Magnetized strange quark matter in reconstructed $f(R,T)$ gravity for Bianchi I and V universes with cosmological constant

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Abstract: In this article, we have investigated the behaviors of magnetized strange quark matter distributions for Bianchi I and V universes in reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ gravity (here α_1 and α_2 are constants; $f_3(T)$ is an arbitrary function of T). To get solutions of the field equations we have used a deceleration parameter and the equation of state for strange quark matter. The new represented $f(R,T)$ model includes two models of Harko et al. and transforms to general relativity. When $t \to \infty$, we get the dark energy model ($p = -\rho$) in reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ gravity. However, we obtain zero magnetic field in all $f(R,T)$ gravitation models.

Key words: $f(R,T)$ gravity, Bianchi I universe, Bianchi V universe, magnetized strange quark, deceleration parameter

1. Introduction

As is known, the universe is expanding by accelerating. This expansion has been tried to be explained by some alternative gravitation theories such as $f(R,T)$ gravity [1], Lyra manifold [2], $f(R)$ gravity [3], and Brans–Dicke gravitation theory [4]. For this reason, interest in alternative gravitation theories is increasing day by day. Harko et al. proposed $f(R,T)$ gravitation theory in 2011 [1]. Many researchers have studied the $f(R,T)$ gravitation theory. Aktaş and Aygün studied magnetized strange quark matter (MSQM) for the FRW universe in $f(R,T)$ theory [5]. Agrawal and Pawar investigated magnetized domain wall solutions in $f(R,T)$ gravitation theory [6]. The magnetized string model in the Bianchi III universe was researched by Sarita et al. in $f(R,T)$ gravity [7]. Sharma and Sing investigated the magnetized string model for the Bianchi II metric in $f(R,T)$ theory [8]. Ram and Kumari studied bulk viscous fluid for Bianchi I and V universes in $f(R,T)$ theory [9]. Quark and strange quark matter solutions for Bianchi I and V universes in $f(R)$ gravitation theory were investigated by Yılmaz et al. [10]. Ramesh and Umadevi studied perfect fluid matter distributions with linearly varying deceleration parameter for the FRW universe in $f(R,T)$ theory [11]. Perfect fluid solutions for the FRW universe in $f(R,T)$ theory were researched by Myrzakulov [12]. Momeni et al. analyzed $f(R)$ and $f(R,T)$ gravitational theories with Noether symmetry [13]. Some authors investigated various space-times and matter distributions for reconstructed $f(R,T)$ models [14–17].

In this research we consider MSQM distributions for a model of Bianchi type I and V universes in $f(R,T)$ theory with cosmological constant. The strange quark matter equation of state (EoS) is given by:

$$p = \frac{\rho - 4B_c}{3} \tag{1}$$

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Here $p = p_q - B_c$, $\rho = \rho_q + B_c$, $p_q = \frac{\rho_q}{3}$, and B_c is the bag constant [18,19]. p_q is the quark pressure and ρ_q is the quark energy density [18,19].

This article is organized as follows. In Section 2 we will give the modified field equation of $f(R,T)$ gravitation theory. In Sections 3 and 4, we consider a reconstructed $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, $f(R,T) = 2f(T) + R$ and $f(R,T) = f_2(T) + f_1(R)$ models for Bianchi type I and V universes in $f(R,T)$ theory. Finally, conclusions are given with the transform from $f(R,T)$ theory to general relativity in the reconstructed model for Bianchi type I and V universes.

