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Quark gluon plasma and upsilon suppression

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Abstract: The time-dependent suppression probability P(t) of upsilon particles has been estimated due to the screening when embedded in the medium of quark gluon plasma where the screened potential is represented by an optical potential of type $V(r,\lambda) = -[\frac{V_0+iW_0}{1+exp((r-R)\lambda)}]$. A similar type of variation of suppression probability has been obtained by others. It has been observed that $t_{\frac{1}{2}}$ for melting of Υ is $\sim$ 1.5 fm/c. The variation of the screening radii of heavy mesons $(J/\psi, \Upsilon)$ for 1S and 2S states with respect to plasma radius have also been investigated with the same complex screening potential. We have found that the more highly excited state (2S) is less tightly bound and has a larger effective radius than the ground state.

Key words: Upsilon suppression, quark gluon plasma, optical potential, screening radii, suppression probability

1. Introduction

QCD predicts that strongly interacting matter should at sufficiently high energy density and temperature undergo a transition from hadronic matter to a new phase of QCD matter: the quark gluon plasma (QGP). This prediction of the possible existence of a deconfined QGP phase at high temperature and/or density by QCD opened up a challenging task to detect this deconfined state of strongly interacting matter. QGP (often described as a soup-like medium) is a hot, dense state in which quarks and gluons exist freely. Heavy quarkonia are expected to play an important role in testing QCD and investigating the nature of the QCD phase of quarks and gluons. Melting of a quarkonia-bound state to its constituent is one of the predicted signatures of QGP because in a hot plasma of quarks and gluons, it should be harder for stable hadrons to retain their binding. Thus, one of the specific signatures of the creation of this extreme condition of matter is the detection of a reduced yield of heavy mesons (bound states of heavy quark–antiquark pairs). Melting manifests itself as the suppression of quarkonia production in heavy-ion collisions compared to the level of quarkonia production in collisions between protons. The temperature and energy densities reached in central nucleus collisions at the Relativistic Heavy Ion Collider (RHIC) [1,2] and at the Large Hadron Collider (LHC) [3,4] are predicted to be sufficient for QGP formation. The suppression of quarkonium (Q$\overline{Q}$) states is one of the most promising probes for the properties of QGP. In QGP the confining potential is screened due to the interaction of the heavy Q$\overline{Q}$ with medium partons. The basic mechanism for deconfinement in dense matter is the Debye screening of the quark color charge. When the screening radii become less than the hadron radii the confining force can no longer bind the quark together and deconfinement sets in [5]. Various experimental and theoretical studies have been

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carried out on charmonium and bottomonium suppression. The binding effects in quarkonia at zero temperature can be understood in terms of nonrelativistic potential models [6]. Depending on the success of the potential model at zero temperature and on the idea that color screening indicates modification of the interquark forces, potential models have been used to try to understand quarkonium properties at finite T [7,8]. Chu and Matsui [7] used the concept of color screening [5] and presented a model to analyze the quarkonia suppression in QGP. Mocsy et al [9] calculated quarkonium spectral functions in QGP using a potential model based on full QCD lattice calculations of the free energy of static quark–antiquark pair. Charmonium suppression has been studied in detail both theoretically [10–12] and experimentally at energies reached at CERN (SPS), BNL (RHIC)[13], and the LHC [3,14].

Recently Υ suppression was observed for the first time, both by the solenoidal tracker in the RHIC (STAR) experiment [14] and the CMS experiment at the LHC [15–18]. Nendzig et al. [19] suggested a three-step model to obtain the suppression of the $\Upsilon(1S, 2S, 3S)$ states in PbPb collisions at energies available at the LHC. They argued that the combined effects of screening, gluon-induced dissociation, collisional damping, and reduced feed-down explain most of the sequential suppression of $\Upsilon(nS)$ states that has been observed in PbPb relative to pp collisions at $\sqrt{S_{NN}} = 2.76$ TeV. Srivastava et al. [20] presented a modified color screening model and calculated the anomalous suppression of various states of Υ arising due to QGP medium alone. They obtained the suppression patterns of the different bottomonia states with respect to centrality at various available collision energies. Gunion et al. [21] discussed the implication of the high initial temperature on the screening mass and on the relative suppression of members of the Υ family. They found that expectations for Υ resonance production in Pb+Pb collisions at the LHC depend on the nature and details of the QGP, which may allow us to determine much about the fundamental nature of QGP including the energy density, the initial temperature, the plasma radius, and the temperature-dependent screening mass. Grandchamp et al. [22] showed the QGP suppression of bottomonia in central (b = 1 fm) Pb-Pb collisions at the LHC, using the quasi-free destruction process. Kisslinger et al. [23] estimated the relative probabilities of $\Upsilon(nS)$ production at the LHC and Fermilab in p-p collisions using the mixed hybrid theory for heavy quark states and predicted the cross-section for production of the bottomonium states. Akamatsu et al. [24] discussed the time evolution of $Q\overline{Q}$ analytically in a limiting scenario for the spatial decoherence and provided a qualitative one-dimensional numerical simulation of the real-time dynamics. Das [25] discussed the roles of suppression and regeneration mechanisms as well as the importance of the results of the p-Pb data taking the estimate of cold nuclear matter effects on quarkonia and discussed the perspective for the bottomonia measurements. Ganesh et al. [26] presented a model of bottomonium suppression in QGP medium by combining color screening, gluonic dissociation, and collisional damping. They used temperature-dependent formation time of bottomonium states by employing the quasi-particle model as the equation of state for the QGP. They found that the modification of formation time due to temperature modifies the Υ suppression to a considerable extent in the central region and plays a very crucial role in accurately determining the Υ suppression.

