Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2017) 41: 436 – 446

doi:10.3906/fiz-1704-14

Marder type universe with bulk viscous string cosmological model in $f(R,T)$ gravity

Sezgin AYGÜN*
Department of Physics, Faculty of Arts and Sciences, Çanakkale Onsekiz Mart University, Çanakkale, Turkey

Received: 12.04.2017 • **Accepted/Published Online:** 12.06.2017 • **Final Version:** 10.11.2017

Abstract: In this study, we have investigated homogeneous and anisotropic Marder space-time with bulk viscous string matter distribution in $f(R,T)$ gravity. For this aim we have used the anisotropy feature $(\frac{\sigma_x^x}{\theta})$ of Marder space-time and a deceleration parameter in two different $f(R,T)$ models. We have obtained bulk viscous matter distribution solutions in $f(R,T)$ gravity. Finally, some kinematical and physical properties are discussed.

Key words: Bulk viscous, string matter, Marder universe, deceleration parameter, $f(R,T)$ gravity

1. Introduction

The universe is accelerating [1,2] with unknown forces and scientists continue to work to identify the causes of the acceleration of the universe [3]. In this context, it is very important to investigate alternative gravitation theories in order to find the causes of acceleration. To solve this problem, Harko et al. suggested the $f(R,T)$ theory in 2011 [4]. This theory has recently been studied by many other scientists [5–13]. Myrzakulov and his colleagues investigated $f(R,T)$ gravitation theory for various matter contents and various space-time models [14–16]. The thermodynamics properties [17], the energy conditions [18], and anisotropic space-times were investigated with scalar field and perfect fluid matter distributions [19] in $f(R,T)$ gravitation theory by Sharif and Zubair. In 2014, Reddy et al. studied the Kantowski–Sachs bulk viscous model including strings in Harko et al.'s [4] new theory of gravity [20]. Kiran and Reddy researched the Bianchi type III universe model with bulk viscous string (BVS) in $f(R,T)$ theory [21]. Reddy et al. studied $f(R,T)$ gravity with Kaluza–Klein space-time and bulk viscosity with strings [22]. The LRS Bianchi type II universe model was also researched in $f(R,T)$ gravity for bulk viscosity and strings [23]. Çağlar and Aygün studied self-creation cosmology for BVS cloud and strange quark matter distribution [24]. A string cloud model with bulk viscosity was also researched in Brans–Dicke cosmology [25]. Samanta et al. investigated BVS models in higher dimensions for the Saez–Ballester theory [26]. Pradhan obtained magnetized BVS solutions with the Λ term [27]. Yadav et al. magnetized BVS solutions for the Bianchi type I universe model in general relativity [28]. The Kantowski-Sachs (KS) BVS model was studied in Lyra geometry [29] and KS BVS solutions in an alternative theory of gravity were researched [30].

The Marder space-time model has some characteristics for sensing the evaluation of the initial universe

*Correspondence: saygun@comu.edu.tr

[31] and viscosity has a significant role in cosmology [21]. In this paper we will research Marder space-time with the $f(R,T)$ theory for the BVS cosmological model.

2. Modified *f(R,T)* theory and Marder space-time

The action of the new modified $f(R,T)$ theory is served by [4]:

$$S=\int\left(\frac{f(R,T)}{16\pi G}+L_m\right)\sqrt{-g}d^4x. \tag{1}$$

Here trace T is the stress-energy tensor of the matter, $T_{\alpha\beta}$, and g is the determinant of metric tensor $g_{\alpha\beta}$ while $f(R,T)$ is a function of the Ricci scalar, R. L_m indicates the Lagrangian [4]. The matter's energy-momentum tensor is given by [4]:

$$T_{\alpha\beta}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\alpha\beta}}, \tag{2}$$

where L_m is the Lagrangian density and depends only on the metric tensor, and the relation is given by [4]:

$$T_{\alpha\beta}=g_{\alpha\beta}L_m-\frac{2\partial L_m}{\partial g^{\alpha\beta}}. \tag{3}$$

By varying the action of $f(R,T)$ given in Eq. (1), we get:

$$(g_{\alpha\beta}\nabla^{\alpha}\nabla_{\alpha}-\nabla_{\alpha}\nabla_{\beta})f_R(R,T)+f_R(R,T)R_{\alpha\beta}-\frac{1}{2}f(R,T)g_{\alpha\beta}$$

$$=-f_T(R,T)(T_{\alpha\beta}+\Xi_{\alpha\beta})+8\pi T_{\alpha\beta}. \tag{4}$$

Here $f_T(R,T)$ indicates the derivative with respect to T and $f_R(R,T)$ indicates the derivative with respect to R [4]. ∇_α is the covariant derivative [4]. We show $\Xi_{\alpha\beta}$ in Eq. (4) as follows [4]:

$$\Xi_{\alpha\beta}=-2T_{\alpha\beta}+g_{\alpha\beta}L_m-2g^{ik}\frac{\partial^2L_m}{\partial g^{\alpha\beta}g^{ik}}. \tag{5}$$

