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Theory and application of hydrogen formation in proton-alkali atom collision

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Abstract: The possibility of producing more hydrogen during p-alkali atom collisions is discussed. The coupled static approximation is modified for the first time to make it applicable to the multichannel problem of the collisions of p-alkali atoms. The formation of H (1s) and excited H (in 2s- and 2p- states) in the scattering of p-Li atoms is treated to test the convergence of our method. The modified method is used to calculate the total cross-sections of seven partial waves in a range of energy between 50 and 1000 keV. Our p-Li results are compared with earlier ones.

Key words: Proton-alkali, hydrogen formation, excited hydrogen formation, cross-sections

1. Introduction

The most interesting phenomenon in quantum mechanics is the intermediate states that appear in a nuclear reaction. Most theoretical and experimental studies of proton-atom interactions have been discussed in the last decade by many authors. They calculated the total cross-sections of the interaction. Banyard and Shirtcliffe [1] discussed p-Li scattering using continuum distorted wave (CDW) approximation. Ferrante and Fiordilino [2] studied p-alkali atoms using eikonal approximations. Daniele et al. [3] reported the total cross-sections for p-alkali atom collision using eikonal approximation. Ferrante et al. [4] also investigated the total H-formation cross-sections in p-alkali atoms using Oppenheimer–Brinkman–Kramers (OBK) approximation. Fritsch and Lin [5] studied p-H atom collisions using the coupled-state calculations method. Choudhury and Sural [6] studied p-alkali atom (Na, K, Rb, Cs) collisions in the wave formation of impulse approximation at energies ranging from 50 to 500 keV. Tiwari [7] reported the differential and total cross-sections in H-formation in the collision of p-Li and p-Na atoms using Coulomb-projected Born approximation.

The present work explores the possibility of producing more hydrogen through p-alkali atom collisions. In the present paper, the coupled static approximation (CSA) method, which is used by Elkilany [8–11], is modified to make it applicable to discuss the multichannel coupled static approximation (MCSA) problem ($n = 4$) of the collision of p-Li atoms at intermediate energies of the projectile. A numerical procedure is generalized to solve the obtained multicoupled equations. Throughout this paper Rydberg units are used and the total cross-sections are expressed in units of $\pi a_0^2 (= 8.8 \times 10^{-17} cm^2)$ and energy units of keV.

2. Theoretical formalism

The MCSA of protons scattered by alkali atoms may be written as (see Figure 1):

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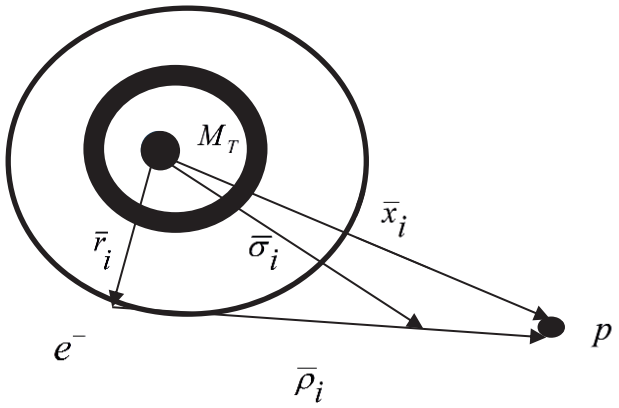

Figure 1. Configuration space of p-atom scattering: $\bar{x}_i$ and $\bar{r}_i$ are the vectors of the proton and the valence electron of the target with respect to the center of mass of the target, $\bar{\rho}_i$ is the vector of the proton with respect to the valence electron of the target, $\bar{\sigma}_i$ is the vector of the center of mass of H from the target, and M_T is the mass of the nucleus of the target.

$$p + A = \begin{bmatrix} p + A & \text{Elastic channel (first channel)} \\ H(n\ell) + A^+ & H(n\ell) \text{ formation channels } ((n-1) - \text{channels}) \end{bmatrix}, \tag{1}$$

where p is the proton, A is an alkali target atom, $H(n\ell)$ is hydrogen formation in $n\ell$-states, and n is the number of open channels.

