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Second law of gravitational thermodynamics in the locally rotationally symmetric Bianchi type-II universe

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Abstract: In this work, under the assumption that the universe is locally rotationally symmetric (LRS) Bianchi type-II and filled with anisotropic dark fluid (dark energy and dark radiation), we investigate whether the generalized second law of thermodynamics (GSLT) is still valid. We also consider another important condition for LRS Bianchi type-II spacetimes whereby the model expansion θ is proportional to the shear σ, which leads to $B = A^n$ (where n is a constant and A and B are the metric potentials). Based on this, we derive a general circumstance for the GSLT and find that its validity depends on the anisotropies of both the dark fluid and the metric potentials. We conclude that the GSLT still holds at all times even if these two anisotropic conditions are removed from the results.

Key words: Dark fluid, thermodynamics, Bianchi II

1. Introduction

Although general relativity is now more than 100 years old, there remain many unexplained topics within its theoretical framework. The expansion of the universe has been explained and measured on the basis of numerous cosmological observations and tools, including type-Ia supernova luminosities, temperature fluctuations in the cosmic microwave background radiation, baryon acoustic oscillations, measurements of the universe's large-scale structure, and the Hubble constant and the Sloan Digital Sky Survey [1–5]. Following the major results of these studies, modified gravitation theories have gained significant interest as an alternative to dark energy [6–18] in answering the question 'Why is the universe expanding faster than it should be?' There are currently three primary models for explaining this phenomenon: a cosmological constant, the presence of dark energy, and modified gravity. Spacetimes in which dark energy interacts with dark and other cosmic matter are well understood [19–23]. Observations and theory suggest that our spatially flat universe comprises 76% dark energy, 20% dark matter, and 4% other cosmic matter. To the extent that they are supported by observational evidence, conventional theories of dark energy remain attractive [24–34].

Gravitational thermodynamics is another area of great interest in modern cosmology. It is well understood that the Clausius relation $-dE{=}T_A dS_A$, where (T_A) is the Hawking temperature and S_A is the Bekenstein–Hawking entropy, can be used to derive the Friedmann equation from the first law of thermodynamics. Here $T_A = 1/(2\pi r_A)$, where r_A is the apparent horizon, and S_A is proportional to the area of the horizon and Newton's constant of gravitation [35,36]. Furthermore, the first law of thermodynamics is satisfied by the temperature, entropy, and mass of a black hole [37], and it has been shown that the generalized second law of

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thermodynamics (GSLT) agrees with the cosmological holographic principle for isotropic flat and open spacetime models [38]. Wu et al. showed further that, under Lovelock and braneworld gravity, the GSLT can always be satisfied [15], and Sadjadi pointed out that the GSLT must be satisfied by the modified Gauss–Bonnet theory of gravity (f(R,G)) derived from a quasi-de Sitter model and by a universe with power law expansion [39]. In f(T) gravity, the GSLT is satisfied from early to present times but is violated for special ranges of the torsion scalar [40]. Black hole thermodynamics play an important role in current modifications of Einstein's field equations and are well documented in the literature [41–49].

Based on the above, we will develop a concise dark energy scenario in the next section. In Section 3, we will investigate the validity of the GSLT in an LRS Bianchi type-II spacetime model, and a discussion will be provided in the final section.

2. Preliminaries: dark energy scenario

The generalized energy-momentum tensor defining an anisotropic dark fluid is given as

$$T^{\mu}_{\nu}=T^{\mu(m)}_{\nu}+T^{\mu(e)}_{\nu}+T^{\mu(r)}_{\nu}, \tag{1}$$

where dark matter, anisotropic dark energy, and anisotropic dark radiation are represented by m, e, and r, respectively. Based on this, the nonvanishing energy-momentum tensors are given as follows:

$$T^{\mu(m)}_{\nu}=(-\rho^m P^m P^m P^m), \tag{2}$$

$$T^{\mu(e)}_{\nu}=(-\rho^e P^e_x P^e_y P^e_z), \tag{3}$$

$$T^{\mu(r)}_{\nu}=(-\rho^r P^r_x P^r_y P^r_z). \tag{4}$$

The equations of state for these components can be written using the following relations:

$$P^m=\rho^m\omega^m \tag{5}$$

$$P^e_i=\rho^e\omega^e_i \tag{6}$$

$$P^r_i=\rho^r P\omega^r_i \tag{7}$$

using the space coordinate indices 'i=xyz'. By defining four distinct skewness parameters, the anisotropy of pressure densities can be easily parameterized as follows [50]:

$$\lambda_1=\frac{1}{3\rho^e}(P^e_x-P^e_y), \quad \lambda_2=\frac{1}{3\rho^e}(P^e_z-P^e_x), \tag{8}$$

$$\eta_1=\frac{1}{3\rho^r}(P^r_x-P^r_y), \quad \eta_2=\frac{1}{3\rho^r}(P^r_z-P^r_x). \tag{9}$$