In the present work we have studied the time-dependent suppression probability $P(t)$ of Υ in plasma where the screening as well as the absorption effect have been incorporated through the optical-type complex potential used in the nuclear scattering process. It has been assumed that the $b\bar{b}$ system is immersed into the deconfining medium at time t = 0. We have also analyzed the behavior of the screening radii of J/ψ and the Υ meson for 1S and 2S states with respect to the plasma radii.

2. Time dependence of survival probability of upsilons in QGP

The bottomonium system has been described nonrelativistically in detail in the framework of the statistical model [27–29]. The ground state wave function associated with the $b\bar{b}$ meson in context of the linear type of potential can be described as:

$$\Phi_{\Upsilon}(r) = A(r_0 - r)^{\frac{3}{4}}\Theta(r_0 - r), \tag{1}$$

where $A = \sqrt{\frac{315}{64\pi r_0^{\frac{9}{2}}}}$, r_0 is the size of the $b\bar{b}$ meson, and Θ is the usual step function. We have assumed that at the very birth of the Υ-particle when QGP is not present,which corresponds to $t = 0$, the $b\bar{b}$ system may be described by the wave packet represented by $\psi(r, 0)$. With the passage of time as Υ starts to melt at $t = 0$ (when it is embedded in the medium of QGP), the time evolution of this wave packet is described by the corresponding time-dependent Schrödinger equation. As precise knowledge for the initial wave function is not yet available [30], we assume $\psi(r, t = 0)$ to be [31]

$$\psi(r, t = 0) = \Phi_{\Upsilon}(r) \tag{2}$$

as the initial boundary condition in the current investigation. The deconfining medium at which the created $b\bar{b}$ pair is embedded may be either QGP above some critical temperature or the gas of quarks and gluons with a large density where the screening of color charges sets in. We assume that this screening as well as the absorption of Υ in the medium may be described by a complex potential of the form

$$V(r) = -[v(r) + i\omega(r)] = -[\frac{V_0 + iW_0}{1 + exp(\frac{r-R}{a})}] \tag{3}$$

in analogy with the optical potential conventionally used for nuclear scattering processes. V_0 and W_0 are the strength parameters representing the real and imaginary part of the potential. W_0, representing the imaginary part of the potential, provides the quarkonium width and is closely related to the mean lifetime. R is the typical size of the plasma and the plasma is assumed to be spherical, r is the transverse distance from the center of the plasma, and a is the inverse of the screening parameter. The exact analytical solution for the S-wave scattering state for the optical potential is obtained from the work of Bencze [32]. The Schrödinger equation runs as:

$$\frac{d^2u}{dr^2} + [K_n^2 + \frac{p^2}{1 + exp(\frac{r-R}{a})}]u = 0, \tag{4}$$

where $K_n^2 = m_q E_n$ (m_q being the quark mass, E_n the eigenenergy for the nth state). We have arrived at the corresponding bound state wave function according to the suggestions made by Bencze [32]:

$$\Phi_{nl}(r) = \frac{U(r)}{r} = \frac{C_{nl}}{r}Y^{\lambda}(1 + Y)^n {}_2F_1(-n, -n - 1, -3 - 2K_n a; \frac{Y}{1 + Y}), \tag{5}$$

where $Y = \exp[\frac{r-R}{a}]$, C_{nl} is a constant, n = 1,2,3....etc., and $\lambda^2 = a^2(K_n^2 - p^2)$. ${}_2F_1$ is the hypergeometric function of four arguments. Extracting the zeros of the scattering matrix $S_0(K)$ on the negative imaginary axis of the complex K-plane and solving the resulting equation [32], we have:

