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Prompt fission neutron energy spectrum of ^{252}Cf in center of mass and laboratory systems

Mehmet KOÇAK[*], **Humbat AHMADOV**
Department of Engineering Physics, Faculty of Engineering, Gaziantep University, Gaziantep, Turkey

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Abstract: In this work we discuss the influence of different forms of prompt fission neutron energy spectra in the fission fragment center of mass system on the laboratory spectrum of neutrons in spontaneous fission of ^{252}Cf. We show that the Le Couteur spectrum, which takes into account multiple neutron emission of neutrons in the center of mass system, describes the observed neutron energy spectrum well when transformed to the laboratory system.

Key words: Spontaneous fission, neutron spectrum, energy distribution, nuclear data

PACS: 25.85.Ca

1. Introduction

Studies on prompt fission neutron energy spectra have great importance for the design of nuclear reactors and scientific research [1, 2]. ^{252}Cf spontaneous fission as regard to shape study of the spectrum is used as standard in this manner. The center of mass energy spectrum of neutrons depends on many effects, such as multiple neutron emissions from fission fragments, initial excitation energy distribution of fission fragments, fission fragment charge and mass distribution, competition between neutron and γ-emissions from lower excitation fragments, and so on [3] - [10]. However, in studying theoretical problems it is preferred to have a simple representation of the laboratory energy spectrum.

Almost all of the effects mentioned above lead to the softening of the center of mass neutron energy spectrum as compared with the Weisskopf evaporation spectrum [11]. Weisskopf evaporation theory gives the following expressions for the neutron energy distribution in the range $\epsilon, \epsilon + d\epsilon$:

$$\Phi(\epsilon)d\epsilon = const.\ \epsilon\ \sigma_c(\epsilon)\ W(\epsilon_{max} - \epsilon)d\epsilon, \tag{1}$$

where $\sigma_c(\epsilon)$ is the compound nucleus formation cross section at the energy ϵ and W is the energy level density of residual nucleus while ϵ_{max} is the maximum possible energy of emitted neutron. Relating the nuclear level density with the thermodynamic temperature [11], [12] of residual nucleus, $T(\epsilon_{max})$, Eq. (1) is transformed to the form of

$$\Phi(\epsilon)d\epsilon = const.\ \epsilon\ \sigma_c(\epsilon)\ e^{-\epsilon/T} d\epsilon. \tag{2}$$

If we take $\sigma_c(\epsilon) = constant$, then one gets the Weisskopf's neutron energy spectrum as

$$\Phi_{Wf}(\epsilon)\ d\epsilon = const.\ \epsilon\ e^{-\epsilon/T} d\epsilon \tag{3}$$

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[*]Correspondence: kocak@gantep.edu.tr

where "Wf" refers to the Weisskopf spectrum. When $\sigma_c(\epsilon) \sim 1/\sqrt{\epsilon}$, ($1/v$ law), then the Maxwell form of the energy spectrum is obtained as

$$\Phi_M(\epsilon)\ d\epsilon = const.\ \sqrt{\epsilon}\ \ e^{-\epsilon/T} d\epsilon. \tag{4}$$

The spectrum in Eq. (4) is softer than that in in Eq. (3), i.e. the portion of low energy neutrons is greater. This is due to pre-exponential energy dependence of the spectrum, which, in general form, is proportional to ϵ^a, where a is 1 in Eq. (3) and 1/2 in Eq. (4). Therefore, lower a leads to a softer spectrum. For the so-called Le Couteur spectrum [13, 14] $a = 5/11$. This spectrum was calculated in the framework of cascade neutron evaporation from a highly excited nucleus and compared with experimental results for the neutron evaporation mechanism in a 190 MeV proton bombardment experiment. The Le Couteur form of neutron energy spectrum was expressed in the form

$$\frac{1}{\sigma_c}\Phi(\epsilon) d\epsilon = const.\ \epsilon^{\ell-1} e^{-\epsilon/T} d\epsilon, \tag{5}$$

where $\ell \approx 16/11$, $T \approx (11/12)\ T_m$ within the Fermi-gas model and T_m is the temperature corresponding to first neutron emission. The expression in Eq. (5) can be written in the usual form of

$$\Phi(\epsilon) = const.\ \epsilon^{5/11} e^{-\epsilon/T}. \tag{6}$$

Here we note that the excitation energy distribution of initial fission fragments and energy dependent cross section of the inverse process cause similar softening effects like the Le Couteur effect. Thus, the spectrum in Eq. (6) may be used in description of the fission neutron spectrum.

