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Study of the energetic field characteristics of the *TE*-modal waves in waveguides

Fatih ERDEN*
Department of Electronics Engineering, Turkish Naval Academy, Tuzla, İstanbul, Turkey

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Abstract: The energetic characteristics of cylindrical medium-free time-domain waveguide fields are solved within the framework of the *evolutionary approach to electromagnetics*. Solving the boundary-eigenvalue problem for transverse Laplacian yields a configurational basis in the waveguide cross section. Elements of the basis depend on *transverse* coordinates, whereas the modal amplitudes depend on the *longitudinal* coordinate, z, and time, t. Solving the resulted Klein–Gordon equation yields a basis for analysis of the modal amplitudes. Exact solutions for the amplitudes of *TE*-modes are obtained, and the energetic field characteristics are derived in accordance with the causality principle.

Key words: Time domain, waveguide, electrodynamics, Maxwell's equations, evolutionary approach, Klein–Gordon equation

1. Introduction

There are two goals in this article. One is to obtain the exact solutions for the amplitudes of the *TE*-modes in a cylindrical medium-free waveguide within the framework of the *evolutionary approach to electromagnetics* (EAE) [1]. The second goal is to derive the energetic field characteristics as the functions of coordinates and time. These goals need to address the problem in the time-domain *directly*, without postulating time dependence of the fields proportionally to $\exp(i\omega t)$.

The background history of time-domain electromagnetics has been discussed in our previous publications: for a hollow cavity [2] and for a cavity filled with dispersive medium [3]. An elegant time-domain method relying on wave splitting technique and representing the propagation of transient electromagnetic waves in waveguides was performed earlier by Kristensson [4]. Presenting the fields as a convolution of a Green function and a source function, his method yields loss of accuracy when t and z become relatively large. In a later study [5], excitation of waves in a rectangular waveguide was considered by Geyi in the time domain.

The present study is based on solving Maxwell's equations in a transverse-longitudinal form convenient for transient analysis of electromagnetic waves in waveguides. Herein the waveguide fields are considered in SI units and presentable as the *real-valued* quantities. Therefore, this approach enables us also to study the energetic characteristics of electromagnetic fields.

The medium-free waveguide has perfect electric conductor surfaces. Its cross-section domain, S, is regular along the axis, Oz, and bounded by a closed singly connected contour, L. A right-handed triplet $(\mathbf{z}, \mathbf{l}, \mathbf{n})$ of mutually orthogonal unit vectors is used as $\mathbf{z} \times \mathbf{l} = \mathbf{n}$, where $\mathbf{n}$ is the outer normal to S, $\mathbf{l}$ is tangential to the L vector, and $\mathbf{z}$ is oriented along the axis, Oz. A point of observation within the waveguide is specified by a

*Correspondence: ferden@dho.edu.tr

three-component vector, $\mathbf{R}=\mathbf{r}+\mathbf{z}\,z$, where component $\mathbf{r}$ varies within S, z is a projection of $\mathbf{R}$ on Oz, and t denotes observation time.

2. Formulation of the problem

We have to solve the system of Maxwell's equations with time derivative ∂_t, that is,

$$\nabla\times\mathcal{E}\left(\mathbf{R},t\right) = -\mu_0\partial_t\mathcal{H}\left(\mathbf{R},t\right) \tag{1a}$$

$$\nabla\times\mathcal{H}\left(\mathbf{R},t\right) = \epsilon_0\partial_t\mathcal{E}\left(\mathbf{R},t\right), \tag{1b}$$

where $\mathcal{E}\left(\mathbf{R},t\right)$ and $\mathcal{H}\left(\mathbf{R},t\right)$ are the electric and magnetic strength vectors with physical dimensions of *volt per meter,* $\left\lfloor \mathrm{Vm}^{-1}\right\rfloor$, and *ampere per meter,* $\left\lfloor \mathrm{Am}^{-1}\right\rfloor$, respectively, μ_0 and ϵ_0 are the free-space constants specified as

