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Cycloid experiment for freshmen physics labs

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Abstract: We establish an instructive experiment to investigate the minimum time curve traveled by a small billiard ball rolling in a grooved track under gravity. Our intention is to popularize the concept of minimum time curves anew for pedagogical reasons and to propose it as a feasible physics experiment both for freshmen and sophomore classes. The experiment is simplified enough to provide motivation for the students. We observed that even the students who were not physics majors enjoyed such a cycloid experiment.

Key words: Cycloid, minimum time curve, experiment

1. The minimum time of descent under gravity

The minimum time of descent under gravity has historical importance in connection with Fermat's principle, a problem that remains ever popular to the readers of general physics matters [1–5]. Our aim here is to propose an experiment for the introductory mechanics laboratory such that the students explore the minimum time curve known as a cycloid (Figure 1) themselves. A small billiard ball rolls from rest under gravity from an initial (fixed) point O to a final (fixed) point A along different paths (Figures 2 and 3). The relation between its speed v and vertical position y can easily be found from the energy conservation, i.e. $T + V = const.$ or $\Delta T = -\Delta V$, in which T and V are the kinetic and the potential energy, respectively.

Out of an infinite number of possible paths joining O to A we are interested in the one that takes the minimum time. This is one of the typical extremal problems encountered in mechanics [6] under the title of Brachistochrone problems, whose solution is given in almost all books of mechanics. The time of slide between O and A is given by

$$\Delta t = \int_O^A \frac{ds}{v}, \tag{1}$$

in which $v = \sqrt{2gy}$ and ds is the element of the arc length along the path (Eq. (2) below). Note that for a billiard ball, as an extended object with inertia, the relation between v and y modifies into $v = \sqrt{\frac{10}{7}gy}$, which does not change the nature of the minimum time curve. We shall state the result simply: the curve is a cycloid

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expressed mathematically in parametric form,

$$x = a\,(\varphi - \sin\varphi)\,,$$
$$y = a\,(1 - \cos\varphi)\,, \tag{2}$$
$$(0 \le \varphi \le 2\pi)\,,$$

where $2a$ is the maximum point $y_{\max}$ since y is a downward coordinate along the curve. For different paths the pathlength of the curve S_{OA} can be obtained from the integral expression

$$S_{OA} = \int_O^A ds = \int_{\varphi=0}^{\varphi=\pi} \sqrt{dx^2 + dy^2}. \tag{3}$$

Mathematically it is not possible to evaluate this arc length unless we know the exact equation for the curve. Two exceptional cases are the straight line and the cycloid. As a curve the cycloid has the property that at O/A it becomes tangent to the vertical/horizontal axis. Although the lower point (i.e. A) could be chosen anywhere before the tangent point is reached, for experimental purposes we deliberately employ the half cycloid, so that identification of the Brachistochrone becomes simpler. By using a string and ruler we can measure each path length with great accuracy. The experimental data will enable us to identify the minimum time curve, namely the cycloid.

2. Apparatus and experiment

1) A thin, grooved track made of a long flexible metal bar (or hard plastic) fixed by clamps. Extra length of the track is chosen deliberately to make it deformable. In analogy with the grooved track of a projectile motion, such bars can be designed easily in introductory mechanics laboratories.

2) A small billiard ball.

3) A digital timer connected to a fork-type light barrier.

4) String and ruler to measure arc lengths.

The experimental set up is seen in Figure 2. We note that the track must be at least 2 m long both for a good demonstration and to detect significant time differences. The track is fixed at A by a screw while the other end of the track passing through the fixed point O is variable. This gives us the freedom to test different paths, with the crucial requirement that in each case the starting point O at which the timer is triggered electronically remains fixed. This particular point is the most sensitive part of the experiment, which is addressed by using a fork-type light barrier (optic eyes) both at O and A. As the path varies manually the light flash can be tolerated to intersect any point of the ball with negligible error. Let us also add that an extra piece of track at A is necessary to provide proper flattening at the minimum of the inverted half cycloid. From Figure 3, path P_1 is identified as a straight line, which is added here for comparison with the otherwise curved paths. As we change the path down from P_1 to P_8 we record the time of each descent by a digital timer. We observe that as we go from P_1 to P_4 with exact tangential touches to the axes, the time decreases, reaching a minimum at P_4. From P_5 on, the time starts to increase again toward P_8 with almost tangential touches at A. In this way we verify experimentally that P_4 can be identified as the minimum time curve. By using a string and ruler we measure

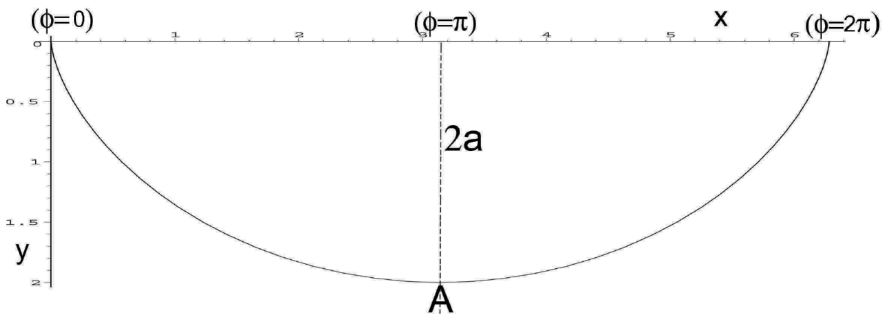

Figure 1. A cycle in an inverted cycloid for $0 \leq \phi \leq 2\pi$, with maximum height $2a$.