$$\lambda + K_n a \simeq -n, \tag{6}$$

which gives us the bound state energy eigenvalues E_n. The time-dependent wave function $\psi(r,t)$, which describes the evolution of the bottomonium wave-packet in the plasma, may be expanded in terms of the complete set of eigenfunctions $\Phi_{nl}(r)$ corresponding to the above complex potential [33] as

$$\psi(r,t) \simeq \sum_n \sum_l a_{nl} \exp[(-iE_n - \omega(r)t)/\hbar]\Phi_{nl}(r), \tag{7}$$

and with the limit of $r \to r_0$ the coefficients a_{nl} are obtained by

$$a_{nl} = \int \Phi_{\Upsilon}(r)\Phi^*_{nl}(r)d^3r. \tag{8}$$

The time-dependent survival probability of the Υ particle corresponding to the rate of melting of it within the medium of QGP is thus given by [31]:

$$P(t) = |\int d^3r\Phi^*_{\Upsilon}(r)\psi(r,t)|^2. \tag{9}$$

However, since $\Phi_{\Upsilon}(r)$ represents the S-state eigenfunctions, only the S-states would contribute to the integral of Eqs. (8) and (9). With the normalization condition

$$\int_0^R |\Phi_{\Upsilon S}(r)|^2 d^3r = 1, \tag{10}$$

we have evaluated C^2_{1S} and C^2_{2S}.

With the help of the standard integral of the form [34]

$$\int_0^u r^{\nu-1}(u-r)^{\mu-1}\exp(\beta r)dr = B(\mu,\nu)u^{\mu+\nu-1}{}_1F_1(\nu,\mu+\nu;\beta\mu) \tag{11}$$

and with Re$\mu > 0$, Re$\nu > 0$, the coefficients a_{1S} and a_{2S} have been evaluated using the integral of Eq. (8) for the 1S and 2S states, respectively.

The number of eigenstates being introduced in the present calculation is given by the following condition:

$$K_n^2 a^2(\frac{V_0}{E_n} - 2) > r^2. \tag{12}$$

With $V_0 = \frac{0.6}{a}$ in GeV [31], a = 10 GeV^{-1}, m_b=4.2 GeV [35], we have evaluated the bound state energies E_n and the choice of a = 10 GeV^{-1} restricts the number of eigenstates to two given by the condition in Eq. (12). Considering r_0 = 2.076 GeV^{-1} from Pineada and Segovia [36] and using R = 5 GeV^{-1}, we have evaluated the coefficients a_{1S} and a_{2S}. W_0 is taken to be 0.38 GeV as in previous studies [37,38]. Cugnon et al. [37] used the imaginary part of the optical model potential $V(r) - i\omega(r)$, as $\omega(r) = W_0\Theta(r-L)$. W_0 is a constant, $\Theta(r-L) = 1$ if r > L and zero otherwise to study the width of the heavy quarkonium states, which is based on the string fragmentation picture. Moreover, a possible application of the optical model-type picture has also been done [37], which deals with the evolution of J/Ψ propagating through matter. On the other hand, unlike Cugnon et al. [37], we have presumed a variation of $\omega(r)$ in the form of $W_0/[1+exp[(r-R)/a]]$ with W_0 = 0.38 GeV as in the previous studies [37,38] in the present calculation.

The survival probability of Υ in the plasma, i.e. $P(t)$ in Eq. (9), has been evaluated and we come across the following analytical expression for P(t):

$$P(t) \simeq \exp[\frac{-2W_0 t}{1+\exp(-R/a)}] \times [a_{1S}^4 + a_{2S}^4 + 2a_{1S}^2 a_{2S}^2 \cos(E_1 - E_2)t] \ for \ t > 0, \tag{13}$$
$$P(t) = 1 \quad \text{for} \quad t = 0.$$

The term $\exp[-2W_0 t/(1+\exp(-R/a))]$ in the expression of $P(t)$ in Eq. (13) simulates the dissipative effects in the process of the dissolution of Υ in the plasma. With the choice of parameters mentioned above, the rate of falling of $P(t)$ with time has been investigated and is displayed in Figure 1. It may be noted that the exponential damping part of Eq. (13), which is a manifestation of the absorption or dissipative effects of Υ in the plasma, is dominating over the region for large values of time, whereas for short time periods the cosine dependence along with the sum of the constants a_n^4 in Eq. (13) plays a role in computing P(t). This cosine part in Eq. (13) not only incorporates the screening effect in the plasma; it also simulates the absorption effects of Υ. The variation of P(t) with time has been investigated in detail and is shown in Figure 1. We also include in Figure 1 the variation of P(t) with time from the work of Grandchamp et al. [22] for comparison. From Figure 1 we obtain a typical time of $t_{\frac{1}{2}} \simeq 1.5$ fm/c for half of the dissolution of the initial contents of Υ in the plasma.

