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# Linear instability of dust acoustic waves in a magnetized gravitating plasma in the presence of dust streaming

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**Abstract:** Linear instability of dust acoustic waves (DAWs) has been theoretically studied in a collisionless gravitating dust-ion magnetized plasma including the effect of dust streaming. A linear dispersion relation has been derived by using the method of normal mode analysis. Gravitational effect is found to make the low-frequency DAW mode linearly unstable in certain parametric regions. The dependence of the growth rate of the instability on various plasma parameters such as streaming of dust particles, size of the dust grain, strength of the magnetic field, obliqueness of propagation, and number density of dust particles has been numerically analyzed and presented graphically.

**Key words:** Linear instability, dust acoustic wave, dust streaming, gravitating dusty plasma

## 1. Introduction

In recent years the study of dusty plasmas has become an important topic of plasma research. The dust acoustic wave (DAW) propagation in dusty plasmas plays an important role in many astrophysical, laboratory, and space environments such as cometary tails, asteroid zones, planetary rings, interstellar medium, and Earth's environment [1–5].

Wave propagation is significantly modified by the presence of charged dust grains due to their extremely large mass as compared to that of ions and electrons. The electron density in the background of the dusty plasmas can be neglected in many practical situations. Most of the electrons, because of their lower inertia, may get attached to the dust grain surface and then we can use the dust-ion plasma model [6,7]. This model has relevance to Saturn's F-ring [8], the surroundings of Halley's comet [9], and some laboratory plasma environments [10]. The charge on the dust grains influences their motion in electromagnetic fields and also affects the coagulation rate of dust particles into large bodies [11]. Different wave modes are predicted in the linear and nonlinear theories of magnetized and unmagnetized dust-ion plasmas [12–19]. One of the most useful modes is DAW mode.

There are many practical situations in space and astrophysical plasma where dust streaming can be found [20]. Dust streaming is found to have considerable effect on the propagation of DAWs in dusty plasma. Some authors have investigated the effect of dust streaming on the different eigenmodes of the wave propagation in magnetized and unmagnetized dusty plasmas [21–25]. Recently Sahoo et al. [26] studied the effect of dust streaming on linear and nonlinear properties of the DAWs in magnetized dust-ion plasma.

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In usual dusty plasma the electromagnetic force dominates over the gravitational force, but as the size of the dust grains increases, the gravitational effect begins to play a significant role. In some of astrophysical plasmas the gravitational force dominates over the electromagnetic force [27] and the dynamics of charged clouds and the formation of stars are controlled by the self-gravitational force [28]. The formation of the large-scale structure of DAWs is attributed to the gravitational condensation [29]. In the case of dusty plasma with significant dust size, it is important to consider the effect of the gravitational force. As far as we know, no one has studied DAWs in magnetized dust-ion plasma including the effect of gravitational force. The purpose of the present paper is to investigate the linear propagation of DAWs in magnetized dust-ion plasma including the effect of gravitational force and dust streaming. It is shown that inclusion of gravitational effect makes the wave linearly unstable in certain parametric regions.

The paper is organized as follows. In Section 2 we present the basic equations governing the dynamics of DAW magnetized dust-ion plasma and derive a general type of linear dispersion relation. In section 3 we discuss the results.

## 2. Basic formulation

We consider two-component homogeneous magnetized dust-ion plasma consisting of positively charged ions and negatively charged massive micron-size dust grains. In this two-component dust-ion plasma model we assume that ions obey the Boltzmann distribution and most of the electrons of the plasma are attached to the surface of the dust grains. The condition $n_{e0} \langle\langle\, z_d n_{d0}$ is satisfied where $n_{e0}$, $n_{d0}$, and $z_d$ are respectively the unperturbed electron number density, dust particle number density, and number of electrons attached to the dust grain surface. We also assume that the dust particles are streaming with some constant velocity ($\vec{v}_{d0} = \hat{e}_z v_{d0}$) along the z-direction and there is an external uniform magnetic field $\vec{B} = B_0\hat{e}_z$ along the z-direction. The basic equations governing the dynamics of the ions and dust grains in such magnetized dust-ion plasma including the effects of gravitational force are the following:

$$\frac{\partial n_d}{\partial t} + \vec{\nabla}.\left(n_d \vec{v}_d\right) = 0, \tag{1}$$

$$\frac{\partial \vec{v}_d}{\partial t} + \left(\vec{v}_d.\vec{\nabla}\right)\vec{v}_d = \left(\frac{ez_d}{m_d}\right)\vec{\nabla}\phi - \vec{\nabla}\psi - \omega_{cd}\left(\vec{v}_d \times \widehat{e}_z\right), \tag{2}$$