$$\mu_0=4\pi\times10^{-7}\ \left\lfloor \mathrm{NA}^{-2}=\mathrm{kgms}^{-2}\mathrm{A}^{-2}\right\rfloor \tag{2}$$

$$\epsilon_0=\tfrac{1}{\mu_0\mathrm{c}^2}\ \left\lfloor \mathrm{Fm}^{-1}=\mathrm{A}^2\mathrm{s}^4\mathrm{kg}^{-1}\,\mathrm{m}^{-3}\right\rfloor \tag{3}$$

where $\mathrm{N}=\mathrm{kgms}^{-2}$ is the force unit, *Newton,* $\mathrm{F}=\mathrm{A}^2\mathrm{s}^4\mathrm{kg}^{-1}\mathrm{m}^{-2}$ is the electric capacitance, *farad,* and c is the numerical value of the speed of light in a vacuum.

In solving the time-domain waveguide problems, equations (1) should be supplemented with the boundary conditions over the waveguide surface as

$$\mathbf{n}\cdot\mathcal{H}|_{\mathbf{r}\in L}=0,\quad \mathbf{l}\cdot\mathcal{E}|_{\mathbf{r}\in L}=0,\quad \mathbf{z}\cdot\mathcal{E}|_{\mathbf{r}\in L}=0. \tag{4}$$

Since Maxwell's equations (1) belong to the hyperbolic type of the partial differential equations (*PDE*), problems (1) – (4) should be supplemented with appropriate *initial conditions* given at a fixed instant when needed.

2.1. The $TE-$ time-domain modal fields

Consider the waveguide with the circular cross section where the domain S is describable in polar coordinates, (r,φ), as $(0\le r\le a,\ 0\le\varphi\le2\pi)$. The position vector, $\mathbf{r}$, in S is $\mathbf{r}=\mathbf{r}_0\,r+\varphi_0\,\varphi$. The triplet given above, $\mathbf{z}\times\mathbf{l}=\mathbf{n}$, should be read as $\mathbf{z}\times\varphi_{\mathbf{0}}=\mathbf{r}_0$. We shall operate with the *real-valued* functions only.

2.1.1. Configurational basis for the $TE-$ modal fields

Derivation of the $TE-$ modal fields (normalized appropriately) starts via solving the *Neumann boundary eigenvalue problem* for transverse Laplacian, $\nabla^2_\perp$, namely,

$$\begin{aligned}\left(\nabla^2_\perp+\nu_n^2\right)\ \Psi_n\left(r,\varphi\right) &= 0,\ \mathbf{n}_0\cdot\nabla_\perp\Psi_n|_{\mathbf{r}\in L}=0\\ \frac{\nu_n^2}{S}\int_S\Psi_n^2\,ds &= 1,\end{aligned} \tag{5}$$

where $\Psi_n\left(r,\varphi\right)$ are the eigenfunctions corresponding to the eigenvalues, ν_n^2, $n=1,2,3,\ldots$. The set $\left\{\Psi_n\left(r,\varphi\right)\right\}_{n=0}^{\infty}$ is complete. We suppose that potential, $\Psi_n\left(r,\varphi\right)$, is dimension-free, physically. The spectrum of the eigenvalues, ν_n^2, is countable $(n=1,2,\ldots)$ because the domain S is finite. Solving the problem (5) by separation of the variables, (r, φ), yields

$$\nu_n^2 \equiv \nu_{qp}^2 = j_{qp}^2/a^2 > 0 \tag{6a}$$

$$\Psi_n \equiv \Psi_{qp}(r,\varphi) = \mathcal{N}_{qp} J_q(j_{qp} r/a) \sin(q\varphi + c_q) \tag{6b}$$