In the following section we discuss three spectra, given by Eqs. (3), (4), and (6) in normalized to unity forms. Temperature parameters are determined using the equality of average energies for all spectra. These spectra are regarded as fission fragment center of mass neutron spectra. Later, we transform center of mass spectra to the laboratory system. Comparisons of calculated laboratory spectra with experimental results for ^{252}Cf spontaneous fission prompt neutron spectra are carried out.

2. Center of mass and laboratory spectra

The energy spectral forms of Weisskopf, Maxwell, and Le Couteur normalized to unity in the energy range of emitted neutrons, $(0, \infty)$, are

$$\Phi_{Wf}(\epsilon) = \frac{\epsilon}{T_{Wf}^2} e^{-\epsilon/T_{Wf}}, \tag{7}$$

$$\Phi_M(\epsilon) = \frac{2\sqrt{\epsilon}}{\sqrt{\pi T_M^3}} e^{-\epsilon/T_M}, \tag{8}$$

$$\Phi_{LC}(\epsilon) = \frac{\epsilon^{5/11} e^{-\epsilon/T_{LC}}}{\Gamma(16/11) T_{LC}^{16/11}}, \tag{9}$$

where T_{Wf}, T_M, and T_{LC} are corresponding temperature parameters, respectively, and gamma function: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Average energies of these spectra are, correspondingly,

$$\bar{\epsilon}_{Wf} = 2T_{Wf}, \quad \bar{\epsilon}_M = \frac{3}{2} T_M, \quad \bar{\epsilon}_{LC} = \frac{16}{11} T_{LC}. \tag{10}$$

The laboratory spectrum for isotropic distribution of neutrons in the center of mass system has the form

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\Phi(\epsilon)d\epsilon}{4\sqrt{\epsilon E_f}}, \tag{11}$$

where E_f is the kinetic energy of fission fragment per nucleon. Laboratory spectra corresponding to the center of mass system, given by Eqs. (7), (8), and (9), have respectively the forms of

$$N_F(E) = \frac{1}{4\sqrt{E_f T}}[(-x_1 e^{-x_1^2} + erf(x_1)) - |(-x_2 e^{-x_2^2} - erf(x_2))|] \tag{12}$$

where $x_1 = \sqrt{\frac{E}{T}} + \sqrt{\frac{E_f}{T}}$, $x_2 = \sqrt{\frac{E}{T}} - \sqrt{\frac{E_f}{T}}$ and $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$,

$$N_W(E) = \frac{1}{\sqrt{\pi\, TE_f}} e^{-E_f/T} e^{-E/T} \sinh\left(\frac{2}{T}\sqrt{EE_f}\right) \tag{13}$$

and

$$N_{LC}(E) = \frac{\Gamma(21/22)}{\Gamma(16/11)} \frac{1}{4\sqrt{E_f T}} \left[\Gamma\left(\frac{(\sqrt{E}+\sqrt{E_f})^2}{T}, \frac{21}{22}\right) - \Gamma\left(\frac{(\sqrt{E}-\sqrt{E_f})^2}{T}, \frac{21}{22}\right)\right], \tag{14}$$

in which $\Gamma(x,a)$ is the incomplete gamma function defined by

$$\Gamma(x,a) = \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt. \tag{15}$$

Here N_F is the Feather spectrum [15] while N_W is the Watt spectrum [16] and N_{LC} is the Le Couteur spectrum that is transformed to the laboratory system.

In nuclear reactor applications it is important to use neutron energy spectra having simple mathematical expressions depending on as few parameters as possible. In this respect, Maxwell and Watt distributions have been mostly preferred for physics-based theoretical spectra so far. For this reason, we compare our calculated results with these spectra. Most of the figures in this work are given as a ratio of calculated spectra to the Maxwell or Watt spectra rather than absolute comparison of spectra to clearly show the consistency or discrepancy between them.