The Hamiltonian of the elastic channel is given by:

$$H = H^{(1)} = H_T - \frac{1}{2\mu_1}\nabla^2_{x_1} + V^{(1)}_{int}(x_1) = -\frac{1}{2\mu_T}\nabla^2_{r_1} - \frac{2}{r_1} + V_c(r_1) - \frac{1}{2\mu_1}\nabla^2_{x_1} + V^{(1)}_{int}(x_1), \tag{2}$$

where H_T is the Hamiltonian of the target atom. μ_T is the reduced mass of the target atom.

$$H = H^{(i)} = H_i - \frac{1}{2\mu_i}\nabla^2_{\sigma_i} + V^{(i)}_{int}(\sigma_i) = -\frac{1}{2\mu_i}\nabla^2_{\rho_i} - \frac{2}{\rho_i} - \frac{1}{2\mu_i}\nabla^2_{\sigma_i} + V^{(i)}_{int}(\sigma_i),\ i = 2, 3, 4, ...n \tag{3}$$

The Hamiltonian of the (n-1)-rearrangement channels are expressed by:

Here, $H_i,\ i = 2, 3, 4, ...n$ are the Hamiltonians of the hydrogen formation atoms, $H(nl)$, respectively. $\mu_i,\ i = 2, 3, 4, ...n$ are the reduced masses of $(n-1)-$ channels, respectively.

$V_c(r_1)$ is a screened potential and $V^{(1)}_{int}(x_1)$ is the interaction potential of the first channel, given by:

$$V_c(r_1) = V_{cCoul}(r_1) + V_{cex}(r_1), \tag{4}$$

where $V_{cCoul}(r_1)$ and $V_{cex}(r_1)$ are the Coulomb and exchange parts of the core potential, respectively (see ref. [11]), and

$$V^{(1)}_{int}(x_1) = \frac{2}{x_1} - \frac{2}{\rho_1} + V_{cCoul}(x_1) \text{ where } V_{cCoul}(x_1) = -V_{cCoul}(r_1), \tag{5}$$

and $V^{(i)}_{int}(\sigma_i)$, is the interaction between the two particles of the considered hydrogen formation and the rest of the target, which is given by:

$$V^{(i)}_{int}(\sigma_i) = \frac{2}{x_i} - \frac{2}{r_i} + V_{cCoul}(x_i) + V_{cCoul}(r_i) + V_{cex}(r_i),\ i = 2, 3, 4, ...n. \tag{6}$$

The total energies E of the n-channels are defined by:

$$E = E_i + \frac{1}{2\mu_i} k_i^2,\ i = 1, 2, 3...n, \tag{7}$$

where $\frac{1}{2\mu_1} k_1^2$ is the kinetic energy of the incident proton relative to the target and $\frac{1}{2\mu_i} k_i^2,\ i = 2, 3, 4, ..., n$ are the kinetic energy of the center of mass of the hydrogen formation atoms, $H(n\ell)$, respectively, relative to the nucleus of the target. E_1 is the binding energy of the target atom, and $E_i,\ i = 2, 3, 4, ..., n$ refer to the binding energies of the hydrogen formation atoms, respectively.

In MCSA, it is assumed that the projections of the vector $(H - E)|\Psi\rangle$ onto the bound state of the n-channels are zero. Thus, the following conditions are satisfied:

$$\langle \Phi_i | (H - E) | \Psi \rangle = 0,\ i = 1, 2, 3, ..., n. \tag{8}$$

The total wave function $|\Psi\rangle$ is expressed by

$$\Psi = \sum_{i=1}^{n} |\phi_i \psi_i\rangle, \tag{9}$$

$$\psi_1 = \sum_{\ell} \ell(\ell + 1) f_\ell^{(1)}(x_1) Y_\ell^0(\hat{x}_1), \tag{10}$$

$$\psi_i = \sum_{\ell} \ell(\ell + 1) g_\ell^{(i)}(\sigma_i) Y_\ell^0(\hat{\sigma}_i),\ i = 2, 3, ...n, \tag{11}$$