If we contract Eq. (4), the connection is obtained between R and T as follows:

$$f_R(R,\ T)(3\nabla^{\alpha}\nabla_{\alpha}+R)-2f(R,T)=-f_T(R,T)(T+\Xi)+8\pi T. \tag{6}$$

Here $\Xi=g^{\alpha\beta}\Xi_{\alpha\beta}$ [4,8]. Using Eqs. (4) and (6), we get the following equation [4]:

$$\left(R_{\alpha\beta}-\frac{1}{3}Rg_{\alpha\beta}\right)f_R(R,T)+\frac{1}{6}g_{\alpha\beta}f(R,T)=8\pi\left(-\frac{1}{3}Tg_{\alpha\beta}+T_{\alpha\beta}\right)$$

$$-f_T(R,T)\left(T_{\alpha\beta}-\frac{1}{3}Tg_{\alpha\beta}\right)-\left(\Xi_{\alpha\beta}-\frac{1}{3}\Xi g_{\alpha\beta}\right)f_T(R,T)+\nabla_\alpha\nabla_\beta f_R(R,T), \tag{7}$$

and we write the perfect fluid matter distribution as follows:

$$T_{\alpha\beta}=(p+\rho)u_\alpha u_\beta-pg_{\alpha\beta}. \tag{8}$$

Here p is the cosmic pressure, $\rho$is the cosmic density, $u_\alpha$and u_β are four-velocities, and $u_\alpha u^\alpha = 1$ [4]. From Eq. (5), we obtain [4]:

$$\Xi = -(2T_{\alpha\beta} + pg_{\alpha\beta}). \tag{9}$$

According to Harko et al. [4], three models of $f(R,T)$ theory are:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}. \tag{10}$$

In this paper we will research the BVS cosmological model with Marder type universe in two different $f(R,T)$ models. The Marder space-time is given by [32]:

$$ds^2 A(t)^2(dt^2 - dx^2) - B(t)^2dy^2 - C(t)^2dz^2 \tag{11}$$

and the energy momentum tensor of the BVS model is given by [20]:

$$T_{\alpha\beta} = (\rho + \bar{p})u_\alpha u_\beta - \bar{p}g_{\alpha\beta} - \lambda x_\alpha x_\beta \tag{12}$$

where

$$\bar{p} = p - 3\xi\, H = \varepsilon\, \rho \tag{13}$$

and

$$\varepsilon = \varepsilon_0 - \beta \quad (0 \le \varepsilon \le 1) \tag{14}$$

$$p = \varepsilon_0 \rho \tag{15}$$

Here ε_0 and β are constants, ρ is energy density, λ is the string tension density, H is the Hubble parameter, and ξ is the coefficient of bulk viscosity [20]. u_α is the four-velocity vector and x_α is the direction of the string, which satisfies the following [20]:

$$x_\alpha u^\alpha = 0, \quad u_\alpha u^\alpha = -x_\alpha x^\alpha = 1. \tag{16}$$

The kinematical quantities for the Marder space-time cosmological model are given by [31,33]:

$$u_\alpha = (0,0,0,-A), \tag{17}$$

$$\theta = \frac{1}{A}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{18}$$

$$\sigma_x^x = \frac{1}{3A}\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{19}$$

$$\sigma_y^y = \frac{1}{3A}\left(\frac{2\dot{B}}{B} - \frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right), \tag{20}$$

$$\sigma_z^z = \frac{1}{3A}\left(\frac{2\dot{C}}{C} - \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right), \tag{21}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{22}$$

$$V = (ABC)^{\frac{1}{3}}, \tag{23}$$

and

$$q = -\frac{V\ddot{V}}{\dot{V}^2}. \tag{24}$$

In Section 3, we will solve modified Einstein field equations in $f(R,T)$ gravitation theory for the $R+2f(T)$ model, and in Section 4, we will obtain exact solutions of the $f(R,T)$ gravitation theory for the $f_1(R)+f_2(T)$ model with BVS cosmological model matter distribution in Marder's universe model.

3. Modified gravitational field equations for $f(R,T) = R + 2f(T)$ model

For the $f(R,T)$ modified gravitation theory, the gravitational field equations are given by [4]:

$$G_{\alpha\beta} = g_{\alpha\beta}[f(T) + 2p\, f'(T)] + [2f'(T) + 8\pi]\, T_{\alpha\beta}, \tag{25}$$

where the prime indicates differentiation w.r.t. the argument [4]. If we take $f(T) = \mu T$, where μ is a constant, in Eq. (25), we get [4]:

$$G_{\alpha\beta} = [\mu\rho - p\mu]g_{\alpha\beta} + [8\pi + 2\mu]T_{\alpha\beta}, \tag{26}$$

and field equations are given by:

$$\frac{\ddot{C}}{A^2C} + \frac{\ddot{B}}{A^2B} + \frac{\dot{B}\dot{C}}{A^2BC} - \frac{\dot{B}\dot{A}}{A^3B} - \frac{\dot{C}\dot{A}}{A^3C} = -8\pi\lambda - 3\mu\lambda + 3\mu\,\bar{p} - \mu\rho + 8\pi\,\bar{p}, \tag{27}$$