$$n_i = n_{i0}\exp\left(\frac{-e\phi}{k_B T_i}\right), \tag{3}$$

$$\nabla^2\psi = 4\pi G m_d n_d, \tag{4}$$

where $n_i$, $n_d$, $v_d$, and $m_d$ are respectively the ion number density, dust particle number density, dust fluid velocity, and dust particle mass; $\omega_{cd} = (ez_d B_0/m_d)$ is the dust-cyclotron frequency, in which e is the magnitude of electron charge; $T_i$, $k_B$, $G$, $\phi$, and $\psi$ are respectively the ion temperature, Boltzmann constant, gravitational constant, electric potential, and gravitational potential. The plasma quasineutrality condition is:

$$n_i = z_d n_d. \tag{5}$$

In the above equations we have normalized $v_d$ by dust acoustic speed $c_d = \sqrt{z_d k_B T_i/m_d}$, time (t) by $\omega_{cd}^{-1}$, distance by $c_d/\omega_{cd}$, $\phi$ by $k_B T/e$, $\psi$ by $z_d k_B T_i/m_d$, and all number densities by the equilibrium ion number density n$_{i0}$. We now make the following perturbation expansion for the field quantities X$(n_d,\ v_{dz},\ \phi,\ \psi,\ v_{dx},\ v_{dy})$:

$$X = X_0 + X_1 + \cdots \tag{6}$$

where $X_0\,(n_{d0},\, v_{dz0},\, 0,\, 0,\, 0,\, 0)$ represents equilibrium values and $X_1\,(n_{d1},\, v_{dz1},\, \phi_1,\, \psi_1,\, v_{dx1},\, v_{dy1})$ represents first-order perturbed values. Substituting the perturbation expansion of Eq. (6) into Eqs. (1)–(5) and linearizing, we get:

$$\frac{\partial n_{d1}}{\partial t} + n_{d0}\frac{\partial v_{dx1}}{\partial x} + n_{d0}\frac{\partial v_{dy1}}{\partial y} + n_{d0}\frac{\partial v_{dz1}}{\partial z} + v_{dz0}\frac{\partial n_{d1}}{\partial z} = 0 \quad , \tag{7}$$

$$\frac{\partial v_{dx1}}{\partial t} + v_{dz0}\frac{\partial v_{dx1}}{\partial z} = \frac{\partial \phi_1}{\partial x} - \frac{\partial \psi_1}{\partial x} - v_{dy1}, \tag{8}$$

$$\frac{\partial v_{dy1}}{\partial t} + v_{dz0}\frac{\partial v_{dy1}}{\partial z} = \frac{\partial \phi_1}{\partial y} - \frac{\partial \psi_1}{\partial y} + v_{dx1}, \tag{9}$$

$$\frac{\partial v_{dz1}}{\partial t} + v_{dz0}\frac{\partial v_{dz1}}{\partial z} = \frac{\partial \phi_1}{\partial z} - \frac{\partial \psi_1}{\partial z}, \tag{10}$$

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial y^2} + \frac{\partial^2 \psi_1}{\partial z^2} = \frac{\omega_{gd}^2}{\omega_{cd}^2} n_{d1}, \tag{11}$$

$$n_{d1} = -\frac{\phi_1}{z_d}, \tag{12}$$

where $\omega_{gd} = \sqrt{4\pi G m_d z_d n_{d0}}$ is called the dust Jeans frequency. Now we assume that the field parameters vary with space and time according to $\exp\left[i\left(\overrightarrow{k}\cdot\overrightarrow{r} - \omega t\right)\right]$, where $\omega$ and $k$ are the normalized wave frequency and wave number, respectively. Following the standard procedure, we finally obtain the linear dispersion relation for an obliquely propagating DAW:

$$(\omega - k_z v_{dz0})^2\left[(\omega - k_z v_{dz0})^2 - 1\right] = \left[k^2(\omega - k_z v_{dz0})^2 - k_z^2\right] n_{d0}\left\{z_d - \left(\frac{\omega_{gd}}{\omega_{cd} k}\right)^2\right\}, \tag{13}$$

where $k^2 = k_x^2 + k_y^2 + k_z^2$ and $k_z = k\cos\theta$, in which $\theta$ is the angle between $\widehat{k}$ and $\vec{B}$. If we neglect the gravitational effect the dispersion relation of Eq. (13) reduces to that given in reference [30]. The dispersion law as given by Eq. (13) can be rewritten as:

$$a\omega^4 + b\omega^3 + c\omega^2 + d\omega + e = 0, \tag{14}$$

where $a = \omega_{cd}^2 k^2$,

$$b = -4\omega_{cd}^2 k^2 k_z v_{dz0},$$

$$c = k^2[n_{d0}\omega_{gd}^2 - \omega_{cd}^2(1 + k^2 n_{d0} z_d - 6k_z^2 v_{dz0}^2)],$$

$$d = 2k^2 k_z v_{dz0}[n_{d0}\left(k^2 z_d \omega_{cd}^2 - \omega_{gd}^2\right) + \omega_{cd}^2\left(1 - 2k_z^2 v_{dz0}^2\right)],$$

$$e = k_z^2[\omega_{cd}^2 k^2 v_{dz0}^2\left(k_z^2 v_{dz0}^2 - 1\right) + n_{d0}\left(k^2 v_{dz0}^2 - 1\right)\left(\omega_{gd}^2 - z_d k^2 \omega_{cd}^2\right)].$$