where $f_\ell^{(1)}(x_1)$ and $g_\ell^{(i)}(\sigma_i),\ i = 2, 3, ...n$ are the radial wave functions of the elastic and the hydrogen formation atoms, respectively, corresponding to the total angular momentum ℓ. $Y_\ell^0(x_1)$ and $Y_\ell^0(\hat{\sigma}_i)\ i = 2, 3, ..., n$ are the related spherical harmonics. $\hat{x}_1$ and $\hat{\sigma}_i,\ i = 1, 2, 3, ..., n$ are the solid angles between the vectors $\hat{x}_1$,$\hat{\sigma}_i,\ i = 2, 3, ...n$ and the z-axis, respectively. $\psi_i,\ i = 1, 2, 3, ...n$ are the corresponding scattering wave functions of the n-channels, respectively. Φ_1 is the wave function for the valence electron of the target atom, which is calculated using ref. [12]. $\Phi_i,\ i = 2, 3, 4, ..., n$ are the wave functions of the hydrogen formation atoms, $H(n\ell)$, respectively, which are defined using a hydrogen-like wave function.

Eq. (8) can be solved by considering differential equations

$$[\frac{d^2}{dx_1^2} - \frac{\ell(\ell + 1)}{x_1^2} + k_1^2] f_\ell^{(1)}(x_1) = 2\mu_1 U_{st}^{(1)}(x_1) f_\ell^{(1)}(x_1) + \sum_{\alpha=2}^{n} Q_{1\alpha}(x_1), \tag{12}$$

$$[\frac{d^2}{d\sigma_i^2} - \frac{\ell(\ell + 1)}{\sigma_i^2} + k_i^2] g_\ell^{(i)}(\sigma_i) = 2\mu_i U_{st}^{(i)}(\sigma_i) g_\ell^{(i)}(\sigma_i) + \sum_{\alpha=1}^{n}{}' Q_{i\alpha}(\sigma_i),\ i = 2, 3, ..., n, \tag{13}$$

where the prime on the sum sign means that $i \neq \alpha$, and

$$Q_{1\alpha}(x_1) = \int_0^\infty K_{1\alpha}(x_1, \sigma_\alpha) g_\ell^{(\alpha)}(\sigma_\alpha) d\sigma_\alpha, \alpha = 2, 3, ..., n, \tag{14}$$

$$Q_{i1}(\sigma_i) = \int_0^\infty K_{i_1}(\sigma_i, x_1) f_\ell^{(1)}(x_1) dx_1, \ i = 2, 3, ..., n, \tag{15}$$

$$Q_{i\alpha}(\sigma_i) = \int_0^\infty K_{i_\alpha}(\sigma_i, \sigma_\alpha) g_\ell^{(\alpha)}(\sigma_\alpha) d\sigma_\alpha, \ i, \alpha = 2, 3, ..., n, i \neq \alpha. \tag{16}$$

Kernels $K_{i\alpha}$, $i = 1, 2, 3, ..., n, i \neq \alpha$ are expanded by:

$$K_{1\alpha}(x_1, \sigma_\alpha) = 2\mu_1 (8 x_1 \sigma_\alpha) \iint \Phi_1(r_1) \Phi_\alpha(\rho_\alpha) [-\frac{1}{2\mu_\alpha} (\nabla^2_{\sigma_\alpha} + k^2_\alpha) + V^{(\alpha)}_{int}] Y^o_\ell(\hat{x}_1) Y^o_\ell(\hat{\sigma}_\alpha) d\hat{x}_1 d\hat{\sigma}_\alpha, \tag{17}$$
$$\alpha = 2, 3, ...n,$$

$$K_{i1}(\sigma_i, x_1) = 2\mu_i (8\sigma_i x_1) \iint \Phi_i(\rho_i) \Phi_1(r_1) [-\frac{1}{2\mu_1} (\nabla^2_{x_1} + k^2_1) + V^{(1)}_{int}] Y^o_\ell(\hat{\sigma}_i) Y^o_\ell(\hat{x}_1) d\hat{\sigma}_i d\hat{x}_1, \ i = 2, 3, ...n, \tag{18}$$

$$K_{i\alpha}(\sigma_i, \sigma_\alpha) = 2\mu_i (8\sigma_i \alpha_\alpha) \iint \Phi_i(\rho_i) \Phi_\alpha(\rho_\alpha) [-\frac{1}{2\mu_\alpha} (\nabla^2_{\sigma_\alpha} + k^2_\alpha) + V^{(\alpha)}_{int}] Y^o_\ell(\hat{\sigma}_i) Y^o_\ell(\hat{\sigma}_\alpha) d\hat{\sigma}_i d\hat{\sigma}_\alpha, \tag{19}$$
$$i, \alpha = 2, 3, ...n, i \neq \alpha.$$