$$\mathcal{N}_n \equiv \mathcal{N}_{qp} = \sqrt[-2]{\left(1 - q^2/j_{qp}^2\right) J_q^2(j_{qp})} \tag{6c}$$

where $\mathcal{N}_n \equiv \mathcal{N}_{qp}$ is a normalizing constant. As far as the domain S is two-dimensional, the subscript n is double as $n \equiv q, p$, where $q = 0, 1, 2, \ldots,$ $p = 1, 2, \ldots$. The subscript q specifies the order of the Bessel function, $J_q(*)$, the numbers $j_{qp} > 0$ are the solutions to equation $\frac{d}{dr} J_q(j_{qp} r/a)|_{r=a} = 0$, which yields the distinct from zero $p-$th roots of derivative of the Bessel function, and c_q in (6b) is an arbitrary real-valued numerical parameter; the value $c_q = 0$ is available.

Every function from the set $\{\Psi_{qp}(r,\varphi)\}$ is specified completely by the *configuration* and *size*, a, of the circular contour L. The complete set of the eigensolutions to the *Neumann problem* suggests introducing a basis for the field distributions in the waveguide cross section. This basis is defined by a configuration of L, only. Therefore, we name this basis *"configurational"* $\{\mathbf{e}_{qp}, \mathbf{h}_{qp}, \mathcal{Z}_{qp}\}$, and specify

$$\mathbf{e}_{qp}(\mathbf{r}) = \nabla_\perp \Psi_{qp} \times \mathbf{z}, \quad \mathbf{h}_{qp}(\mathbf{r}) = \nabla_\perp \Psi_{qp}, \quad \mathcal{Z}_{qp}(\mathbf{r}) = \mathbf{z}\, \nu_{qp} \Psi_{qp}. \tag{7}$$

Note that the differential procedure, $\nabla_\perp$, and factor $\nu_{qp} = \sqrt{\nu_{qp}^2}$ in (7) have physical dimension of *inverse meter*, $\lfloor \mathrm{m}^{-1} \rfloor$. Therefore, all the basis elements in (7) have just this dimension.

A pair of new empirical constants We propose to introduce a pair of new empirical constants, $\mathring{\mu}_0$ and $\mathring{\epsilon}_0$, via combinations of the force unit, N, and the free-space constants as follows: $\mathring{\mu}_0 \overset{def.}{=} \sqrt{\mathrm{N}/\mu_0}$ and $\mathring{\epsilon}_0 \overset{def.}{=} \sqrt{\mathrm{N}/\epsilon_0}$. One can verify that $\mathring{\mu}_0$ has the physical dimension of *ampere*, $\lfloor \mathrm{A} \rfloor$, and $\mathring{\epsilon}_0$ has the dimension of *volt*, $\lfloor \mathrm{V} \rfloor$. Indeed,

$$\mathring{\mu}_0 = \sqrt[-2]{\pi}\, 1.5811 \times 10^3 \lfloor \mathrm{A} \rfloor \tag{8a}$$

$$\mathring{\epsilon}_0 = \sqrt{\pi}\, 1.8961 \times 10^5 \left\lfloor \mathrm{kgm^2 A^{-1} s^{-3}} \equiv \mathrm{V} \right\rfloor. \tag{8b}$$

Identity $\mathring{\mu}_0 \mathring{\epsilon}_0 \overset{def.}{=} 1$ yields the speed of light, $\mathrm{c} \left\lfloor \frac{\mathrm{m}}{\mathrm{s}} \right\rfloor$, as

$$\mathring{\mu}_0 \mathring{\epsilon}_0 = 1 \rightrightarrows \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.9979 \times 10^8 \left\lfloor \frac{\mathrm{AV}}{\mathrm{N}} \right\rfloor \equiv \mathrm{c}. \tag{9}$$