It is a fourth-order algebraic equation in wave frequency $(\omega)$, indicating different modes of wave propagation. The dispersion law as given by Eq. (14) has been solved numerically for different plasma parameters. It is shown that for a particular set of values of $k$, $B_0$, $v_{dz0}$, $n_{d0}$, $z_d$, $\theta$, $m_d$ two roots are real and two are complex. It is also observed that the real roots may be both 'positive' and 'negative'. The positive real roots correspond to the forward-moving wave and negative real roots correspond to the backward-moving wave. The instability (growing/decaying) of the wave when it propagates through the plasma medium is determined by $\omega_I$, where $\omega_I$ is the imaginary part of the complex roots. Positive values of $\omega_I$ correspond to growing instability and negative values of $\omega_I$ correspond to decaying instability of the wave.

## 3. Results and discussion

In this paper we have studied the linear instability of DAWs in dust-ion magnetized plasma including the effects of gravitational force and dust streaming. By using normal mode analysis, a general type of dispersion law for dust acoustic waves has been derived. The linear dispersion equation is of the fourth order in wave frequency, which shows the possibilities of excitation of a number of wave modes. In the absence of gravitational effect all the values of wave frequency $\omega$ turn out to be real. This means that in the absence of a gravitational effect all the wave modes are linearly stable. One of these modes is the DAW mode. Its linear dispersion characteristic is shown in Figure 1.

Including the gravitational effect it is shown through numerical computation that the DAW mode becomes linearly unstable for a wide range of plasma parameters appropriate to astrophysical plasma systems. The growth rate of this instability depends in a significant way on gravitational force and dust streaming. To study the instability characteristics of the DAW, we have plotted $\omega_I$ (which is intimately related to the growth rate of instability) for different plasma parameters that are mostly relevant to astrophysical plasma system [23].

Figure 2 shows the dependence of the growth rate of instability on dust particle size ($m_d$) and dust particle number density ($n_{d0}$). Obviously the growth rate of instability increases significantly with increase in both dust particle size and dust particle number density. Physically an increase in dust particle size/mass or dust particle number density increases the gravitational effect. In self-gravitating dusty plasma there is a

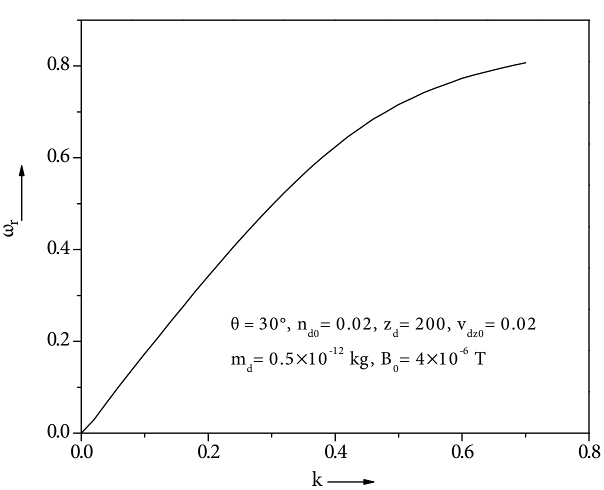


**Figure 1.** Dispersion curve for dust acoustic wave.

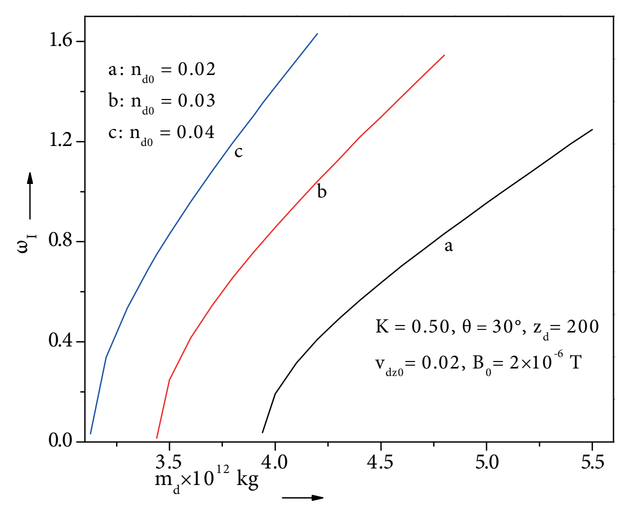


**Figure 2.** Growth rate of instability plotted as a function of the dust mass $m_d$ for different values of dust number density $n_d$.

competition between gravitational self-attraction and electrostatic repulsion between the charged grains, apart from other electromagnetic effects, and hence it is expected that an increase in gravitational influence makes the DAW more unstable.