The static potentials $U^{(1)}_{st}(x_1)$ and $U^{(i)}_{st}(\sigma_i)$, $i = 2, 3, ..., n$ are defined by

$$U^{(1)}_{st}(x_1) = < \Phi_1(r_1) | V^{(1)}_{int} | \Phi_1(r_1) >, \quad U^{(i)}_{st}(\sigma_i) = < \Phi_i(\rho_i) \left| V^{(i)}_{int} \right| \Phi_i(\rho_i) > . \tag{20}$$

Eqs. (12) and (13) are inhomogeneous equations in x_i, and σ_i, $i = 1, 2, 3, ..., n$, and possess the general form

$$(\varepsilon - H_0) |\chi > = |\eta\rangle \tag{21}$$

where ε is $k^2_i (i = 1, 2, ..., n)$. H_0 is $-\frac{d^2}{dx^2_1} + \frac{\ell(\ell+1)}{x^2_1}$ or $-\frac{d^2}{d\sigma^2_i} + \frac{\ell(\ell+1)}{\sigma^2_i}$, $i = 2, 3, ..., n$. $|\chi\rangle$ is $\left| f^{(1)}_\ell(x_1) \right\rangle$ or $\left| g^{(i)}_\ell(\sigma_i) \right\rangle$. $|\eta\rangle$ is the right-hand side of the coupled integro-differential equations, respectively.

The solutions of Eqs. (12) and (13) are given (formally) by the Lippmann–Schwinger equation in the form

$$|\chi > = |\chi_o > + G_o |\eta > , \tag{22}$$

where G_0 is the Green operator $(\varepsilon - H_0)^{-1}$ and $|\chi_0\rangle$ is the solution of the homogeneous equation

$$(\varepsilon - H_0) |\chi_0\rangle = |0\rangle, \tag{23}$$

Using Green operator G_0, the solutions of Eqs. (12) and (13) are given formally by

$$f^{(1,j)}_\ell(x_1) = \{\delta_{j1} + \frac{1}{k_1} \int_0^\infty \tilde{g}_\ell(k_1 x_1) [2\mu_1 U^{(1)}_{st}(x_1) f^{(1,j)}_\ell(x_1) + \sum_{\alpha=2}^n Q^{(j)}_{1\alpha}(x_1)] dx_1 \} \tilde{f}_\ell(k_1 x_1)$$
$$+ \{-\frac{1}{k_1} \int_0^\infty \tilde{f}_\ell(k_1 x_1) [2\mu_1 U^{(1)}_{st}(x_1) f^{(1,j)}_\ell(x_1) + \sum_{\alpha=2}^n Q^{(j)}_{1\alpha}(x_1)] dx_1 \} \tilde{g}_\ell(k_1 x_1), j = 1, 2, 3, ..., n \tag{24}$$

$$g_\ell^{(i,j)}(\sigma_i) = \{\delta_{ji} + \frac{1}{k_i}\int_0^\infty \tilde{g}_\ell(k_i\sigma_i)[2\mu_i U_{st}^{(i)}(\sigma_i)\, g_\ell^{(i,j)}(\sigma_i) + \sum_{\alpha=1}^{n}{}' Q_{i\alpha}(\sigma_i)]d\sigma_i\}\tilde{f}_\ell(k_i\sigma_i)$$

$$+ \{-\frac{1}{k_i}\int_0^\infty \tilde{f}_\ell(k_i\sigma_i)[2\mu_i \overset{(i)}{U}_{st}(\sigma_i) g_\ell^{(i,j)}(\sigma_i) + \sum_{\alpha=1}^{n}{}' Q_{i\alpha}^{(j)}(\sigma_i)]d\sigma_i\}\tilde{g}_\ell(k_i\sigma_i), \tag{25}$$

$$i = 2,3,...,n \quad j = 1,2,3,...,n$$

where δ_{ji}, $i,j = 1,2,3,...,n$ specify two independent solutions for each of $f_\ell^{(1,j)}(x_1)$ and $g_\ell^{(i,j)}(\sigma_i)$, $i = 2,3,...,n$, according to the considered channel. The functions $\tilde{f}_l(\eta)$ and $\tilde{g}_l(\eta)$, $\eta = k_1x_1$, or $\eta = k_i\sigma_i$ $i = 2,3,...,n$ are related to the Bessel functions of the first and second kinds, i.e. $j_l(\eta)\, and\, y_l(\eta)$, respectively, by the relations $\tilde{f}_l(\eta) = \eta\, j_l(\eta)$ and $\tilde{g}_l(\eta) = -m\, y_l(\eta)$.