2.1.2. The $TE-$ time-domain modal fields

We can write down the time-domain field vectors, which are generated by the elements of the basis from (7), as follows:

$$\mathcal{E}_n = \mathcal{A}_n(t,z)\, \mathbf{e}_n(\mathbf{r}), \quad \mathcal{E}_{zn}(\mathbf{r}) = 0 \tag{10a}$$

$$\mathcal{H}_n = \mathcal{B}_n(t,z)\, \mathbf{h}_n(\mathbf{r}) + h_n(t,z)\, \mathcal{Z}_n(\mathbf{r}) \tag{10b}$$

where the factors, $\mathcal{A}_n(t,z)$, $\mathcal{B}_n(t,z)$, and $h_n(t,z)$, have the physical sense of the amplitudes of the field components. The amplitudes are unknown as yet, but we can already say that they should be obtained as *dimension-free* quantities.

Combining the time-domain field vectors that have the physical dimensions of *inverse meter*, $\lfloor \mathrm{m}^{-1} \rfloor$, (10), with the new empirical constants that have the physical dimensions of *ampere*, $\lfloor \mathrm{A} \rfloor$, and *volt*, $\lfloor \mathrm{V} \rfloor$, (8), electric and magnetic field strength vectors retain their needed dimensions, *volt per meter*, $\lfloor \mathrm{Vm}^{-1} \rfloor$, and of *ampere per meter*, $\lfloor \mathrm{Am}^{-1} \rfloor$, respectively:

$$\mathcal{E}\left(\mathbf{R},t\right) = \mathring{\epsilon}_0\, \mathcal{E}_n, \quad \mathcal{H}\left(\mathbf{R},t\right) = \mathring{\mu}_0\, \mathcal{H}_n. \tag{11}$$

2.1.3. Evolutionary equations for the $TE-$ modes

Substituting the field vectors $\mathcal{E}_n$ and $\mathcal{H}_n$ into (1) results in the Klein–Gordon equation (KGE) as

$$\left(\frac{\partial^2}{\mathrm{c}^2 \partial t^2} - \frac{\partial^2}{\partial z^2} + \nu_n^2\right) \theta_n\left(t,z\right) = 0, \tag{12}$$

where the first coefficient, ν_n^2, is the eigenvalue in the Neumann problem (5) and thereby that carry all information about contour L of the waveguide cross section. Solution to the KGE generates the modal amplitude, h_n, of the longitudinal component of *magnetic* field, $\mathcal{H}_n$, in (10b) as

$$h_n\left(t,z\right) = \theta_n\left(t,z\right). \tag{13}$$

In turn, this function, $h_n\left(t,z\right)$, specifies the amplitudes of transverse field components in (10a) – (10b) as

$$\mathcal{A}_n\left(t,z\right) = -\tfrac{\partial}{\nu_n \mathrm{c}\partial t} h_n\left(t,z\right) \qquad \mathcal{B}_n\left(t,z\right) = \tfrac{\partial}{\nu_n \partial z} h_n\left(t,z\right). \tag{14}$$

The KGE is invariant under the relativistic Lorentz transformations. Taking into account this fact, *Miller* investigated the KGE within the framework of group theory [6]. He discovered ten "orbits of symmetry," which allow solving the KGE via separation of the variables in various manners dependently on a chosen orbit of symmetry. The list of these orbits is cited in Appendix A of [7]. Only the lowest, *orbit 1*, corresponds to the classical time-harmonic electromagnetics. All the others open very wide possibilities for development of electrodynamics in the time domain. Herein we consider what the *orbit 1* and *orbit 2* result in. Miller's other orbits open up a new way for analytical studies in time-domain electromagnetics.