3. Calculations

To compare different spectra with each other and with the experimental spectrum for the spontaneous fission of ^{252}Cf we must define firstly the temperature parameters for each of Eqs. (7)–(9). These parameters are chosen due to the equality of average energies to that of the observed laboratory spectrum, which is 2.13 MeV. Average energy in the laboratory system for the fragment of given mass, charge, and kinetic energy is defined as

$$E(Z,M) = E_f(Z,M) + \overline{\epsilon}(Z,M). \tag{16}$$

This formula may be written for the average light and heavy fragments approximately as

$$\overline{E} = \overline{E}_f + \overline{\epsilon}. \tag{17}$$

Our calculations are carried out both for the average single fission fragment and for the two complementary average fission fragments. For the average single fission fragment we take $\overline{E}_f = 0.768\ MeV$ as in the work [7]. Then the temperature parameters in the equations (7) to (9) are $T_{Wf} = 0.681\ MeV$, $T_M = 0.91\ MeV$, and $T_{LC} = 0.936\ MeV$, respectively. The following first five figures are drawn by using these parameters. Figure 1 gives the center of mass spectra calculated from the equations (7) to (9) and Figure 2 illustrates the ratio of these spectra to the Maxwell spectrum in the center of mass system. As expected, the Le Couteur spectrum in the center of mass is closer to the Maxwell spectrum than the Weisskopf spectrum is to the Maxwellian.

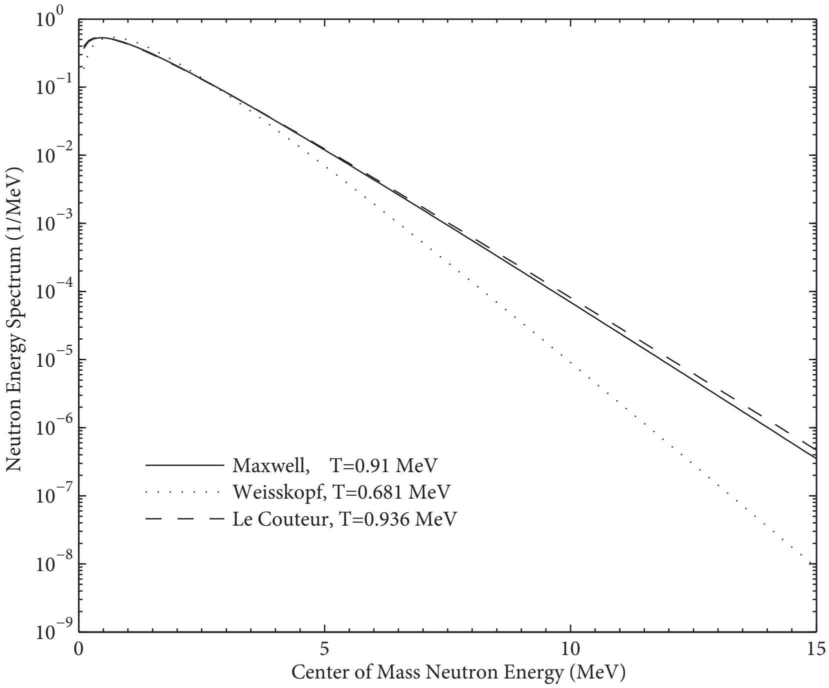

Figure 1. Comparisons of center of mass neutron energy spectra.

In Figure 3, the laboratory spectra, calculated by using Eqs. (12) to (14), are illustrated. Figure 4 shows the ratio of laboratory spectra to Watt spectrum. As seen in Figure 4, the laboratory spectrum of Le Couteur is closer to the Watt spectrum than the Le Couteur spectrum to the Maxwell spectrum in center of mass in Figure 2. This is due to the averaging effect in fission fragment motion.

Figure 5 illustrates the comparison of laboratory spectra given by Eqs. (12) to (14) with the Maxwell spectrum with $T = 1.42\ MeV$ temperature value, which is recommended by ENDF, Evaluated Nuclear Data File, for the spontaneous prompt fission neutron spectrum of ^{252}Cf. As it is evident from Figure 5, the Le Couteur in laboratory spectrum is closer to the Maxwellian than the Watt spectrum is to the Maxwellian.