$$\frac{\ddot{C}}{A^2C} + \frac{\ddot{A}}{A^3} - \frac{\dot{A}^2}{A^4} = 3\mu\,\bar{p} - \mu\lambda - \mu\rho + 8\pi\,\bar{p}, \tag{28}$$

$$\frac{\ddot{B}}{A^2B} + \frac{\ddot{A}}{A^3} - \frac{\dot{A}^2}{A^4} = 3\mu\,\bar{p} - \mu\lambda - \mu\rho + 8\pi\,\bar{p}, \tag{29}$$

$$\frac{\dot{B}\dot{C}}{A^2BC} + \frac{\dot{A}\dot{B}}{A^3B} + \frac{\dot{A}\dot{C}}{A^3C} = -8\pi\rho - 3\mu\rho + \mu\,\bar{p} - \mu\lambda. \tag{30}$$

In Eqs. (27)–(30), we have six unknowns, i.e. $\bar{p}$, ρ, λ, A, B, and C. To solve the field equations, we use a homogeneous and anisotropic universe model because the study of homogeneous and anisotropic cosmological models is significant to understand the large-scale behavior of the universe. For the solutions of modified field equations, first we assume that the anisotropy parameter $\frac{\sigma}{\theta} \neq 0$ [31,34–36]. With the characteristic of the anisotropy for Marder space-time, we obtain

$$\frac{\sigma_x^x}{\theta} = \kappa, \tag{31}$$

where κ is a constant and $0 \leq \kappa \leq 1$. From Eqs. (18), (19), and (31) we get

$$A = (B\,C)^m, \tag{32}$$

where $m = \frac{3\kappa+1}{2-3\kappa}$ is a constant. Secondly, we will use the deceleration parameter because the deceleration parameter (q) has a significant effect on the universe. Using this parameter we could say whether our universe accelerates or not [37]. The deceleration parameter is given as follows:

$$q = -\frac{V\ddot{V}}{\dot{V}^2} = const. \tag{33}$$

From Eqs. (32) and (33) we obtain,

$$B = \frac{(at+b)^{\frac{1}{\eta}}}{C}. \tag{34}$$

Here, a, b, and η are constants. Using Eqs. (28), (29), and (32), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = 0. \tag{35}$$

From Eqs. (32)–(35) we get metric potentials A, B, and C as follows:

$$A = (at+b)^{\frac{m}{\eta}}, \tag{36}$$

$$B = e^{\frac{2(at+b)^{\frac{\eta-1}{\eta}} a^{\frac{1}{\eta}} c_1\eta - a - 2c_2\eta}{2a(\eta-1)}} (at+b)^{\frac{1}{2\eta}}, \tag{37}$$

$$C = e^{\frac{-2(at+b)^{\frac{\eta-1}{\eta}} a^{\frac{1}{\eta}} c_1\eta + a + 2c_2\eta}{2a(\eta-1)}} (at+b)^{\frac{1}{2\eta}}. \tag{38}$$

Here c_1 and c_2 are constants. From Eqs. (13)–(15), (27)–(30), and (36)–(38), we obtain ρ, $\bar{p}$, λ, ξ, and p as follows:

$$\rho = \frac{c_1^2 a^{\frac{2}{\eta}}}{2(4\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}} - \frac{a^2(8\pi m + \eta\mu + 4m\mu + 2\pi)}{8(4\pi+\mu)(2\pi+\mu)\eta^2(at+b)^{\frac{2m+2}{\eta}}}, \tag{39}$$

$$\bar{p} = \frac{c_1^2 a^{\frac{2}{\eta}}}{2(4\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}} - \frac{a^2(8\pi\, m\eta + 4m\eta\mu + 4\pi\eta + \eta\mu - 2\pi)}{8(4\pi+\mu)(2\pi+\mu)\eta^2(at+b)^{\frac{2(m+\eta)}{\eta}}}, \tag{40}$$

$$\lambda = \frac{(2m-1)(1-\eta)a^2}{4(4\pi+\mu)\eta^2(at+b)^{\frac{2(m+\eta)}{\eta}}}, \tag{41}$$

$$p = \frac{\varepsilon_0 c_1^2 a^{\frac{2}{\eta}}}{2(4\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}} - \frac{\varepsilon_0 a^2(8\pi m + \eta\mu + 4m\mu + 2\pi)}{8(4\pi+\mu)(2\pi+\mu)\eta^2(at+b)^{\frac{2m+2}{\eta}}}, \tag{42}$$

$$\xi = \frac{a}{(at+b)^{\frac{\eta+2m}{\eta}}} \left(\frac{(\eta-\varepsilon_0)m}{2\eta(m+1)(4\pi+\mu)} + \frac{(4\eta - 2\varepsilon_0 - 2)\pi - \mu\eta(\varepsilon_0-1)}{8\eta(m+1)(4\pi+\mu)(2\pi+\mu)} \right)$$

$$+ \frac{\eta c_1^2(\varepsilon_0-1)(at+b)^{\frac{\eta-2-2m}{\eta}}}{2(m+1)(4\pi+\mu)a^{\frac{\eta-2}{\eta}}}. \tag{43}$$