3. Solution for evolutionary equations

3.1. Real-valued solutions on orbit 1

Studies of KGE on Miller's *orbit 1* results in time-harmonic solutions in electromagnetics. As opposed to the classical time-harmonic field concept, where the complex-valued exponential, $\exp\left(i\omega t\right)$, plays the conceptual role, we obtained the real-valued solution directly as

$$h_n\left(t,z\right) = \sin\left(\omega t - \gamma_n z + C_n\right) \tag{15a}$$

$$\mathcal{A}_n\left(t,z\right) = -\frac{\partial}{\nu_n \mathrm{c}\partial t} h_n\left(t,z\right) \tag{15b}$$

$$\mathcal{B}_n\left(t,z\right) = \frac{\partial}{\nu_n \partial z} h_n\left(t,z\right), \tag{15c}$$

where ω is a frequency, $\gamma_n = \sqrt{\left(\omega/\mathrm{c}\right)^2 - \nu_n^2}$, and C_n is a real-valued numerical parameter; $C_n = 0$ is admissible. The condition $\gamma_n = 0$ yields the cut-off frequencies for the $TE-$modes as $\omega_n^{\mathrm{TE}} = \nu_n \mathrm{c}$. The modal amplitudes on Miller's *orbit 1* (15) are exhibited graphically in Figure 1.

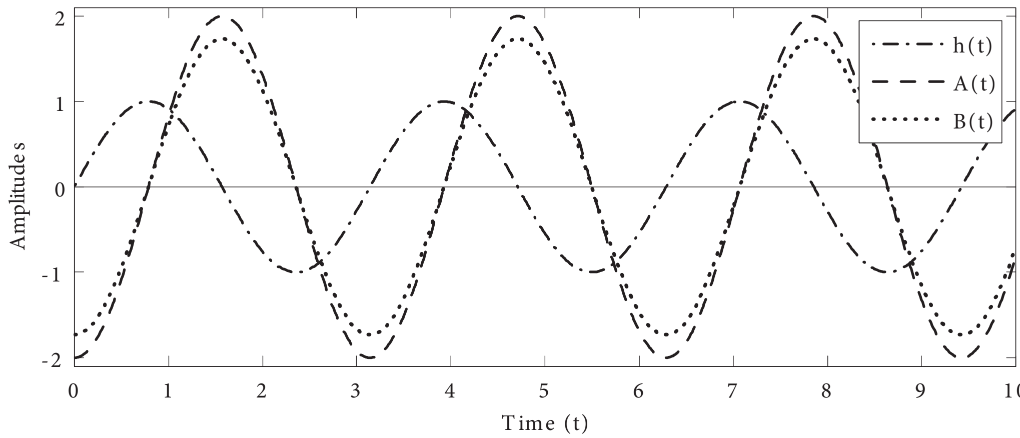

Figure 1. The modal amplitudes of $h_n(t,z), A_n(t,z)$, and $B_n(t,z)$ for *orbit 1* (15).

3.2. Real-valued solutions on orbit 2

Operations on Miller's *orbit 2* yield the solutions to KGE (12) as

$$\theta_n^m(t,z) = \left(\frac{\mathrm{c}t - |z|}{\mathrm{c}t + |z|}\right)^{m/2} J_m\left(\nu_n\sqrt{\mathrm{c}^2t^2 - z^2}\right) \tag{16}$$

where $m=0,1,2,\ldots,$ which results in

$$h_n(t,z) \equiv h_n^m(t,z) = \theta_n^m(t,z)\,. \tag{17}$$

Consequently, the amplitudes of transverse field components can be calculated by simple formulae as

$$\mathcal{A}_n^m = -\tfrac{\partial}{\nu_n \mathrm{c}\partial t} h_n^m(t,z)\,, \qquad \mathcal{B}_n^m = \tfrac{\partial}{\nu_n \partial z} h_n^m(t,z)\,. \tag{18}$$

The modal amplitudes on Miller's *orbit 2* (17), (18) are exhibited graphically in Figure 2.