The deviation in pressure measured by these four parameters is along the y- and z-axes, away from the x-axis. These parameters can be used to obtain the following stress-energy tensors [51]:

$$T^{\mu(e)}_{\nu}=[-\rho^e\omega^e\rho^e,(\omega^e+3\lambda_1)\rho^e,(\omega^e+3\lambda_2)\rho^e], \tag{10}$$

$$T_{\nu}^{\mu(r)}= [-\rho^r \omega^r \rho^r, (\omega^r+3\eta_1)\rho^r, (\omega^r+3\eta_2)\rho^r]. \tag{11}$$

The Einstein field equations are given in terms of the generalized stress-energy tensor $T_{\mu\nu}$ (assuming $8\pi G= 1$ and $c= 1$) as follows:

$$G_{\mu}^{\nu}= -T_{\mu}^{\nu} \tag{12}$$

The Bianchi type-II model plays an important role in constructing cosmological spacetimes suitable for describing the early stages of the evolution of the universe [52], and the fundamental roles of Bianchi type-II universes were emphasized by Asseo and Sol [53]. Kumar and Akarsu [54] showed that the anisotropy of the dark energy is constrained to be axially symmetric considering Bianchi type-II models.

In our calculations, we consider the following line-element describing an LRS Bianchi type-II universe [55]:

$$ds^2= -dt^2+B^2(dx+zdy)^2+A^2(dy^2+dz^2), \tag{13}$$

where the scale factors A and B depend on the time coordinate t. Based on this, we have the following metric tensor:

$$g_{\mu\nu}= -\delta_{\mu}^0\delta_{\nu}^0+B^2\delta_{\mu}^1\delta_{\nu}^1+\left(A^2+z^2B^2\right)\delta_{\mu}^2\delta_{\nu}^2+A^2\delta_{\mu}^3\delta_{\nu}^3+zB^2\left[\delta_{\mu}^0\delta_{\nu}^2+\delta_{\mu}^2\delta_{\nu}^0\right] \tag{14}$$

from which the nonvanishing components of the Einstein tensor,

$$G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu} \tag{15}$$

are calculated as follows:

$$G_0^0= -\frac{\dot{A}^2}{A^2}+\frac{B^2}{4A^2}-\frac{2\dot{A}\dot{B}}{AB} \tag{16}$$

$$G_1^1=\frac{3B^2}{4A^4}-\left(\frac{\dot{A}}{A}\right)^2-\frac{2\ddot{A}}{A} \tag{17}$$

$$G_2^1=\frac{3zB^2}{A^4}-z\left(\frac{\dot{A}}{A}\right)^2+\frac{z\dot{A}\dot{B}}{AB}-\frac{z\ddot{A}}{A}+\frac{z\ddot{B}}{B}, \tag{18}$$

$$G_2^2= -\frac{B^2}{4A^4}-\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B} \tag{19}$$

$$G_3^3= -\frac{B^2}{4A^4}-\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B} \tag{20}$$

Considering the line-element of Eq. (13), it is possible to easily obtain the following system of equations:

$$-\frac{\dot{A}^2}{A^2}+\frac{B^2}{4A^2}-\frac{2\dot{A}\dot{B}}{AB}=\rho^m+\rho^e+\rho^r \tag{21}$$

$$\left(\frac{\dot{A}}{A}\right)^2-\frac{3B^2}{4A^4}+\frac{2\ddot{A}}{A}=\omega^m\rho^m+\omega^e\rho^e+\omega^r\rho^r \tag{22}$$

$$\frac{B^2}{4A^4}+\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}=\omega^m\rho^m+(\omega^e+3\lambda_1)\rho^e+(\omega^r+3\eta_1)\rho^r \tag{23}$$

$$\frac{B^2}{4A^4}+\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}=\omega^m\rho^m+\left(\omega^r+3\eta_2\right)\rho^r+\left(\omega^e+3\lambda_2\right)\rho^e \tag{24}$$

where single and double overdotting of the scale factors A and B indicate, respectively, first and second differentiation with respect to cosmic time t.