4. Modified gravitation field equations for the $f(R,T)=f_1(R)+f_2(T)$ model

When we take $f(R,T)=f_1(R)+f_2(T)$ in Eq. (6) with Eqs. (8) and (9), we get the following [4]:

$$f_1'(R)R_{\alpha\beta}-\frac{1}{2}f_1(R)g_{\alpha\beta}+(g_{\alpha\beta}\nabla^{\alpha}\nabla_{\alpha}-\nabla_{\alpha}\nabla_{\beta})f_1'(R)$$

$$=(8\pi+f_2'(T))T_{\alpha\beta}+\left(f_2'(T)p+\frac{1}{2}f_2(T)\right)g_{\alpha\beta}. \tag{44}$$

In $f(R,T)$ theory, if we choose $f_1(R)=\mu R$ and $f_2(T)=\mu T$ in Eq. (44), we get [4]

$$G_{\alpha\beta}=\left(\frac{8\pi+\mu}{\mu}\right)T_{\alpha\beta}+\left(\frac{\rho-p}{2}\right)g_{\alpha\beta}, \tag{45}$$

and we can write the modified field equations for this model as follows:

$$\frac{\ddot{C}}{A^2C}+\frac{\ddot{B}}{A^2B}+\frac{\dot{B}\dot{C}}{A^2BC}-\frac{\dot{B}\dot{A}}{A^3B}-\frac{\dot{C}\dot{A}}{A^3C}=\frac{8\pi(\bar{p}-\lambda)}{\mu}-\lambda+\frac{3\bar{p}-\rho}{2}, \tag{46}$$

$$\frac{\ddot{C}}{A^2C}+\frac{\ddot{A}}{A^3}-\frac{\dot{A}^2}{A^4}=\frac{8\pi\,\bar{p}}{\mu}+\frac{3\bar{p}-\rho}{2}, \tag{47}$$

$$\frac{\ddot{A}}{A^3}-\frac{\dot{A}^2}{A^4}+\frac{\ddot{B}}{A^2B}=\frac{8\pi\,\bar{p}}{\mu}+\frac{3\bar{p}-\rho}{2}, \tag{48}$$

$$\frac{\dot{B}\dot{C}}{A^2BC}+\frac{\dot{A}\dot{B}}{A^3B}+\frac{\dot{A}\dot{C}}{A^3C}=-\frac{8\pi\rho}{\mu}+\frac{\bar{p}-3\rho}{2}. \tag{49}$$

From Eqs. (12)–(15), (36)–(38), and (46)–(49), we get energy density ρ, pressure $\bar{p}$ and string tension density λ, coefficient of bulk viscosity ξ, and p for the second $f(R,T)$ model as follows:

$$\rho=\frac{\mu c_1^2 a^{\frac{2}{\eta}}}{(8\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}}-\frac{(32\pi m+2\mu\eta m+\mu\eta++6m\mu+8\pi+\mu)a^2\mu}{8(4\pi+\mu)\eta^2(8\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}}, \tag{50}$$

$$\bar{p}=\frac{(32\pi\eta m+6\mu\eta m+16\pi\eta+3\eta\mu+2m\mu-8\pi-\mu)a^2\mu}{8\eta^2(4\pi+\mu)(8\pi+\mu)(at+b)^{\frac{2m+2\eta}{\eta}}}+\frac{\mu c_1^2 a^{\frac{2}{\eta}}}{(8\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}}, \tag{51}$$

$$\lambda=\frac{(1-\eta)\,(2m-1)\,a^2\mu}{2\,(8\pi+\mu)\,\eta^2(at+b)^{\frac{2m+2\eta}{\eta}}}, \tag{52}$$

$$p=\frac{\mu c_1^2 a^{\frac{2}{\eta}}\varepsilon_0}{(8\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}}-\frac{(32\pi m+2\mu\eta m+\mu\eta++6m\mu+8\pi+\mu)a^2\mu\varepsilon_0}{8\eta^2(4\pi+\mu)(8\pi+\mu)(at+b)^{\frac{2m+2}{\eta}}}, \tag{53}$$

$$\xi=\frac{\mu a(\eta(2m+1)(16\pi+3\mu-\mu\varepsilon_0)-(6m\varepsilon_0-2m+\varepsilon_0+1)\mu-(32m\varepsilon_0+8\varepsilon_0+8)\pi}{8(m+1)(4\pi+\mu)(8\pi+\mu)(at+b)^{\frac{2m+\eta}{\eta}}}$$

$$+\frac{\eta c_1^2\mu(\varepsilon_0-1)a^{\frac{2-\eta}{\eta}}}{(8\pi+\mu)(m+1)(at+b)^{\frac{2m-\eta+2}{\eta}}}. \tag{54}$$

However, we find the same metric potentials given by Eqs. (36)–(38) in this model.