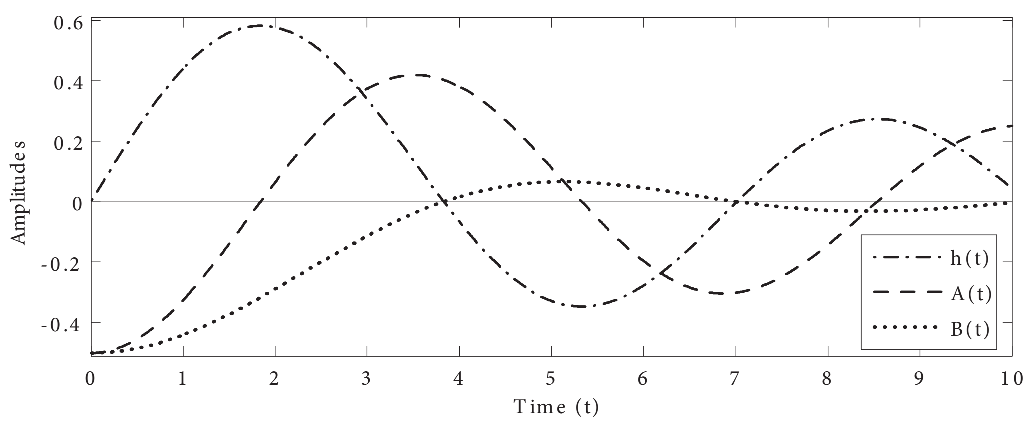

Figure 2. The modal amplitudes of $h_n(t,z)$, $A_n^m(t,z)$, and $B_n^m(t,z)$ for *orbit 2* (17), (18).

4. Energetic functions of the waveguide modes

For convenience of calculations, we replace t and z variables in (13) – (14) by their dimensionless equivalents as $\tau=\nu_n \mathrm{c}t$ and $\xi=\nu_n z$. Calculation of the power flow density, $\mathcal{P}_{z\,n}$, averaged over the waveguide cross section

(ξ is fixed) is written in (19a). Averaged value of the energy density, $\mathtt{W}_n$, in the same cross section is presented in (19b):

$$\mathcal{P}_{z\,n} = \mathbf{z}\,c\,\mathtt{S}_n(\tau,\xi), \quad \mathtt{S}_n(\tau,\xi) = \mathcal{A}_n(\tau,\xi)\,\mathcal{B}_n(\tau,\xi) \tag{19a}$$

$$\mathtt{W}_n = \frac{1}{2}\left[\mathcal{A}_n^2(\tau,\xi) + \mathcal{B}_n^2(\tau,\xi) + h_n^2(\tau,\xi)\right] \tag{19b}$$

The conservation of energy law is used to check the calculations

$$d\mathtt{W}_n = \frac{1}{2}\left[\mathcal{A}_n^2(\tau,\xi) - \mathcal{B}_n^2(\tau,\xi)\right], \quad \mathtt{w}_n = \frac{1}{2}h_n^2(\tau,\xi). \tag{20}$$

where $d\mathtt{W}_n$ is the surplus of energy of transverse fields, and $\mathtt{w}_n$ is the energy density of the longitudinal field component. The normalized instant velocity of transportation of the modal field energy through any fixed waveguide cross section is specified as

$$\mathtt{v}_n(\tau,\xi) = \mathtt{V}_n(\tau,\xi)/\mathrm{c}, \quad \mathtt{V}_n(\tau,\xi) = \mathtt{S}_n/\mathtt{W}_n \tag{21}$$

in accordance with Umov's theorem [8].

In Figures 3 and 4, graphical examples are exhibited for Miller's *orbit 1*, and in Figures 5 and 6 for *orbit 2* (19) – (21). Our calculations show that $0 \le \mathtt{v}_n(\tau,\xi) \le 1$ for Miller's *orbit 1*. This means that the velocity of transportation of modal field energy varies from *zero* up to the light velocity, c. The causality condition from the Einstein postulates, that the electromagnetic field can not transfer energy more than the speed of the light in the vacuum [9], is verified via varying the form of the coefficients in the formulae (17), (18).