Now we do calculations for two complementary average fission fragments with parameters $\overline{E}_f^L = 0.984\ MeV$ and $\overline{E}_f^H = 0.553\ MeV$ as in [7] and using the expression for the average energy in the laboratory system,

$$\overline{E} = \frac{1}{2}(\overline{E}_f^L + \overline{E}_f^H) + \frac{1}{2}(\overline{\epsilon}^L + \overline{\epsilon}^H), \tag{18}$$

where indices L and H indicate light and heavy fragments. Eq. (18) yields $\overline{\epsilon}^L + \overline{\epsilon}^H = 2.74\ MeV$. From the

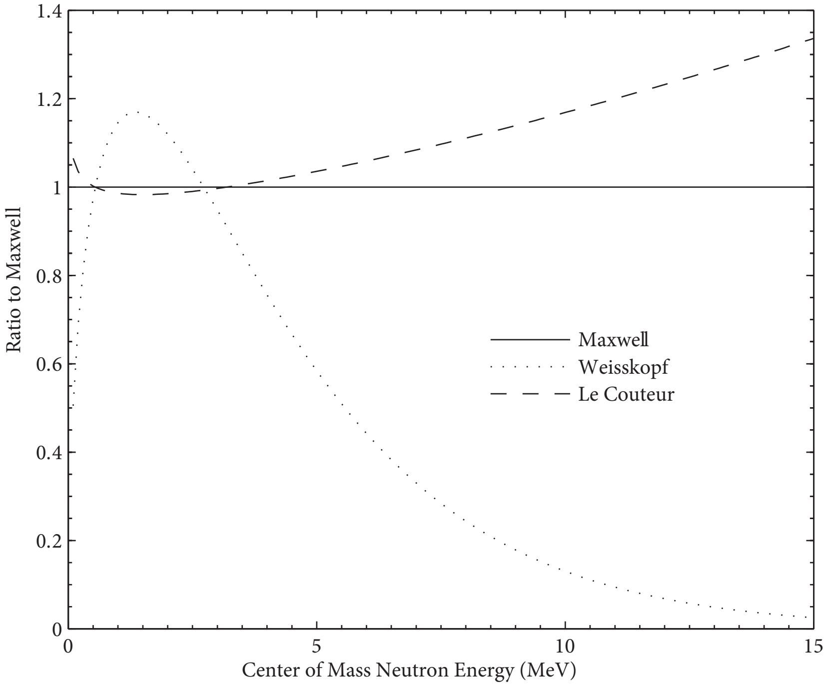

Figure 2. Ratio of the Weisskopf and Le Couteur spectra to the center of mass Maxwell spectrum.

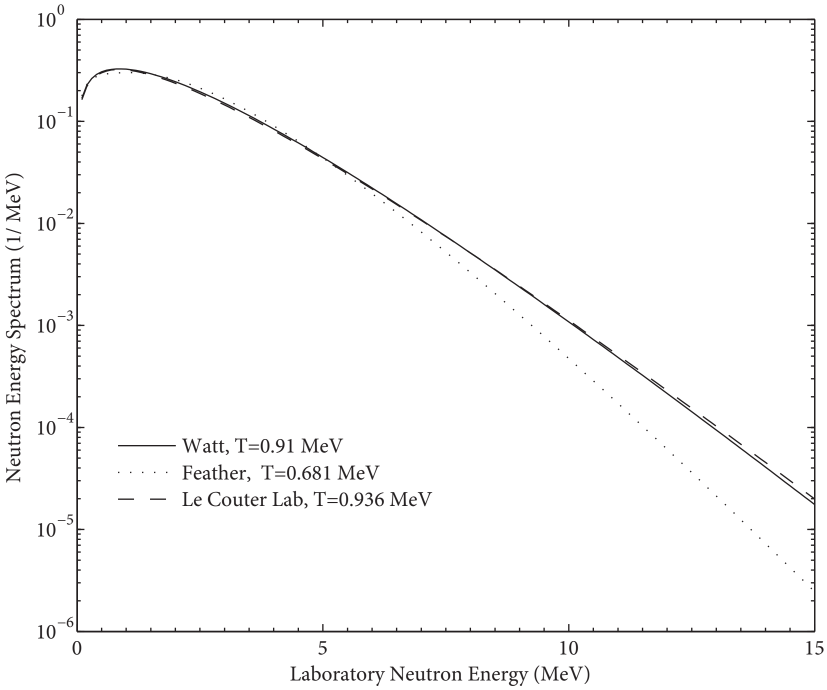

Figure 3. Comparisons of laboratory neutron energy spectra for $E_f = 0.768\ MeV$.