5. Discussion

The anisotropic and homogeneous space-times play significant roles for describing the early universe and these models have some essential properties for understanding the early universe, such as the formation of galaxies [38]. On the other hand, alternative theories seem to be very appropriate for exploring the accelerating of our universe. Therefore, in this research we have studied BVS matter distribution for Marder space time that is homogeneous and anisotropic in $f(R,T)$ gravity with the $R+2f(T)$ and $f_1(R)+f_2(T)$ models given by [4]. To obtain exact solutions of $f(R,T)$ theory, we have handled the anisotropy feature of the Marder universe model and a deceleration parameter in two models. From Eqs. (36)–(38), the metric in Eq. (11) can be written as:

$$ds^2 = (at+b)^{\frac{2m}{\eta}}(dt^2-dx^2)-(at+b)^{\frac{1}{\eta}}\left(e^{\frac{2(at+b)^{\frac{\eta-1}{\eta}}a^{\frac{1}{\eta}}c_1\eta-a-2c_2\eta}{a(\eta-1)}}dy^2-e^{\frac{-2(at+b)^{\frac{\eta-1}{\eta}}a^{\frac{1}{\eta}}c_1\eta+a+2c_2\eta}{a(\eta-1)}}dz^2\right).$$

Kinematical parameters such as cosmic expansion, shear, Hubble parameter, deceleration, and anisotropy parameters (A_m) are given by

$$\theta=\frac{a(m+1)}{m(at+b)^{\frac{m+\eta}{\eta}}}, \tag{55}$$

$$\sigma^2=\frac{c_1^2a^{\frac{2}{\eta}}}{(at+b)^{\frac{2m+2}{\eta}}}+\frac{a^2(2m-1)^2}{12\eta^2(at+b)^{\frac{2m+2\eta}{\eta}}}, \tag{56}$$

$$H=\frac{a(m+1)}{3\eta(at+b)}, \tag{57}$$

$$q=-\frac{3\eta-m-1}{m+1}, \tag{58}$$

$$A_m=\frac{6\eta^2c_1^2(at+b)^{\frac{2\eta-2}{\eta}}}{(m+1)^2a^{\frac{2\eta-2}{\eta}}}+\frac{(2m-1)^2}{2(m+1)^2}. \tag{59}$$

For $t=0$ all physical and kinematical parameters are constant. When $t\to\infty$, the scalar expansion, shear, and Hubble parameter go to zero in the Marder universe model. The string tension density, coefficient of bulk viscosity, cosmic energy density, and pressure tend to zero when $t\to\infty$ in different $f(R,T)$ models. These results are shown in Figures 1–4 for two different $f(R,T)$ models, i.e. the $R+2f(T)$ and $f_1(R)+f_2(T)$ models. Using Eq. (58), to obtain the superexponential expansion model $(q<-1)$, m must be $m>0$ or $m<-1$. For the accelerating power law expansion model $(-1<q<0)$, m must be $m>-\frac{1}{4}$ or $m<0$. To get the decelerating expansion model $(q>0)$, m must be $m<-\frac{1}{4}$ or $m>-1$ in $f(R,T)$ gravitation theory.

In 2013, Naidu et al. [39] investigated the anisotropic and homogeneous Bianchi type V universe with BVS cosmological model in $f(R,T)$ theory. Kiran and Reddy [21] researched the Bianchi type III anisotropic

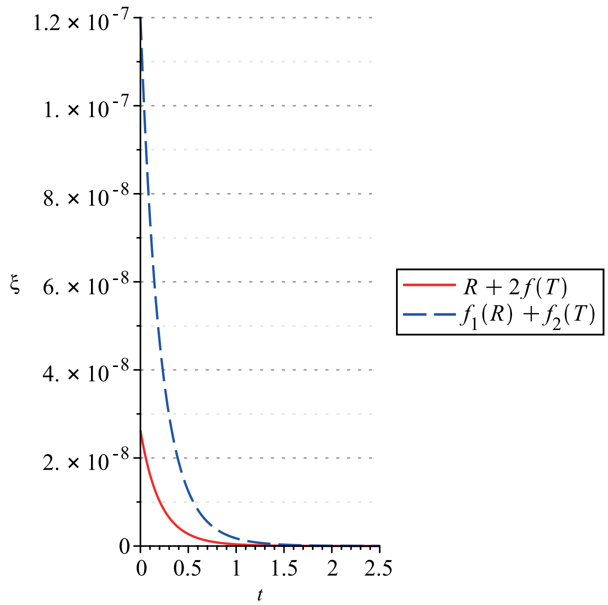

Figure 1. The variation of bulk viscosity for $R + 2f(T)$ and $f_1(R) + f_2(T)$ models.

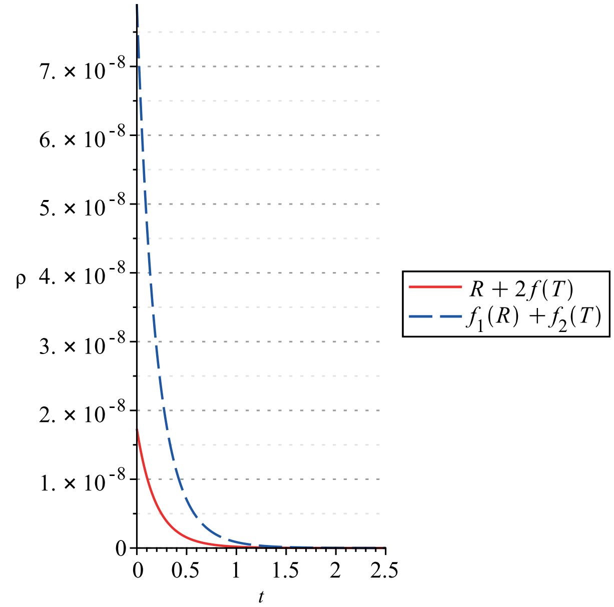

Figure 2. The variation of density for $R + 2f(T)$ and $f_1(R) + f_2(T)$ models.