The dependence of the growth rate of instability on dust streaming is shown in Figure 3. Dust streaming acts as a source of free energy and helps the instability to grow. For this, the growth rate of instability increases with increase in the dust streaming velocity ($v_{dz0}$). In Figure 4 we show the dependence of the linear instability of the wave on the strength of magnetic field ($B_0$) and the angle propagation ($\theta$) of the wave with respect to the magnetic field. From Figure 4 we find that the wave becomes less unstable with an increase in the strength of the magnetic field and the angle of propagation of the wave with respect to the magnetic field. The magnetic field forces the charged particles to gyrate about the magnetic field lines. The motion of charged particles is therefore almost free along the magnetic field but is restricted in the direction perpendicular to the magnetic field. Thus, with the increase in the strength of the magnetic field or the angle of propagation of the wave with respect to the direction of magnetic field, the motion of charged particles becomes more restricted and consequently the DAW tends to become less unstable.

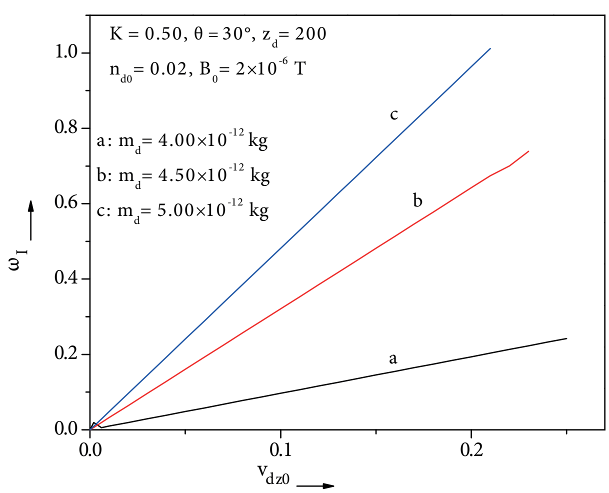


**Figure 3.** Dependence of the growth rate of instability on dust streaming velocity $v_{dzo}$ for different values of dust mass ($m_d$).

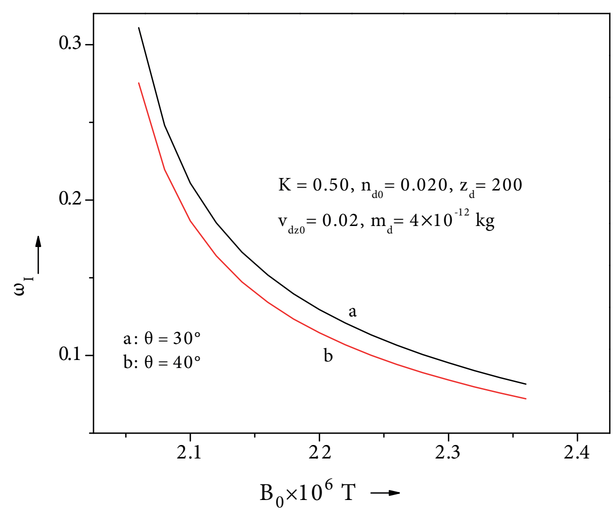


**Figure 4.** Dependence of the growth rate of instability on the strength of magnetic field $B_0$ and the angle of propagation $\theta$.

To summarize, we have studied the linear instability of DAWs in a self-gravitating dust-ion magnetized plasma including the effect of dust streaming. It is shown that inclusion of gravitational effect makes a DAW linearly unstable in magnetized dust-ion plasma and the growth rate of this instability depends sensitively on the dust particle size, dust particle density, dust particle streaming velocity, strength of the external magnetic field, and angle of propagation of the wave with respect to the magnetic field. The results presented in this paper might be helpful in understanding the instability of DAWs in some space and astrophysical plasmas such as cometary tails, planetary rings, or interstellar clouds where the dust-ion plasma model is applicable.

Finally, we would like to point out that our dust-positive ion plasma model can be generalized to include the effect of negative ions, which plays an important role in determining the linear and nonlinear properties of dust acoustic waves in plasmas used for industrial processing and in plasmas in Earth's upper atmosphere. For example, the negative ions can be an important factor in fixing the charge on the dust particles [31]; this could open up the possibility of positive and negative solitary potential structures to coexist [32].

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