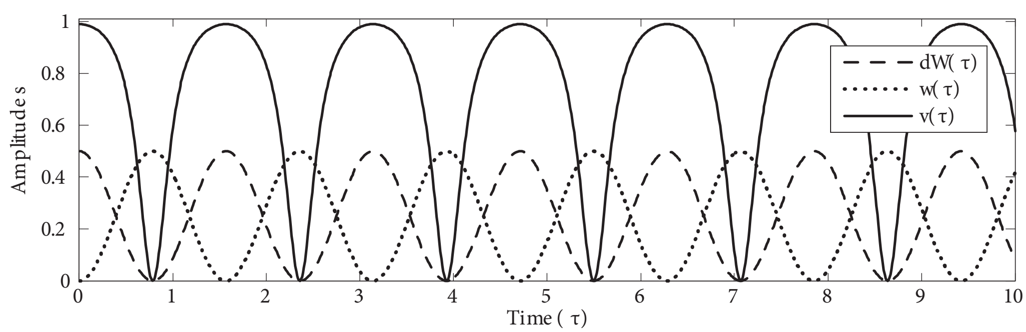

Figure 3. The modal amplitudes of $\mathtt{S}_n(\tau,\xi)$, $\mathtt{W}_n(\tau,\xi)$, and $\mathtt{v}_n(\tau,\xi)$ for *orbit 1*.

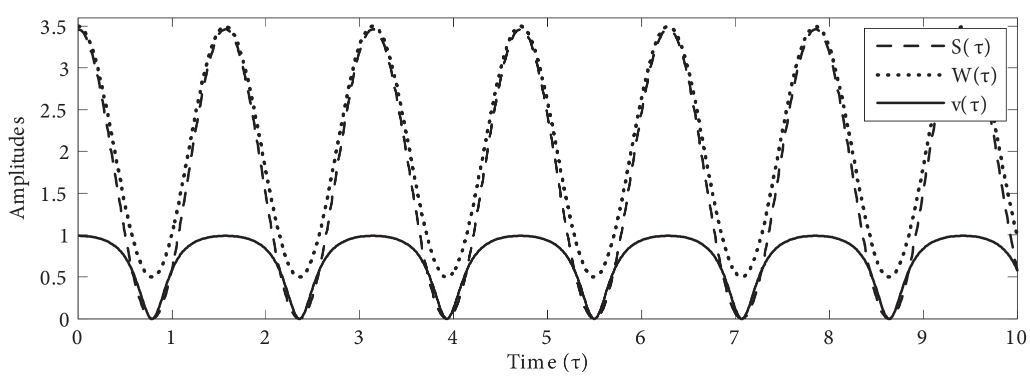

Figure 4. The modal amplitudes of $d\mathtt{W}_n(\tau,\xi)$, $\mathtt{w}_n(\tau,\xi)$, and $\mathtt{v}_n(\tau,\xi)$ for *orbit 1*.

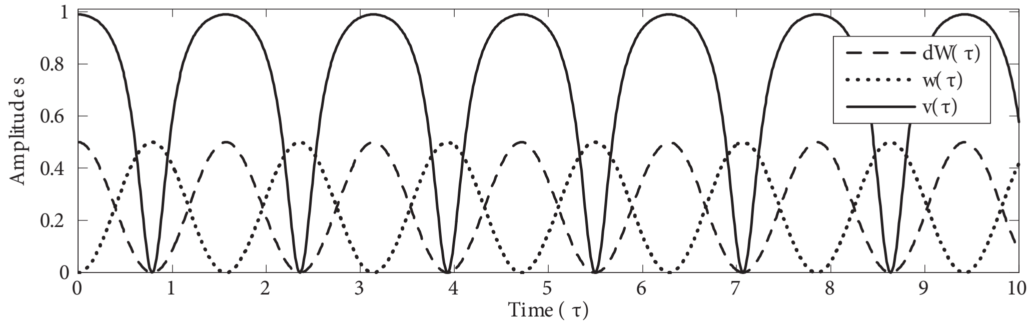

Figure 5. The modal amplitudes of $S_n(\tau,\xi)$, $W_n(\tau,\xi)$, and $v_n(\tau,\xi)$ for *orbit 2*.