2. Field equations of $f(R,T)$ gravitation theory

The action of $f(R,T)$ theory is given as follows [1]:

$$S = \int \frac{1}{16\pi} \left(\frac{f(R,T)}{G} + L_m \right) \sqrt{-g} d^4 x \tag{2}$$

Here $f(R,T)$ is an arbitrary function of T and R, T is the trace of T_{ik}, R is the Ricci scalar, L_m is the Lagrangian, and g is the determinant of g_{ik} [1]. T_{ik} is defined by:

$$T_{ik} = L_m g_{ik} - \frac{2\partial L_m}{\partial g^{ik}} \tag{3}$$

From Eq. (1), we have [1]:

$$f_R(R,T) R_{ik} - \frac{1}{2} f(R,T) g_{ik} + (g_{ik} \nabla_a \nabla^a - \nabla_i \nabla_k) f_R(R,T) = 8\pi T_{ik} - f_T(R,T) T_{ik} - f_R(R,T) \Xi_{ik} - \Lambda g_{ik} \tag{4}$$

Here $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ and $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$. Also, ∇_i is the covariant derivative and Ξ_{ik} is given by [1]:

$$\Xi = -2T_{ik} - pg_{ik} \tag{5}$$

For the solution of modified field equations we will use Bianchi type I and V universe models in $f(R,T)$ theory. The Bianchi type V universe model is given by:

$$ds^2 = -A(t)^2 dx^2 - e^{2mx} B(t)^2 dy^2 - e^{2mx} C(t)^2 dz^2 + dt^2 \tag{6}$$

where m is a constant. The energy-momentum tensor of MSQM is given by [20]:

$$T_{ik} = \left(\rho + p + h^2 \right) u_i u_k - \left(p + \frac{h^2}{2} \right) g_{ik} - h_i h_k \tag{7}$$

where p is the pressure, ρ is the energy density, h^2 is the magnitude of the field's magnetic component, and u_i is a four-velocity vector [20]. u_i and h_i are four-velocity vectors that satisfy $u_i u^i = 1$ and $u_i h^i = 0$ conditions [20].

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3. Field equation for $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model in Bianchi V universe

If we choose $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ in Eq. (4), we reconstruct the $f(R,T)$ gravity model as follows:

$$\alpha_1 R_{ik} - \frac{\alpha_1}{2} R g_{ik} = \left(8\pi - \alpha_2 f_3'\right) T_{ik} - \alpha_1 \Xi_{ik} + \frac{\alpha_2 f_3}{2} g_{ik} - \Lambda g_{ik} \tag{8}$$

where $f_3' = \frac{df_3(T)}{dT}$. From Eqs. (6)–(8) we get the reconstructed $f(R,T)$ field equations as follows:

$$\alpha_1 \left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2}\right) = -8\pi p + 4\pi h^2 + \frac{\alpha_2 f_3' h^2}{2} + \frac{\alpha_2}{2} f_3 + \Lambda, \tag{9}$$

$$\alpha_1 \left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2}\right) = -8\pi p - 4\pi h^2 - \frac{\alpha_2 f_3' h^2}{2} + \frac{\alpha_2}{2} f_3 + \Lambda, \tag{10}$$

$$\alpha_1 \left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2}\right) = -8\pi p - 4\pi h^2 - \frac{\alpha_2 f_3' h^2}{2} + \frac{\alpha_2}{2} f_3 + \Lambda, \tag{11}$$

$$\alpha_1 \left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3m^2}{A^2}\right) = 8\pi \rho + 4\pi h^2 + (p + \rho + \frac{h^2}{2})\alpha_2 f_3' + \frac{\alpha_2 f_3}{2} + \Lambda, \tag{12}$$

$$2m\frac{\dot{A}}{A} - m\frac{\dot{B}}{B} - m\frac{\dot{C}}{C} = 0. \tag{13}$$

In this study, we have five field equations and seven unknowns. In order to solve Eqs. (9)–(13) we can use the deceleration parameter:

$$q = -\frac{V\ddot{V}}{\dot{V}^2} = constant \tag{14}$$

If we solve Eq. (14), we get metric potential as follows:

$$A = \frac{((q+1)(at+b))^{\frac{3}{q+1}}}{BC} \tag{15}$$

where a and b are integral constants. From Eqs. (9)–(13) and (15), we get metric potentials $A(t)$, $B(t)$, and $C(t)$ as follows:

$$A = \frac{((q+1)(at+b))^{\frac{3}{q+1}}}{c_1} \tag{16}$$

$$B = (q+1)^{\frac{2}{q+1}} (at+b)^{\frac{1}{q+1}} e^{\frac{(q+1)\left(c_3 - c_2 a^{\frac{3}{q+1}} (at+b)^{\frac{q-2}{q+1}}\right) + a}{a(q-2)}} \tag{17}$$

and

$$C = c_1 (at+b)^{\frac{1}{q+1}} e^{\frac{(q+1)\left(c_2 a^{\frac{3}{q+1}} (at+b)^{\frac{q-2}{q+1}} - c_3\right) - a}{a(q-2)}} \tag{18}$$

Here c_1, c_2, and c_3 are integral constants. From Eqs. (1), (9)–(13), and (16)–(18) we get the values of the pressure, energy density, magnetic field, and cosmological constant for the reconstructed $f(R,T)$ theory in the Bianchi type V universe model as follows:

$$p = \frac{\alpha_1 a^2}{2(q+1)(\alpha_2 f_3' + 8\pi)(at+b)^2} - \frac{\alpha_1 c_2^2 a^{\frac{6}{q+1}}}{2(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{\alpha_1 c_1^2 m^2}{2(q+1)^{\frac{2}{q+1}}(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{2}{q+1}}} - B_c \quad (19)$$

$$\rho = \frac{3\alpha_1 a^2}{2(q+1)(\alpha_2 f_3' + 8\pi)(at+b)^2} - \frac{3\alpha_1 c_2^2 a^{\frac{6}{q+1}}}{2(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{3\alpha_1 c_1^2 m^2}{2(q+1)^{\frac{2}{q+1}}(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{2}{q+1}}} + B_c \quad (20)$$

$$h^2 = 0, \quad (21)$$

and cosmological constant

$$\Lambda = \frac{\alpha_1 c_2^2 (\alpha_2 f_3' + 4\pi) a^{\frac{6}{q+1}}}{(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{\alpha_1 c_1^2 m^2 (\alpha_2 f_3' + 12\pi)}{(q+1)^{\frac{2}{q+1}}(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{2}{q+1}}} - \frac{\alpha_1 \alpha_2 a^2 f_3' (2q-1)}{(q+1)^2 (\alpha_2 f_3' + 8\pi)(at+b)^2}$$

$$- \frac{12\alpha_1 a^2 \pi (q-1)}{(q+1)^2 (\alpha_2 f_3' + 8\pi)(at+b)^2} - 8\pi B_c - \frac{\alpha_2 f_3}{2} \quad (22)$$

In the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, if we take $\alpha_1 = 1$, $\alpha_2 = 2\mu$, and $f_3(T) = T$, we get the first model of Harko et al. [1]. From Eqs. (19)–(22) we obtain the pressure, energy density, magnetic field, and cosmological constant as follows:

$$p = \frac{a^2}{4(q+1)(\mu + 4\pi)(at+b)^2} - \frac{c_2^2 a^{\frac{6}{q+1}}}{4(\mu + 4\pi)(at+b)^{\frac{6}{q+1}}} - \frac{c_1^2 m^2}{4(\mu + 4\pi)(q+1)^{\frac{2}{q+1}}(at+b)^{\frac{2}{q+1}}} - B_c \quad (23)$$

$$\rho = \frac{3a^2}{4(q+1)(\mu + 4\pi)(at+b)^2} - \frac{3c_2^2 a^{\frac{6}{q+1}}}{4(\mu + 4\pi)(at+b)^{\frac{6}{q+1}}} - \frac{3c_1^2 m^2}{4(\mu + 4\pi)(q+1)^{\frac{2}{q+1}}(at+b)^{\frac{2}{q+1}}} + B_c \quad (24)$$

$$h^2 = 0, \quad (25)$$

and cosmological constant

$$\Lambda = \frac{c_2^2 (\mu + 2\pi) a^{\frac{6}{q+1}}}{(\mu + 4\pi)(at+b)^{\frac{6}{q+1}}} - \frac{c_1^2 m^2 (\mu + 6\pi)}{(q+1)^{\frac{2}{q+1}}(\mu + 4\pi)(at+b)^{\frac{2}{q+1}}} - \frac{\mu a^2 (2q-1)}{(q+1)^2 (\mu + 4\pi)(at+b)^2}$$

$$- \frac{6a^2 \pi (q-1)}{(q+1)^2 (\mu + 4\pi)(at+b)^2} - 4(2\pi + \mu) B_c \quad (26)$$