Next, the continuity equations based on $T^{\mu\nu}_{;\nu}=0$ in the LRS Bianchi II spacetimes are given as:

$$\dot{\rho}^m+3H(1+\omega^m)\rho^m=\Sigma, \tag{25}$$

$$\dot{\rho}^e+3H(1+\omega^e+\Gamma_1)\rho^e=-\Sigma', \tag{26}$$

$$\dot{\rho}^r+3H(1+\omega^r+\Gamma_2)\rho^r=\Sigma'-\Sigma. \tag{27}$$

Here, we have defined an average scale factor as

$$R(t)=(A^2B)^{1/3} \tag{28}$$

the Hubble parameter as

$$H=\frac{\dot{R}}{R} \tag{29}$$

and

$$\Gamma_1=\frac{\dot{A}}{AH}(\lambda_1+\lambda_2),\quad \Gamma_2=\frac{\dot{A}}{AH}(\eta_1+\eta_2). \tag{30}$$

We further introduce Σ and Σ' to reflect the mutual interaction of two principal components of the universe. Assuming [56]

$$\Sigma=\Xi^m\rho^e,\quad \Sigma'=\Xi^e\rho^e \tag{31}$$

we can take the ratios of the energy densities [51] as

$$\alpha_1=\frac{\rho^m}{\rho^e},\quad \alpha_2=\frac{\rho^r}{\rho^e} \tag{32}$$

Using these definitions, the continuity equations become

$$\dot{\rho}^m+3H(1+\omega^m_{``eff"})\rho^m=0, \tag{33}$$

$$\dot{\rho}^e+3H(1+\omega^e_{``eff"})\rho^e=0, \tag{34}$$

$$\dot{\rho}^r+3H(1+\omega^r_{``eff"})\rho^r=0, \tag{35}$$

where

$$\omega^m_{``eff"}=\omega^m-\frac{\Xi^m}{3H\alpha_1} \tag{36}$$

$$\omega^{e}_{"eff"}=\omega^{e}+\frac{\dot{A}}{AH}(\lambda_1+\lambda_2)+\frac{\Xi^{e}}{3H} \tag{37}$$

$$\omega^{r}_{"eff"}=\omega^{r}+\frac{\dot{A}}{AH}(\eta_1+\eta_2)+\frac{\Xi^{m}-\Xi^{e}}{3H\alpha_2} \tag{38}$$

3. Generalized second law of thermodynamics

In 1973, Bekenstein showed that the thermodynamics of a black hole are related to the size and dynamics of its event horizon [57]. He suggested that the area of the event horizon is a function of the black hole's entropy. In 2006, Wang et al. [58] investigated the first and second laws of thermodynamics in an accelerating universe driven by dark energy and demonstrated that both laws are satisfied.

Here, we will assess whether the GSLT is valid in an LRS Bianchi type-II spacetime model bounded by an apparent horizon of size F. In a flat geometry, F coincides with the Hubble horizon:

$$H=\frac{1}{F}. \tag{39}$$

In our galaxy, the ratio of shear to the Hubble parameter is given as [52,59,60]:

$$\frac{\sigma}{H}=0.3. \tag{40}$$

Thus, we can assume that the shear tensor σ_i is proportional to the expansion scalar θ [53]. This condition gives the following relation:

$$B=A^{n}, \tag{41}$$

where n is a constant. In our subsequent calculations, we will consider this important relation.

The first law of thermodynamics is given by

$$TdS=PdV+dE \tag{42}$$

which can be rearranged as

$$dS=\frac{PdV+dE}{T} \tag{43}$$

where S, P, E, and T are the entropy, pressure, internal energy, and temperature of the system, respectively. Time derivation of Eq. (43) yields

$$\dot{S}=\frac{P\dot{V}+\dot{E}}{T} \tag{44}$$

Reverting to the discussion of dark fluid in Section 2, the corresponding time derivatives of the entropies for dark matter, anisotropic dark energy, and anisotropic dark radiation, respectively, become

$$\dot{S}^{m}=\frac{P^{m}\dot{V}+\dot{E}^{m}}{T} \tag{45}$$

$$\dot{S}^{e}=\frac{P^{e}\dot{V}+\dot{E}^{e}}{T} \tag{46}$$

$$\dot{S}^r = \frac{P^r \dot{V} + \dot{E}^r}{T} \tag{47}$$

Here it is assumed that each system component has the same temperature [58]:

$$T = \frac{G_s}{2\pi}, \tag{48}$$

where G_s describes the surface gravity of a black hole. The thermodynamic quantities are defined in terms of the cosmological quantities as follows:

$$P^m = \omega^m_{eff} \rho^m, \quad E^m = R^3 \rho^m \tag{49}$$

$$P^e = \omega^e_{eff} \rho^e, \quad E^e = R^3 \rho^e \tag{50}$$

$$P^r = \omega^r_{eff} \rho^r, \quad E^r = R^3 \rho^r \tag{51}$$

where $V = R^3$ is the volume of the system.