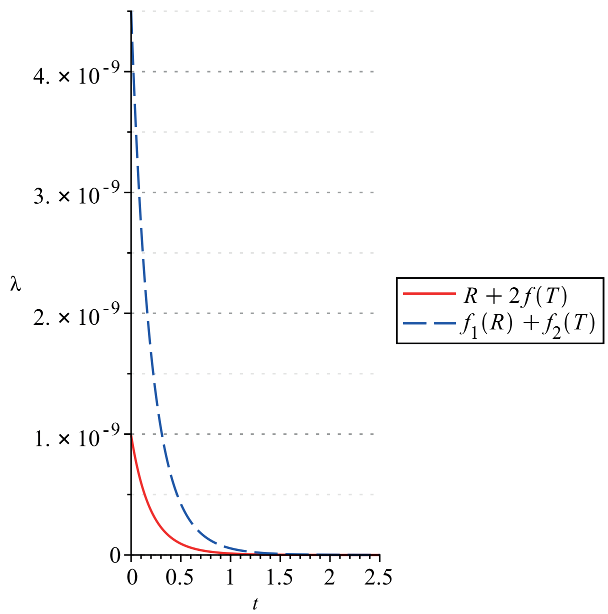

Figure 3. The variation of string tension density for $R + 2f(T)$ and $f_1(R) + f_2(T)$ models.

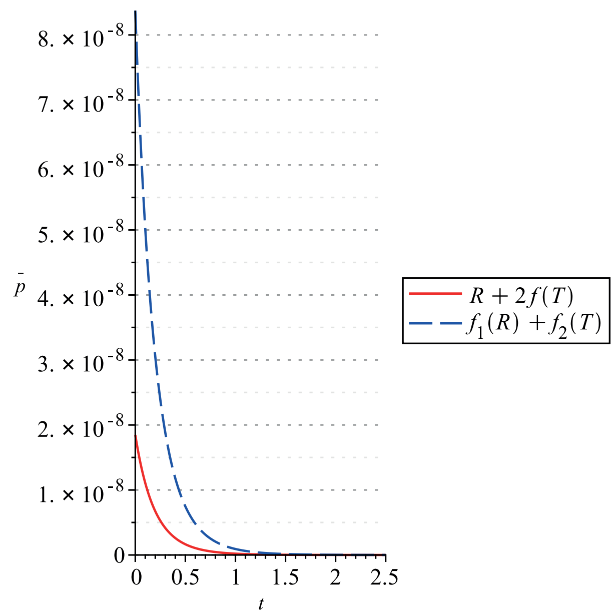

Figure 4. The variation of cosmic pressure for $R + 2f(T)$ and $f_1(R) + f_2(T)$ models.

and homogeneous universe with BVS in $f(R,T)$ gravity and they obtained zero string tension density for homogeneous and anisotropic universe models. In this study we obtain nonzero string tension density for a homogeneous and anisotropic Marder universe. If we take $m = \frac{1}{2}$ or $\eta = 1$ in Eqs. (41) and (52), we obtain zero string tension density in the $R + 2f(T)$ and $f_1(R) + f_2(T)$ models. However, for $\kappa = 0$ we get $m = \frac{1}{2}$ and $\frac{\sigma_x^x}{\theta} = 0$ and our model becomes isotropic. In this situation bulk viscous string energy-momentum solutions transform to just bulk viscous energy-momentum solutions in $f(R,T)$ gravity for Marder space-time. We obtain the same metric potentials for bulk viscous matter in two different $f(R,T)$ gravitation models as follows:

$$A = (at+b)^{\frac{1}{2\eta}}, \tag{60}$$

$$B = (at+b)^{\frac{1}{2\eta}} e^{\frac{2(at+b)^{\frac{\eta-1}{\eta}} a^{\frac{1}{\eta}} c_1 \eta - 2\eta c_2 - a}{2a(\eta-1)}}, \tag{61}$$

$$C = (at+b)^{\frac{1}{2\eta}} e^{\frac{-2(at+b)^{\frac{\eta-1}{\eta}} a^{\frac{1}{\eta}} c_1 \eta + 2\eta c_2 + a}{2a(\eta-1)}}, \tag{62}$$

and other bulk viscous energy-momentum results in $f(R,T)$ theory for $R+2f(T)$ and $f_1(R)+f_2(T)$ models are given below.