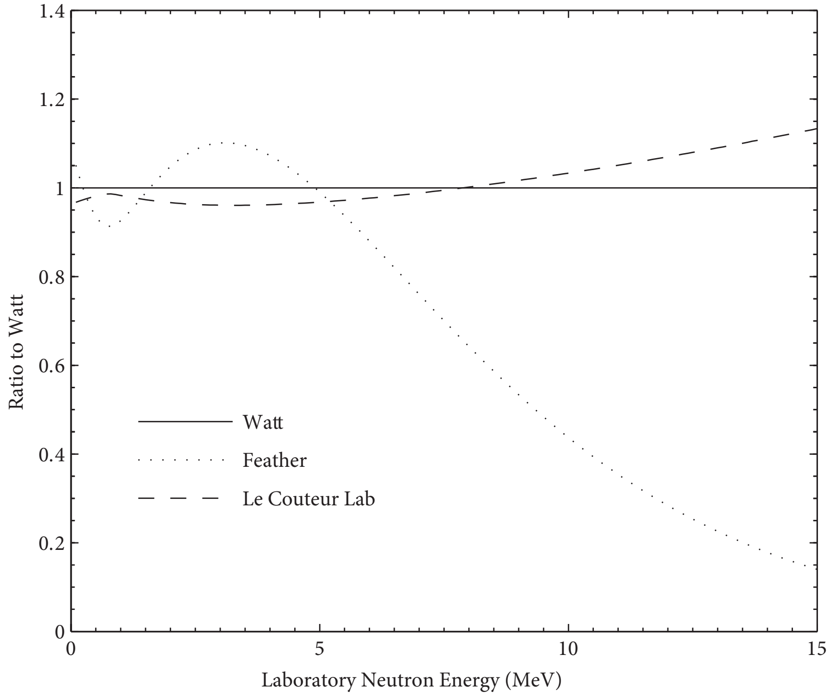

Figure 4. Ratio of the Feather and laboratory Le Couteur spectra to the Watt spectrum.

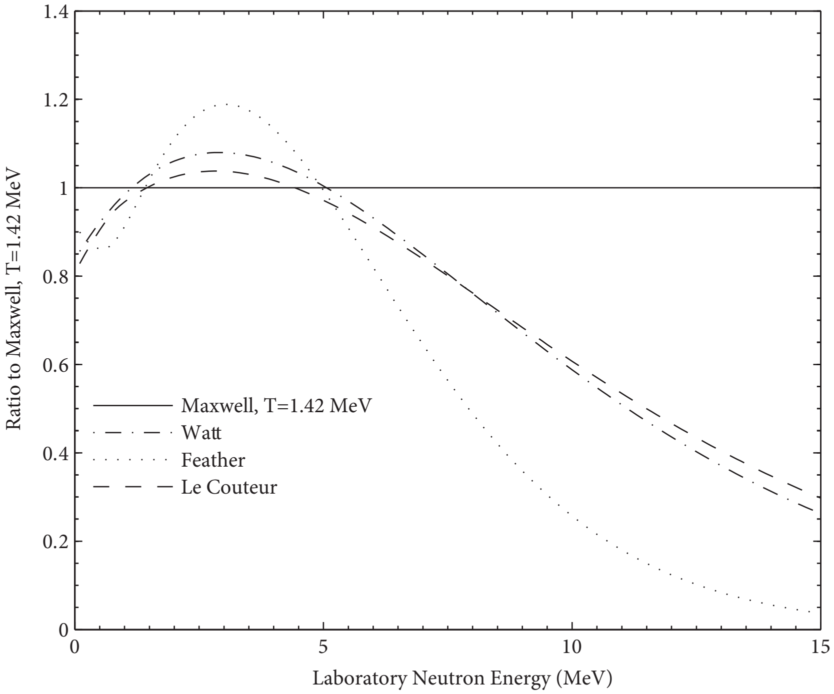

Figure 5. Ratio of the laboratory spectra to the Maxwellian spectrum with $T = 1.42\ MeV$.

experimental data of Bowman et al. [17], $\bar{\epsilon}^L = 1.42\ MeV$ and $\bar{\epsilon}^H = 1.3\ MeV$ for the complementary average fission fragments $\overline{A}^L = 108$ and $\overline{A}^H = 144$, respectively. The sum of these data gives $2.72\ MeV$, which is very close to the calculated result. From the average energies we define the temperature parameters for the light and heavy fragments, respectively, as, $T^L_{Wf} = 0.71\ MeV$, $T^L_M = 0.947\ MeV$, $T^L_{LC} = 0.976\ MeV$ and $T^H_{Wf} = 0.65\ MeV$, $T^H_M = 0.87\ MeV$, $T^H_{LC} = 0.89\ MeV$.