Using $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$ in the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we have the second model of Harko et al. [1]. If we use Eqs. (19)–(22), we get the pressure, energy density, magnetic field, and cosmological constant as follows:

$$p = \frac{\mu a^2}{2(q+1)(\mu + 8\pi)(at+b)^2} - \frac{\mu c_2^2 a^{\frac{6}{q+1}}}{2(\mu + 8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{\mu c_1^2 m^2}{2(q+1)^{\frac{2}{q+1}}(\mu + 8\pi)(at+b)^{\frac{2}{q+1}}} - B_c \quad (27)$$

$$\rho = \frac{3\mu a^2}{2(q+1)(\mu+8\pi)(at+b)^2} - \frac{3\mu c_2^2 a^{\frac{6}{q+1}}}{2(\mu+8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{3\mu c_1^2 m^2}{2(q+1)^{\frac{2}{q+1}}(\mu+8\pi)(at+b)^{\frac{2}{q+1}}} + B_c \qquad (28)$$

$$h^2 = 0, \qquad (29)$$

and cosmological constant

$$\Lambda = \frac{\mu c_2^2 (\mu+4\pi) a^{\frac{6}{q+1}}}{(\mu+8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{\mu c_1^2 m^2 (\mu+12\pi)}{(q+1)^{\frac{2}{q+1}}(\mu+8\pi)(at+b)^{\frac{2}{q+1}}} - \frac{\mu^2 a^2 (2q-1)}{(q+1)^2 (\mu+8\pi)(at+b)^2}$$
$$- \frac{12\mu\, a^2 \pi (q-1)}{(q+1)^2 (\mu+8\pi)(at+b)^2} - 4(2\pi+\mu) B_c \qquad (30)$$

4. Field equation for $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model in Bianchi I universe

If we take $m=0$ in Eq. (6), we get the Bianchi type I universe as follows:

$$ds^2 = -A(t)^2 dx^2 - B(t)^2 dy^2 - C(t)^2 dz^2 + dt^2 \qquad (31)$$

If we use Eqs. (9)–(12) and (14) we get same the metric potentials in Eqs. (16)–(18). In this case, the values of pressure, energy density, magnetic field, and cosmological constant for the reconstructed $f(R,T)$ theory in the Bianchi type I universe model are as follows:

$$p = \frac{\alpha_1 a^2}{2(q+1)(\alpha_2 f_3' + 8\pi)(at+b)^2} - \frac{\alpha_1 c_2^2 a^{\frac{6}{q+1}}}{2(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{6}{q+1}}} - B_c \qquad (32)$$

$$\rho = \frac{3\alpha_1 a^2}{2(q+1)(\alpha_2 f_3' + 8\pi)(at+b)^2} - \frac{3\alpha_1 c_2^2 a^{\frac{6}{q+1}}}{2(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{6}{q+1}}} + B_c \qquad (33)$$

$$h^2 = 0, \qquad (34)$$

and cosmological constant

$$\Lambda = \frac{\alpha_1 c_2^2 (\alpha_2 f_3' + 4\pi) a^{\frac{6}{q+1}}}{(\alpha_2 f_3' + 8\pi)(at+b)^{\frac{6}{q+1}}} - \frac{\alpha_1 \alpha_2 a^2 f_3' (2q-1)}{(q+1)^2 (\alpha_2 f_3' + 8\pi)(at+b)^2} - \frac{12\alpha_1 a^2 \pi (q-1)}{(q+1)^2 (\alpha_2 f_3' + 8\pi)(at+b)^2} - 8\pi B_c - \frac{\alpha_2 f_3}{2} \qquad (35)$$