Figure 2. Our complete experimental apparatus/setup on display.

the length of each path as soon as we record its time of descent. The length S_1 of the straight line path P_1, for example, will be (recall from Eq. (2)) from the simple hypotenuse rule that (for $\phi = \pi$)

$$S_1 = \sqrt{x^2 + y^2} = 2a\sqrt{1 + \left(\frac{\pi}{2}\right)^2}, \tag{4}$$

where $2a = 73.4$ cm stands for the maximum height measured empirically. From Figure 1 and the Table we see that S_1 is calculated theoretically (≈ 136.5 cm) and experimentally (≈ 137.1 cm) and is acceptable within the limits of error analysis. Addition of errors involved in the readings of arc lengths, time, and averaging results will minimize the differences. It should also be taken care that, while rolling, the ball does not distort the track.

Table. Data for the length and the time taken, corresponding to each path. Students are expected to fill in this table with their own data.

Path	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Length S_i (cm)	137.1	142.1	144.1	146.2	150.2	154.3	158.5	162.6
Time (s)	0.740	0.663	0.654	0.652	0.654	0.656	0.668	0.677

The path length of the cycloid from Eq. (3) is obtained as

$$S_4 = 4a. \tag{5}$$

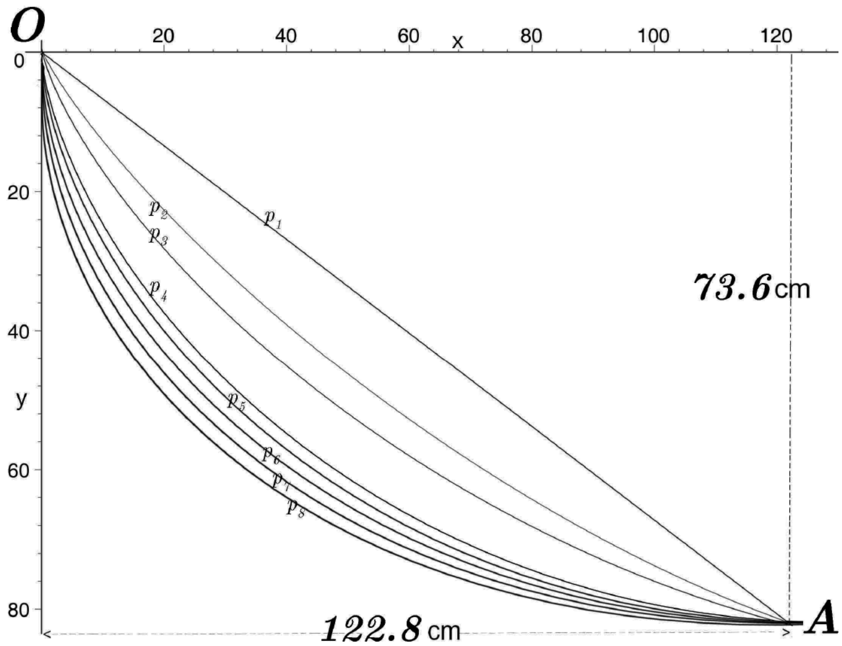

Figure 3. Rolling of a billiard ball from O to A. We consider 8 different paths, labeled as $P_1, P_2...P_8$, which are plotted from the data taken in the Table. Each path is curved/changed by hand since the track is flexible/deformable at O, with fixed final point A. Due to this required deformation the length of the track is chosen longer than appearing in between O and A.

Experimentally, all one has to do after taking each time record is to check that the minimum time curve satisfies Eq. (5), and it is tangent at O/A, which characterizes nothing but the Brachistochrone problem. Theoretically we have $S_4 = 146.8$ cm while the experimental value is 146.2 cm, which implies an error of less than one percent.

As an alternative method that we also tried to convince ourselves, we suggest using a digital camera to take a picture of each path and locate them on a common paper for comparison. As noticed, in performing the experiment, we have used only half of the cycloid $0 \leq \varphi \leq \pi$. If space is available, a longer track can be used to cover the second half, $\pi \leq \varphi \leq 2\pi$, as well. Owing to the symmetry of a cycloid, however, this is not necessary.

3. Conclusion

Theoretically it is known from the variational principles of mechanics that under constant gravity the minimum sliding time curve for a mass coincides with a cycloid. Once this fact is known a priori, the data constructed from a set of curves must concentrate around the cycloid. This naturally will facilitate the experiment. As we increase the data points in the Table it will be observed that as the rolling time decreases curves will accumulate and ultimately converge on the cycloid whose arc length is $4a$. A cycloid arises in many aspects of life. It is the curve generated by a fixed point on the rim of a circle rolling on a straight line. Diving of birds or jet fighters toward their targets and watery sliding platforms in aqua parks are some of the examples in which minimum time curves and therefore cycloids are involved. In comparison with a circle and ellipse, a cycloid is a less familiar curve at the introductory level of mathematics and geometry. The unusual nature comes from the fact that both the angle φ and its trigonometric function arise together so that the angle cannot be inverted in terms of coordinates in easy terms. However, the details of mathematics that are more appropriate for sophomore classes can easily be suppressed. Changing the track each time before rolling the ball, measuring both time of fall and length of the curve is easy and instructive to conduct as a physics experiment. The main task the students are supposed to do is to fill in the data in the Table. It will not be difficult for students to see that the cycloid is truly the minimum time curve of a fall under a constant gravitational field. Let us complete our analysis by commenting that a simple extension of our experiment can be done by using variable initial points. Namely, instead of the fixed point O, the ball can be released from any other point between O and A, which does not change the time of the fall. This introduces the students pedagogically to the problem of a tautochrone, which can also be studied separately.

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