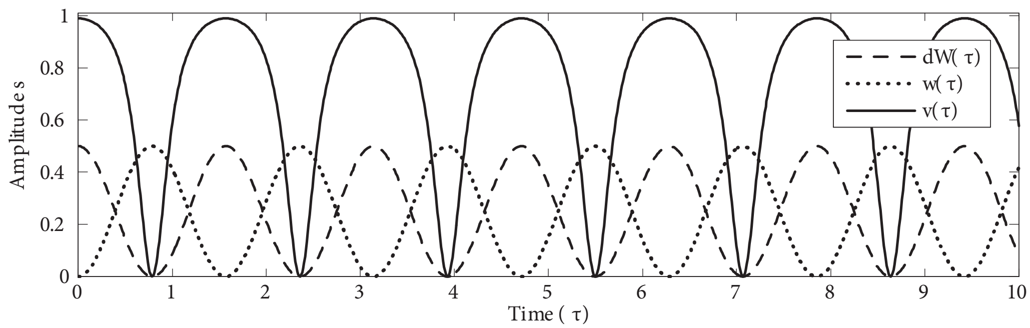

Figure 6. The modal amplitudes of $d\ W_n(\tau,\xi)$, $w_n(\tau,\xi)$, and $v_n(\tau,\xi)$ for *orbit 2*.

Regarding the negative-valued velocities in Figures 5 and 6, the direction of energy transportation is opposite to that of waveform propagation. This situation is similar to the phenomenon in backward oscillators in electronics. For both Miller's *orbits* of *1* and *2*, the causality condition holds for the transportation of energy by a wavefront as well, since it is obtained that $\mathtt{V}_n(\tau,\xi) = c$ at the front of the signals.

5. Conclusion

1) In the standard Maxwell's equation in SI units, the field vectors, $\mathcal{E}$ and $\mathcal{H}$, have the physical dimensions of *volt per meter*, $\lfloor \mathrm{Vm}^{-1} \rfloor$, and of *ampere per meter*, $\lfloor \mathrm{Am}^{-1} \rfloor$, respectively. That is why the free-space constants, μ_0 and ϵ_0, have been installed therein empirically. We propose to introduce a pair of new constants as $\mathring{\mu}_0 = \sqrt{\mathrm{N}/\mu_0}$ and $\mathring{\epsilon}_0 = \sqrt{\mathrm{N}/\epsilon_0}$, where N is the force unit. Then the first constant, $\mathring{\mu}_0$, has the dimension of ampere, $\lfloor \mathrm{A} \rfloor$, and the second one, $\mathring{\epsilon}_0$, has the dimension of volt, $\lfloor \mathrm{V} \rfloor$.

2) Derivation of the energetic field quantities as the *real-valued* functions of time needs to solve the waveguide problem in the time domain *directly*. The postulation that the electromagnetic fields are proportional to the complex valued exponential $\exp(i\omega t)$ is nonusable. Thereby, the solution is obtained herein within the framework of the *evolutionary approach to electrodynamics* (EAE), which was recognized recently as an alternative to the classical time-harmonic field method [10].

3) Mathematical aspects of the EAE have been developed in our previous publications. The Klein–Gordon equation (KGE) plays the central role in the waveguide problem of this approach. Miller discovered *ten* orbits of symmetry of the KGE. Herein, we considered the solutions that correspond to Miller's *orbit 1* and *orbit 2*. One can find: *(i)* the derivation of the modal basis for a waveguide in [11] and [12], *(ii)* the presentation of the solutions to the KGE corresponding to Miller's *orbit 3* in [13] and *orbit 7* in [14].

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