In the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, if we take $\alpha_1 = 1$, $\alpha_2 = 2\mu$, and $f_3(T) = T$, we get the first model of Harko et al. [1]. Using Eqs. (32)–(35), we get the pressure, energy density, and string tension density as follows:

$$p = \frac{a^2}{4(q+1)(\mu+4\pi)(at+b)^2} - \frac{c_2^2 a^{\frac{6}{q+1}}}{4(\mu+4\pi)(at+b)^{\frac{6}{q+1}}} - B_c \qquad (36)$$

$$\rho = \frac{3a^2}{4(q+1)(\mu+4\pi)(at+b)^2} - \frac{3c_2^2 a^{\frac{6}{q+1}}}{4(\mu+4\pi)(at+b)^{\frac{6}{q+1}}} + B_c \qquad (37)$$

$$h^2 = 0, \tag{38}$$

and cosmological constant

$$\Lambda = \frac{c_2^2 \left(\mu + 2\pi\right) a^{\frac{6}{q+1}}}{\left(\mu + 4\pi\right)\left(at + b\right)^{\frac{6}{q+1}}} - \frac{\mu a^2 \left(2q - 1\right)}{\left(q + 1\right)^2 \left(\mu + 4\pi\right)\left(at + b\right)^2} - \frac{6 a^2 \pi \left(q - 1\right)}{\left(q + 1\right)^2 \left(\mu + 4\pi\right)\left(at + b\right)^2} - 4\left(2\pi + \mu\right) B_c \tag{39}$$

Using $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$ in the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we have the second model of Harko et al. [1]. From Eqs. (32)–(35) we have pressure, energy density, magnetic field, and cosmological constant as follows:

$$p = \frac{\mu a^2}{2(q+1)(\mu + 8\pi)(at + b)^2} - \frac{\mu c_2^2 a^{\frac{6}{q+1}}}{2\left(\mu + 8\pi\right)\left(at + b\right)^{\frac{6}{q+1}}} - B_c \tag{40}$$

$$\rho = \frac{3\mu a^2}{2(q+1)(\mu + 8\pi)(at + b)^2} - \frac{3\mu c_2^2 a^{\frac{6}{q+1}}}{2\left(\mu + 8\pi\right)\left(at + b\right)^{\frac{6}{q+1}}} + B_c \tag{41}$$

$$h^2 = 0, \tag{42}$$

and cosmological constant

$$\Lambda = \frac{\mu c_2^2 \left(\mu + 4\pi\right) a^{\frac{6}{q+1}}}{\left(\mu + 8\pi\right)\left(at + b\right)^{\frac{6}{q+1}}} - \frac{\mu^2 a^2 \left(2q - 1\right)}{\left(q + 1\right)^2 \left(\mu + 8\pi\right)\left(at + b\right)^2} - \frac{12\mu a^2 \pi \left(q - 1\right)}{\left(q + 1\right)^2 \left(\mu + 8\pi\right)\left(at + b\right)^2} - 4\left(2\pi + \mu\right) B_c. \tag{43}$$