Figures 6 and 7 illustrate the ratio of the laboratory spectra for light and heavy fragments to the Watt spectrum, respectively, for the parameters discussed above. These figures illustrate the difference between calculated spectra in the whole range of neutron energy.

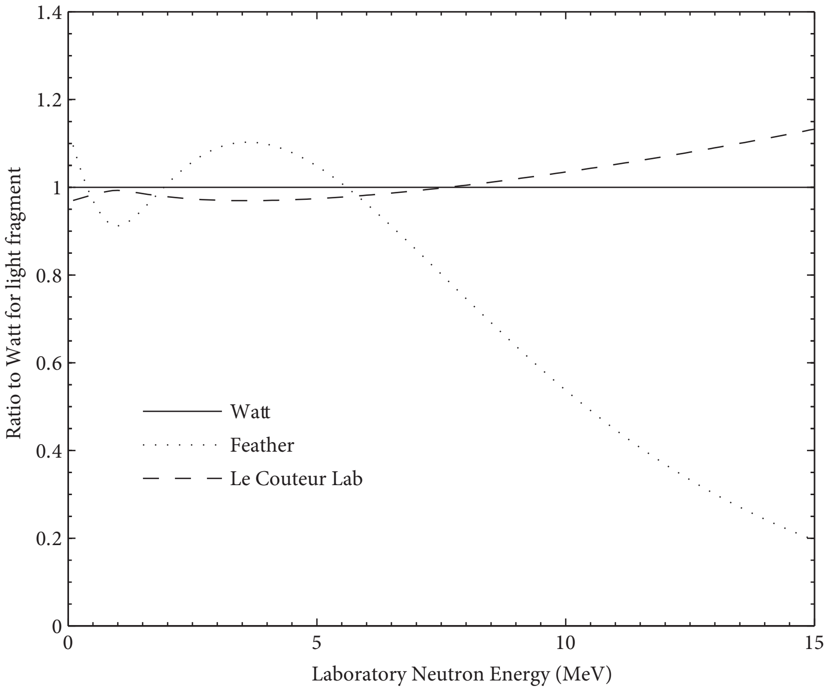

Figure 6. Light fragment laboratory spectra; ratio to the Watt spectrum.

Figure 8 illustrates the ratio of the mean laboratory spectra calculated by

$$N = \frac{1}{2}(N_L + N_H) \tag{19}$$

to the Maxwellian with $T = 1.42\ MeV$. Here N_L and N_H are light and heavy fission fragment neutron spectra, calculated from Eqs. (12) to (14). Comparison of Figure 8 with Figure 5 shows that the mean laboratory spectra approximate the Maxwellian at $T = 1.42\ MeV$ better than the single average laboratory spectra.

The comparison of calculated mean laboratory neutron energy spectra with evaluated experimental data of Mannhart [18] is illustrated in Figure 9. As seen in this figure, the laboratory Le Couteur spectrum describes the experimental data well compared with the Watt spectrum in a wide energy range. In Figure 10, we illustrate the comparison of calculated mean laboratory Le Couteur spectrum with the experimental data of Starostov et al. [19], approximated with Maxwellian of $T = 1.428\ MeV$. The agreement between the data and calculation results is satisfactory in a wide region of neutron energies except for at low energy values ($< 0.7 MeV$).