The entropy of the horizon is [51]

$$S_h = \frac{k\Lambda}{4} \tag{52}$$

where Λ is the surface area of the black hole, which is given as

$$\Lambda = 4\pi F^2 \tag{53}$$

and k is the Boltzmann constant. However, as we have

$$\dot{S}_h = 2k\pi F \dot{F} \tag{54}$$

we obtain for the total entropy

$$\dot{S}_{total} = -2k\pi \frac{\dot{H}}{H^3} \tag{55}$$

where $\dot{S}_{total}$ is the entropy of all matter, radiation, and energy sources inside the horizon. Furthermore, Eq. (21) takes the following form:

$$H^2 = -\frac{(n-2)^2}{9(2n+1)} \left[\rho^m + \rho^e + \rho^r + \frac{1}{4} A^{2n-2} \right] \tag{56}$$

Making use of this equation, we obtain

$$\dot{S}_{total} = \frac{k\pi (n-2)^2}{9(2n+1)H^4} \left[\dot{\rho}^m + \dot{\rho}^e + \dot{\rho}^r + \frac{n-1}{2} A^{2n-3} \right] \tag{57}$$

Finally, taking into account Eqs. (33)–(38), we find

$$\dot{S}_{total} = \frac{k\pi (n-2)^2}{9\,(2n+1)\,H^4} [\frac{n-1}{2} A^{2n-3} - 3H\,(\rho + P) + \frac{3\rho^e}{n-2}\,(\lambda_1 + \lambda_2)$$

$$+\frac{3\rho^r}{n-2}(\eta_1+\eta_2)]. \tag{58}$$

This important result implies that if $\dot{S}_{total}\geq 0$, the GSLT holds. Eq. (58) and the condition $\dot{S}_{total}\geq 0$ give us

$$3H(\rho+P) \leq \frac{n-1}{2}A^{2n-3}+\frac{3\rho^e}{n-2}(\lambda_1+\lambda_2)+\frac{3\rho^r}{n-2}(\eta_1+\eta_2). \tag{59}$$

It is easy to see that the validity of the GSLT depends on the anisotropies of the metric potentials and skewness parameters.

Case I: $n = 1$

By choosing $n= 1$, we can remove the anisotropy of the expansion, reducing the validity to dependence on the skewness parameters.

Case II: $n = -1/2$

In this case $\dot{S}_{total}$ goes to infinity, which means that the entropy of universe takes its maximum value. Therefore, the useable energy of the universe will be transformed into an unusable energy form describing a case called the heat death of a system. If we encounter this case, then thermodynamic free energy will be subtracted from our universe and motion or life will be over.

Case III: $n = 2$

Under this condition we get $\dot{S}_{total} = 0$. This means that the GSLT holds all the time and yields the following relationship: $B(t) = A^2(t)$. Moreover, this case corresponds to a reversible adiabatic expansion phase of our universe.

Case IV: $n = 3$

Here, writing $n = 3$ in Eq. (59) and then using it with Eqs. (21)–(24) leads to the following condition between the metric potential and skewness parameters to hold the GSLT:

$$\left[\frac{25}{12}A\dot{A} - 15\left(\frac{\dot{A}}{A}\right)^2 - 20\frac{\dot{A}\ddot{A}}{A^2}\right] \leq \rho^e(\lambda_1+\lambda_2)+\rho^r(\eta_1+\eta_2). \tag{60}$$

It is important to mention here that our results extend the investigation of Mubasher et al. [61] and agree with the previous works by Salti [45] and Sharif et al. [51].

4. Conclusion

Here we focus on the GSLT of an LRS Bianchi type-II spacetime containing dark energy interacting with dark matter and radiation, and we assume that the boundary of spacetime is enclosed by a dynamical apparent horizon with Hawking temperature. In the general case, we conclude that the GSLT in thermal equilibrium depends on the anisotropies of the metric potentials and skewness parameters. We also see that the GSLT is satisfied in both phantom and quintessence phases under some special conditions that include noninteracting cases and isotropic spacetime models. Furthermore, we discuss the specific limiting conditions of the GSLT in order to interpret our results cosmologically. It is concluded that our investigation agrees with some other papers published in the literature previously.

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