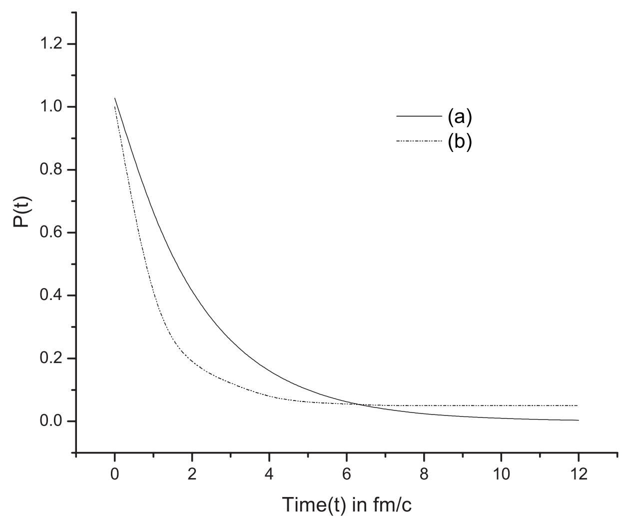

Figure 1. Variation of P(t) of upsilon particles in QGP with time: a) our work; b) work of Grandchamp et al. [22].

3. Estimation of the screening radii of heavy mesons (J/ψ and Υ) for 1S and 2S states

To estimate the expectation value of the radius of heavy quark bound states using the optical potential as the $q\bar{q}$ potential, the normalized wave function of a meson [39] for the ground and first excited states has been used. The expectation value of the screening radius $\langle R_n \rangle$ is:

$$\langle R_n \rangle = \int_0^\infty |\Psi_0|^2 r 4\pi r^2 dr \tag{14}$$

$$= 4\pi c_n^2 \int_0^\infty \frac{1}{r^2} (e^{\frac{r-R}{a}})^{2\lambda} (1 + e^{\frac{r-R}{a}})^{2n} r^3 dr, \tag{15}$$

and with a = 10 GeV^{-1} the expressions for the 1S and 2S states of J/ψ and Υ are obtained as:

$$\langle R_1\rangle_{J/\psi} = 4\pi c_1^2 e^{0.643R}[0.8933 + 2.0135e^{-\frac{R}{10}} + 1.1374e^{-\frac{R}{5}}], \quad (16)$$

$$\langle R_1\rangle_{\Upsilon} = 4\pi c_1^2 e^{1.6R}[0.3297 + 0.72644e^{-\frac{R}{10}} + 0.40124e^{-\frac{R}{5}}] \quad (17)$$

and

$$\langle R_2\rangle_{J/\psi} = 4\pi c_2^2 e^{0.7772R}[0.7604 + 5.7967e^{-\frac{R}{5}} + 1.2342e^{-\frac{2R}{5}} + 3.4323e^{-\frac{R}{10}} + 4.3624e^{-\frac{3R}{10}}], \quad (18)$$

$$\langle R_2\rangle_{\Upsilon} = 4\pi c_2^2 e^{0.676R}[0.8958 + 6.8811e^{-\frac{R}{5}} + 1.4848e^{-\frac{2R}{5}} + 4.0491e^{-\frac{R}{10}} + 5.2129e^{-\frac{3R}{10}}]. \quad (19)$$

For different values of plasma radii (R) (ranging from 5 GeV^{-1} to 10 GeV^{-1}) and with $m_c = 1.71$ GeV [40] and $m_b = 4.2$GeV [35], the corresponding values of $\langle R_1 \rangle$ and $\langle R_2 \rangle$ for J/ψ and Υ are calculated using Eq. (16), (17), (18), and (19) and the variations are displayed in Figures 2 and 3.

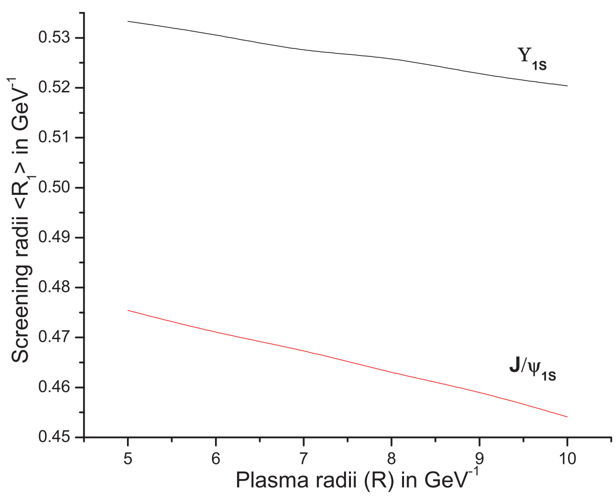

Figure 2. Variation of screening radii $< R_1 >$ with plasma radii R for J/ψ_{1S} and Υ_{1S} state.