5. Discussion

To overcome the expansion of the universe, Harko et al. [1] suggested an alternative gravitation theory, namely $f(R,T)$ gravity. In this theory the $f(R,T)$ function is presented as $f(R,T) = R + 2\mu T$, $f(R,T) = \mu R + \mu T$ (here μ is a constant), and $f(R,T) = f_1(R) + f_2(R) f_3(T)$ [1]. In this study, we have researched the reconstructed $f(R,T)$ model for MSQM in Bianchi type I and V universes with cosmological constant. In this research, we proposed a new $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model for the solutions of modified Einstein field equations in $f(R,T)$ gravitation theory. The variations of cosmic pressure (p), energy density (ρ), and cosmological constant (Λ) with time for the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model in Bianchi I and V universes are given in Figures 1–3. If we take $\alpha_1 = 1$, $\alpha_2 = 2\mu$, and $f_3(T) = T$, we get the first model of Harko et al., i.e. $f(R,T) = R + 2\mu T$ [1]. For $\alpha_1 = \alpha_2 = \mu$ and $f_3(T) = T$, we obtain the second model of Harko et al., i.e. $f(R,T) = \mu R + \mu T$ [1]. For these reasons, this model includes the first and second models of Harko et al. [1]. If $\alpha_1 = 1$ and $f_3(T) = 0$, the $f(R,T)$ gravitation theory is reduced to the general relativity theory. For the solutions of the modified field equations, we have used a constant deceleration parameter, which is an important parameter that determines the fate of the universe. From Eqs. (21), (25), (29), (34), (38), and (42), we get zero magnetic field for Bianchi type I and V universes in all $f(R,T)$ gravity models. From this result, we could say that a magnetic field is not observed in anisotropic and homogeneous cosmological models in $f(R,T)$ theory. From Eqs. (19), (20), (23), (24), (27), and (28) for the Bianchi type V universe and also from Eqs. (32), (33), (36), (37), (40), and (41) for the Bianchi type I universe model we obtain dark energy solutions ($p = -\rho$) with $t \to \infty$. We get the constant and same cosmological constant value, i.e. $\Lambda = -4\left(\mu + 2\pi\right) B_c$, in Eqs. (22),

(26), and (30) for the Bianchi type V universe and in Eqs. (35), (39), and (43) for the Bianchi type I universe models. These results agree with Aktaş and Aygün's [5] solutions in $f(R,T)$ theory. The bag constant B_c is effective on the cosmological constant. While bag constant B_c causes a decrease in pressure, it causes an increase in energy density for all $f(R,T)$ models.
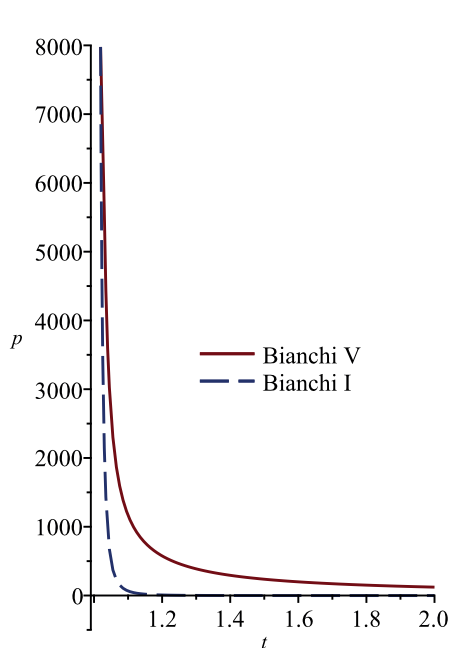

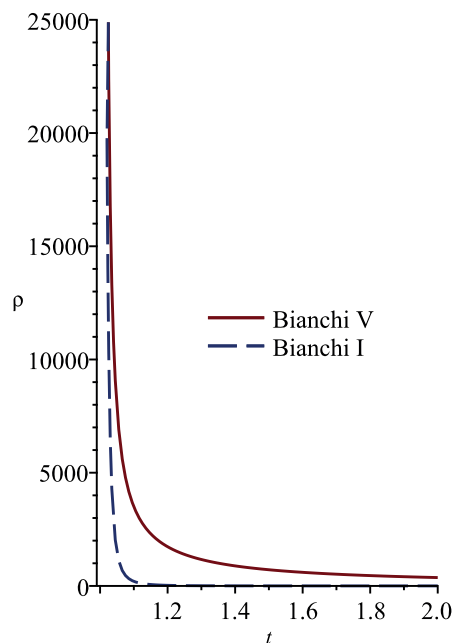**Figure 1.** The variation of cosmic pressure in reconstructed $f(R,T)$ theory for Bianchi V and I universes.