5.1. Bulk viscous matter solutions for $R+2f(T)$ model in $f(R,T)$ gravity

In this situation we give the energy density ρ, the coefficient of bulk viscosity ξ, and $\bar{p}$ for bulk viscous matter solutions in the $R+2f(T)$ model as follows:

$$\rho = \frac{c_1^2 a^{\frac{2}{\eta}}}{2(4\pi+\mu)(at+b)^{\frac{3}{\eta}}} - \frac{a^2(6\pi+\eta\mu+2\mu)}{8(4\pi+\mu)(2\pi+\mu)\eta^2(at+b)^{\frac{3}{\eta}}}, \tag{63}$$

$$\bar{p} = \frac{c_1^2 a^{\frac{2}{\eta}}}{2(4\pi+\mu)(at+b)^{\frac{3}{\eta}}} - \frac{a^2(8\pi\eta+3\eta\mu-2\pi)}{8(4\pi+\mu)(2\pi+\mu)\eta^2(at+b)^{\frac{1+2\eta}{\eta}}}, \tag{64}$$

$$\xi = \frac{\eta c_1^2(\varepsilon_0-1)(at+b)^{\frac{\eta-3}{\eta}}}{3(4\pi+\mu)a^{\frac{\eta-2}{\eta}}} + \frac{a}{(at+b)^{\frac{\eta+1}{\eta}}}\left(\frac{(\eta-\varepsilon_0)}{6\eta(4\pi+\mu)} + \frac{(4\eta-2\varepsilon_0-2)\pi-\mu\eta(\varepsilon_0-1)}{12\eta(4\pi+\mu)(2\pi+\mu)}\right). \tag{65}$$

For $\mu=0$, our results transform to general relativity results with bulk viscous matter distribution in the Marder universe as follows:

$$\rho = \frac{c_1^2 a^{\frac{2}{\eta}}}{(8\pi)(at+b)^{\frac{3}{\eta}}} - \frac{3\pi a^2}{32\pi^2\eta^2(at+b)^{\frac{3}{\eta}}}, \tag{66}$$

$$\bar{p} = \frac{c_1^2 a^{\frac{2}{\eta}}}{8\pi(at+b)^{\frac{3}{\eta}}} - \frac{a^2(4\pi\eta-\pi)}{32\pi^2\eta^2(at+b)^{\frac{1+2\eta}{\eta}}}, \tag{67}$$

$$\xi = \frac{\eta c_1^2(\varepsilon_0-1)(at+b)^{\frac{\eta-3}{\eta}}}{3(4\pi)a^{\frac{\eta-2}{\eta}}} + \frac{a}{(at+b)^{\frac{\eta+1}{\eta}}}\left(\frac{(\eta-\varepsilon_0)}{24\eta\pi} + \frac{(2\eta-\varepsilon_0-)\pi}{48\eta\pi^2}\right). \tag{68}$$

5.2. Bulk viscous matter solutions for $f_1(R)+f_2(T)$ model in $f(R,T)$ gravity

In this section we give the energy density ρ, the coefficient of bulk viscosity ξ, and $\bar{p}$ for bulk viscous matter solutions in the $f_1(R)+f_2(T)$ model as follows:

$$\rho = \frac{\mu c_1^2 a^{\frac{2}{\eta}}}{(8\pi+\mu)(at+b)^{\frac{3}{\eta}}} - \frac{(12\pi+\mu\eta+2\mu)a^2\mu}{4\eta^2(4\pi+\mu)(8\pi+\mu)(at+b)^{\frac{3}{\eta}}}, \tag{69}$$

$$\bar{p} = \frac{(16\pi\eta + 3\mu\eta - 4\pi)a^2\mu}{4\eta^2(4\pi+\mu)(8\pi+\mu)(at+b)^{\frac{2\eta+1}{\eta}}} + \frac{\mu c_1^2 a^{\frac{2}{\eta}}}{(8\pi+\mu)(at+b)^{\frac{3}{\eta}}}, \tag{70}$$

$$\xi = \frac{\mu a(\eta(16\pi + 3\mu - \mu\varepsilon_0) - 2\varepsilon_0\mu - 4(3\varepsilon_0+1)\pi}{6(4\pi+\mu)(8\pi+\mu)(at+b)^{\frac{\eta+1}{\eta}}} + \frac{2\eta c_1^2\mu(\varepsilon_0-1)a^{\frac{2-\eta}{\eta}}}{3(8\pi+\mu)(at+b)^{\frac{3-\eta}{\eta}}}. \tag{71}$$

From Eqs. (63)–(71), we see that energy density, the coefficient of bulk viscosity, and pressure decrease with time. This leads to an inflationary model and these results agree with the study by Reddy et al. [20].

Acknowledgment

I thank the anonymous referees for their careful reading of the manuscript and suggestions.

References

[1] Riess, A. G.; Filippenko, A. V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P. M.; Gilliland, R. L.; Hogan, C. J.; Jha, S.; Kirshner, R. P. et al. *Astron. J.* **1998**, *116*, 1009-1038.

[2] Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R. A.; Nugent, P.; Castro, P. G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D. E. et al. *Astrophys. J.* **1999**, *517*, 565-586.