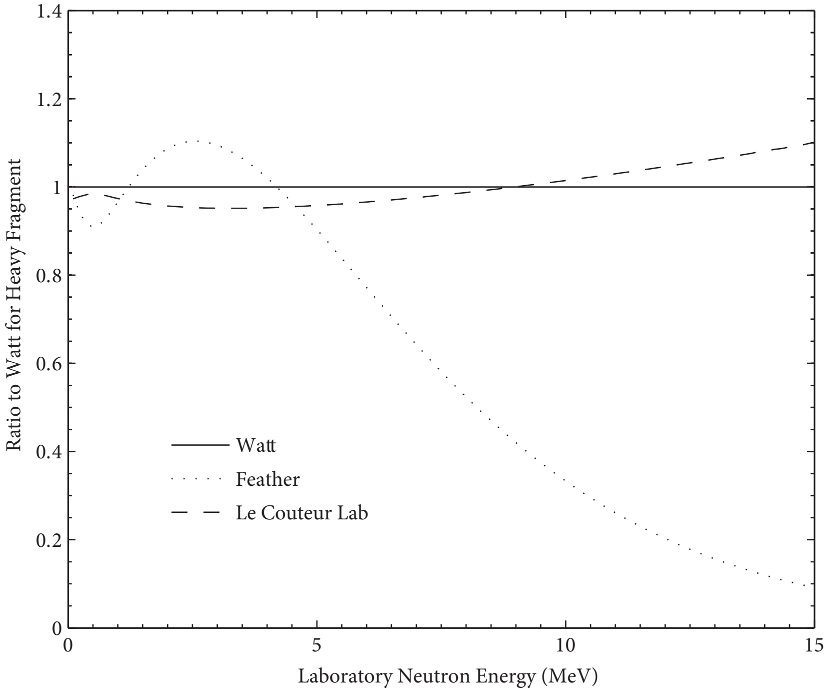

Figure 7. Heavy fragment laboratory spectra; ratio to the Watt spectrum.

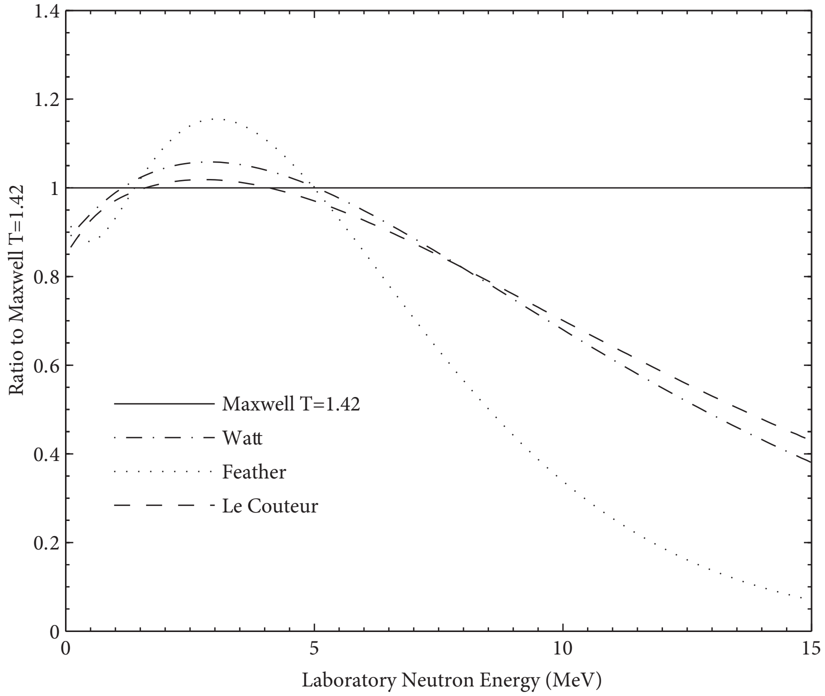

Figure 8. Comparison of mean laboratory spectra (ratio to the Maxwell spectrum with $T = 1.42\ MeV$).

