.3694	–0.0793
0.3843	–0.2799
0.3991	0.3204
0.4139	0.1706
0.4287	–0.0499

0.475, and 0.475 respectively, which correspond to a deviation in the deformation from ^{192}Hg of 2.5% and 0% for ^{190}Hg and ^{194}Hg. Furthermore, for each band in lead isotopes $^{192-198}$Pb, the J_0 values are calculated and listed in Table 1, assuming that C (Z) = 0.04 for the four SD bands, and the extracted β_2 deformations for ^{194,196,198}Pb are found to be smaller than that of ^{192}Pb by 1.4%, 2.7%, and 3%, respectively. This is consistent with the Hartree–Fock calculations plus BCS theory [18], which predicted the occurrence of a SD well for ^{192}Pb.

7. Conclusion

We have shown in this paper that the SD nuclear states can be described in the framework of nuclear softness (NS2) based on the variable moment of inertia model. After adopting the model parameters by using the experimental transition energies, we calculated the transition energies E_γ and the rotational frequencies $\hbar\omega$ of eight yrast SD bands in the A $\sim$ 190 mass region. In all cases the bandhead spins were assigned in our previous works. Excellent agreement between theory and experiment was obtained. The bandhead moments of inertia and the differences between transition energies suggest that the yrast SD bands of ^{192}Hg and ^{194}Pb are IBs and the SD bands in ^{190}Hg and ^{194}Hg have a slightly smaller quadrupole deformation than that of ^{192}Hg, in good agreement with the predictions from Woods–Saxon–Strutinsky calculations. A staggering quantity containing five consecutive E_γ values was used to present the $\Delta I = 2$ staggering in ^{194}Hg (SD1).

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