[3] Spergel, D. N.; Verde, L.; Peiris, H. V.; Komatsu, E.; Nolta, M. R.; Bennett, C. L.; Halpern, M.; Hinshaw, G.; Jarosik, N.; Kogut, A. et al. *Astrophys. J. Suppl.* **2003**, *148*, 175-194.

[4] Harko, T.; Lobo, F. S. N.; Nojiri, S.; Odintsov, S. D. *Phys. Rev. D* **2011**, *84*, 024020.

[5] Rao, V. U. M.; Neelima, D. *Eur. Phys. J. Plus* **2013**, *128*, 35-42.

[6] Ahmed, N.; Pradhan, A. *Int. J. Theor. Phys.* **2014**, *53*, 289-306.

[7] Santos, A. F.; Ferst, C. J. *Mod. Phys. Letters A* **2015**, *30*, 1550214.

[8] Singh, G. P.; Bishi, B. K.; Sahoo, P. K. *Int. J. Geom. Methods M.* **2016**, *13*, 1650058-1038.

[9] Shamir, M. F. *Eur. Phys. J. C* **2015**, *75*, 354.

[10] Sofuoğlu, D. *Astrophys. Space Sci.* **2016**, *361*, 12.

[11] Aygün, S.; Aktaş, C.; Yılmaz, İ. *Astrophys. Space Sci.* **2016**, *361*, 380.

[12] Sahoo, P. K.; Mishra, B.; Sahoo, P.; Pacif, S. K. J. *Eur. Phys. J. Plus* **2016**, *131*, 333-312.

[13] Sahoo, P. K. *Fortschr. Phys.* **2016**, *64*, 414-415.

[14] Myrzakulov, R. *Eur. Phys. J. C* **2012**, *72*, 2203-14.

[15] Mubasher, J.; Momeni, D.; Myrzakulov, R. *Chin. Phys. Lett.* **2012**, *29*, 109801.

[16] Momeni, D.; Myrzakulov, R.; Güdekli, E. *Int. J. Geom. Methods M.* **2015**, *12*, 1550101.

[17] Sharif, M.; Zubair, M. *J. Cosmol. Astropart. Phys.* **2012**, *3*, 028.

[18] Sharif, M.; Rani, S.; Myrzakulov, R. *Eur. Phys. J. Plus* **2013**, *128*, 123.

[19] Sharif , M.; Zubair, M. *J. Phys. Soc. Jpn.* **2012**, *81*, 114005.

[20] Reddy, D. R. K.; Anitha, S.; Umadevi, S. *Eur. Phys. J. Plus* **2014**, *129*, 96.

[21] Kiran, M.; Reddy, D. R. K. *Astrophys. Space Sci.* **2013**, *346*, 521-524.

[22] Reddy, D. R. K.; Naidu, R.L.; Dasu Naidu, K.; Ram Prasad, T. *Astrophys. Space Sci.* **2013**, *346*, 261-265.

[23] Reddy, D. R. K.; Naidu, R. L.; Dasu Naidu, K.; Ram Prasad, T. *Astrophys. Space Sci.* **2013**, *346*, 219-223.

[24] Çağlar, H.; Aygün, S. *IOSR J. Math.* **2015**, *11*, 53-59.

[25] Mahanta, K. L.; Biswal, A. K.; Sahoo, P. K. *Eur. Phys. J. Plus* **2014**, *129*, 141.

[26] Samanta, G. C.; Biswal, S. K.; Sahoo, P. K. *Int. J. Theor. Phys.* **2013**, *52*, 1504.

[27] Pradhan, A. *Commun. Theor. Phys.* **2009**, *51*, 367.

[28] Yadav, M. K.; Pradhan, A.; Singh, S. K. *Astrophys. Space Sci.* **2007**, *311*, 423.

[29] SubbaRao, M. V. *Astrophys. Space Sci.* **2015**, *356*, 149-152.

[30] Ramprasad, T.; Naidu, R. L.; Ramana, K.V. *Astrophys. Space Sci.* **2015**, *359*, 3-5.

[31] Aygün, S.; Aktaş, C.; Yılmaz, İ. *J. Geom. Phys.* **2012**, *62*, 100-106.

[32] Marder, L. *Proc. R. Soc. A* **1958**, *244*, 524-537.

[33] Kılınç, C. B. *Astrophys. Space Sci.* **2004**, *289*, 103-109.

[34] Collins, C. B.; Glass, E. N.; Wilkinson, D.A. *Gen. Relativ. Gravit.* **1980**, *12*, 805-823.

[35] Pradhan, A.; Kumhar, S. S. *Int. J. Theor. Phys.* **2009**, *48*, 1466-1477.

[36] Kanakavalli, T.; Ananda Rao, G. *Astrophys. Space Sci.* **2016**, *361*, 1-6.

[37] Adhav, K. S.; Bansod, A. S.; Munde, S. L. *Open Phys.* **2015**, *13*, 10.

[38] Prakash, S. *Astrophys. Space Sci.* **1985**, *111*, 383-388.

[39] Naidu, R. L.; Reddy, D. R. K.; Rampasad, T.; Ramana, K. V. *Astrophys. Space Sci.* **2013**, *348*, 247-252.