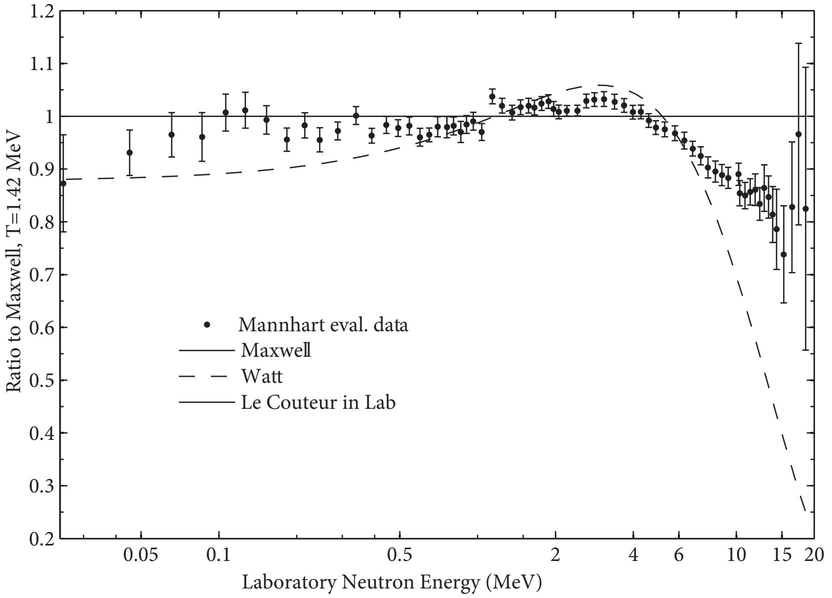

Figure 9. Comparison of calculated mean neutron energy spectra with the evaluated experimental data of Mannhart.

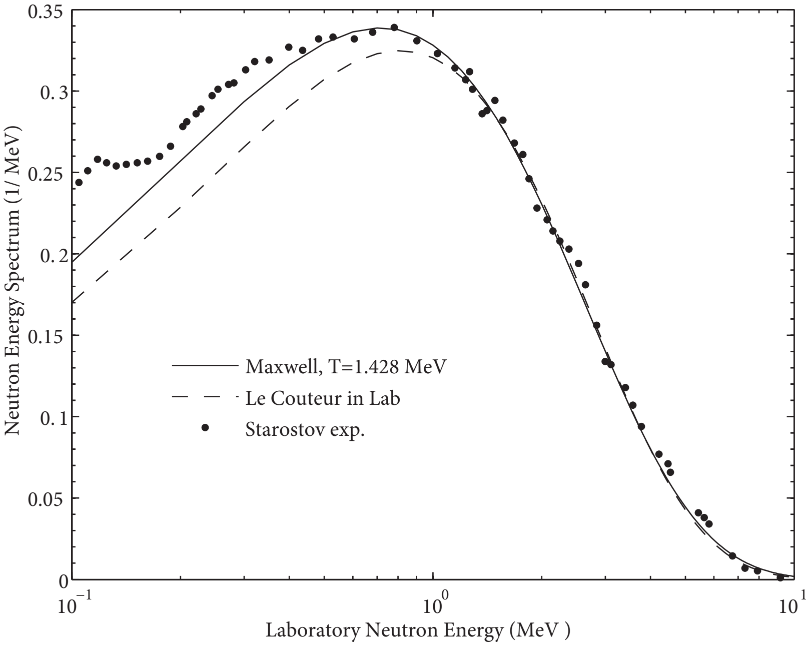

Figure 10. Comparison of calculated mean laboratory Le Couteur spectrum with the experimental data of Starostov and Maxwellian spectrum with $T = 1.428\ MeV$.

4. Discussion and conclusion

In this article we have presented a theoretical analysis of the prompt fission neutron spectrum in spontaneous fission of ^{252}Cf in both the center of mass of fission fragments and the laboratory system.

Using simple representation models, like Maxwell and Watt spectra, it is always desirable for application purposes. In this context, our calculation method considers simple representations of a laboratory spectrum by use of the center of mass spectrum in Le Couteur form. It is shown that while this spectrum has similar behavior with Maxwellian in the center of mass, when transformed into the laboratory system it gives a somewhat better representation for the observed laboratory spectrum than the Watt spectrum.

It should be noted that there is a direct relation between the softening effect and the description of experimental data. As any center of mass spectrum is described in the form of $\epsilon^a\ exp(-\epsilon/T)$, the softening effect can be represented by the value of a. While a is 5/11 for the Le Couteur spectrum it is 1/2 for the Maxwell spectrum. Because the smaller the a the softer the spectrum it can be said that the Le Couteur spectrum is softer than the Maxwell spectrum. Hence, the laboratory form of the Le Couteur spectrum describes the observed neutron energy spectrum better than the Watt spectrum, which is the laboratory form of the Maxwell spectrum.

As a final remark, we particularly emphasize that the formula given by Eq. (14) may be used in addition to the Watt spectrum, for representations of the observed prompt fission neutron spectra in spontaneous fission of ^{252}Cf.

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