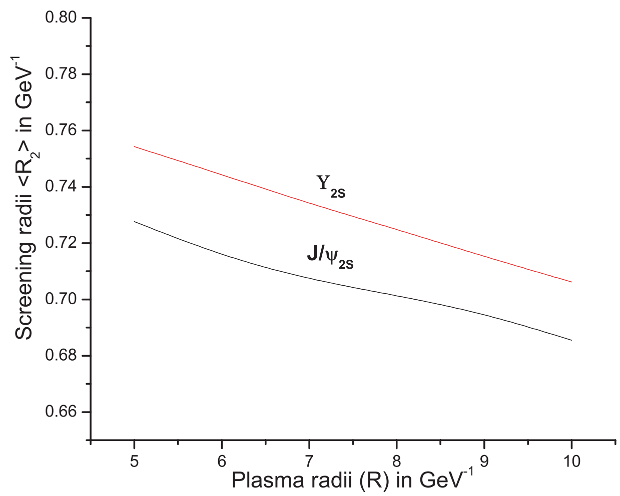

Figure 3. Variation of screening radii $< R_2 >$ with plasma radii R for J/ψ_{2S} and Υ_{2S} state.

4. Discussion

In the present work the optical type of screened potential has been used, which simulates screening as well as the absorption effect in the process of the dissolution of Υ in QGP. We have studied the variation of probability of dissolution of Υ and the variation of screening radii $< R_n >$ with plasma radii (R) for 1S and 2S states of J/ψ and Υ and the results are displayed in Figures 2 and 3. The variation of suppression probability P(t) of Υ with time (t) is plotted in Figure 1 and compared with the work of Grandchamp et al. [22]. It is observed that half of the initial content of Υ dissolved at about time $t_{\frac{1}{2}} \sim 1.5$ fm/c with the optical type of screened potential, whereas Grandchamp et al. [22] estimated $t_{\frac{1}{2}} \sim 0.9$ fm/c.

Borghini et al. [41] studied heavy quarkonia in a medium as a quantum dissipative system. They investigated the time dependence of the populations of a 4-level system and found that the population of the quarkonium state decreases with increasing time. Arts et al. [42] investigated the temperature dependence of bottomonium above deconfinement in lattice nonrelativistic QCD. They studied the time variation of effective mass of Υ for various temperatures and found a gradual decrease in effective mass. In a subsequent work using nonrelativistic dynamics for the bottom quark, Arts et al. [43] investigated the variation of Υ effective mass as a function of Euclidean time for various temperatures and found that effective mass decreases with increasing time. Kim et al. [44] investigated the spectral functions of S-wave and P-wave bottomonium at finite temperature in 2+1 flavor QCD using the NRQCD for bottom quarks and studied the variation of effective masses as a function of time for different values of the gauge coupling, which indicates a gradual fall in effective mass with time. It is interesting to note that the suppression probability P(t) vs. time(t) plot shows a similar type of behavior, which decreases with time, in the current investigation.

It may be mentioned that Liu et al. [45] estimated the critical value of the Debye screening length of J/ψ and Υ states by using different quark binding potentials in a nonrelativistic approximation and found that the critical value of the Debye screening length is highly sensitive to the form of the binding potential of the quark. They observed that the critical screening radius for J/ψ is larger than Υ in the ground state. We have also studied the variation of screening radii with plasma radii for 1S and 2S states of J/ψ and Υ and observed a decrease in screening radii of J/ψ and Υ in QGP with the increase in plasma radii. The free quarks and gluons in the plasma weaken the binding force of the mesons. The more highly excited state (2S) is less tightly bound and has a larger effective radius than the ground state, which indicates being more highly suppressed by the hot plasma [46]. The results could have some dependence on the choice of potential. We will study it with different potentials in the near future.

The current investigations of the probability of dissolution of Υ in QGP along with the binding radii of the 1S and 2S states of J/ψ and Υ can be used as a guide for the experimental investigation of the heavy quark-antiquark potential at finite temperature. As the imaginary part of the complex potential includes loss of probability, screening, and absorption effect along with the scattering, it may be asserted that the screening effect of the optical type of complex potential is evident, resulting in substantial suppression of the Υ in QGP.

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