Figure 2. The variation of energy density in reconstructed $f(R,T)$ theory for Bianchi V and I universes.
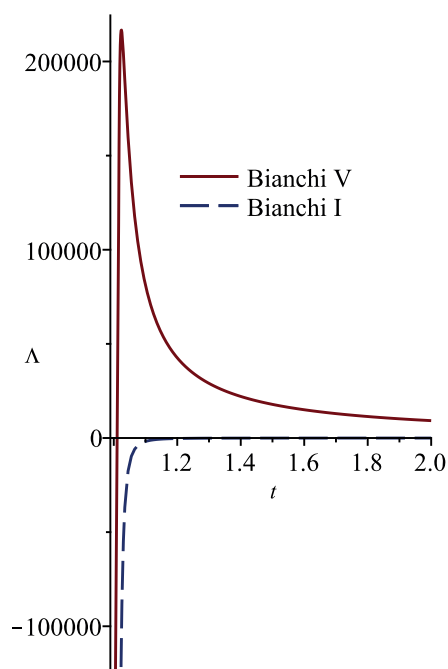

Figure 3. The variation of cosmological constant in reconstructed $f(R,T)$ theory for Bianchi V and I universes.

If we take $\alpha_1 = 1$ and $f_3(T) = 0$, in the $f(R,T) = \alpha_1 R + \alpha_2 f_3(T)$ model, we get general relativity solutions. The pressure (p), energy density (ρ), magnetic field (h^2), and cosmological constant (Λ) are given for Bianchi type I and V universe models in the Table. From the Table we easily see that $h^2 = 0$ in general relativity for Bianchi type I and V universe models. For $t \to \infty$, we get the dark energy model in general relativity. The cosmological constant decreases with time for Bianchi type I and V universe models in general relativity. When $t \to \infty$, we obtain $\Lambda = -8\pi B_c$ for Bianchi type I and V universe models in general relativity. These results agree with $f(R,T)$ solutions.

Table. Values of pressure, energy density, magnetic field, and cosmological constant in general relativity theory for Bianchi V and I universes.

	Bianchi V	Bianchi I
p	$\dfrac{a^2}{16\pi(q+1)(at+b)^2} - \dfrac{c_2^2 a^{\frac{6}{q+1}}}{16\pi(at+b)^{\frac{6}{q+1}}} - \dfrac{c_1^2 m^2}{16\pi(q+1)^{\frac{2}{q+1}}(at+b)^{\frac{2}{q+1}}} - B_c$	$\dfrac{a^2}{16\pi(q+1)(at+b)^2} - \dfrac{c_2^2 a^{\frac{6}{q+1}}}{16\pi(at+b)^{\frac{6}{q+1}}} - B_c$
ρ	$\dfrac{3a^2}{16\pi(q+1)(at+b)^2} - \dfrac{3c_2^2 a^{\frac{6}{q+1}}}{16\pi(at+b)^{\frac{6}{q+1}}} - \dfrac{3c_1^2 m^2}{16\pi(q+1)^{\frac{2}{q+1}}(at+b)^{\frac{2}{q+1}}} + B_c$	$\dfrac{3a^2}{16\pi(q+1)(at+b)^2} - \dfrac{3c_2^2 a^{\frac{6}{q+1}}}{16\pi(at+b)^{\frac{6}{q+1}}} - B_c$
h^2	0	0
Λ	$\dfrac{c_2^2 a^{\frac{6}{q+1}}}{2(at+b)^{\frac{6}{q+1}}} - \dfrac{3c_1^2 m^2}{2(q+1)^{\frac{2}{q+1}}(at+b)^{\frac{2}{q+1}}} - \dfrac{3a^2(q-1)}{2(q+1)^2(at+b)^2} - 8\pi B_c$	$\dfrac{c_2^2 a^{\frac{6}{q+1}}}{2(at+b)^{\frac{6}{q+1}}} - \dfrac{3a^2(q-1)}{2(q+1)^2(at+b)^2} - 8\pi B_c$

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