The iterative solutions of Eqs. (24) and (25) are calculated by:

$$f_\ell^{(1,j,\nu)}(x_1) = \{\delta_{j1} + \frac{1}{k_1}\int_0^{X_1} \tilde{g}_\ell(k_1x_1)[2\mu_1 U_{st}^{(1)}(x_1)\, f_\ell^{(1,j,\nu-1)}(x_1) + \sum_{\alpha=2}^{n} Q_{1\alpha}^{(j,\nu-1)}(x_1)]dx_1\}\tilde{f}_\ell(k_1x_1)$$

$$+ \{-\frac{1}{k_1}\int_0^{X_1} \tilde{f}_\ell(k_1x_1)[2\mu_1 U_{st}^{(1)}(x_1)\, f_\ell^{(1,j,\nu-1)}(x_1) + \sum_{\alpha=2}^{n} Q_{1\alpha}^{(j,\nu-1)}(x_1)]dx_1\}\tilde{g}_\ell(k_1x_1), \tag{26}$$

$$j = 1,2,3,...,n; \nu \geq 1.$$

$$g_\ell^{(i,j,\nu)}(\sigma_i) = \{\delta_{ji} + \frac{1}{k_i}\int_0^{\sum_i} \tilde{g}_\ell(k_i\sigma_i)[2\mu_i U_{st}^{(i)}(\sigma_i)\, g_\ell^{(i,j,\nu)}(\sigma_i) + \sum_{\alpha=1}^{n}{}' Q_{i\alpha}^{(j,\nu)}(\sigma_i)]d\sigma_i\}\tilde{f}_\ell(k_i\sigma_i)$$

$$+ \{-\frac{1}{k_i}\int_0^{\sum_i} \tilde{f}_\ell(k_i\sigma_i)[2\mu_i U_{st}^{(i)}(\sigma_i)\, g_\ell^{(i,j,\nu)}(\sigma_i) + \sum_{\alpha=1}^{n}{}' Q_{i\alpha}^{(j,\nu)}(\sigma_i)]d\sigma_i\}\tilde{g}_\ell(k_i\sigma_i), \tag{27}$$

$$i = 2,3,...,n, j = 1,2,3,...,n; \nu \geq 0.$$

Here, $X_1, \sum_i$, $i = 2,...n$ specify the integration range away from the nucleus over which the integrals of Eqs. (26) and (27) are calculated using Simpson's expansions.

Taylor expansions of $U_{st}^{(1)}(x_1), \tilde{f}_\ell(k_1x_1)$ and $\tilde{g}_\ell(k_1x_1)$ are used to obtain the starting value of $f_\ell^{(1,j,0)}(x_1)$ (see ref. [11]).

Equations (26) and (27) can be abbreviated to the following:

$$f_\ell^{(1,j,\nu)}(x_1) = a_1^{(j,\nu)}\tilde{f}_\ell(k_1x_1) + b_1^{(j,\nu)}\tilde{g}_\ell(k_1x_1), j = 1,2,3,...n; \nu > 0 \tag{28}$$

$$g_\ell^{(i,j,\nu)}(\sigma_i) = a_i^{(j,\nu)}\tilde{f}_\ell(k_i\sigma_i) + b_i^{(j,\nu)}\tilde{g}_\ell(k_i\sigma_i),\ i = 2,...,n, j = 1,2,3,...n; \nu > 0 \tag{29}$$

The preceding coefficients of Eqs. (28) and (29) are elements of the matrices $a^\upsilon\, and\, b^\upsilon$, which are given by:

$$\left.\begin{aligned}(a^\upsilon)_{ij} &= \sqrt{2\mu_{m_i}/k_i}\, a_i^{(j,\upsilon)} \\ (b^\upsilon)_{ij} &= \sqrt{2\mu_{m_i}/k_i}\, b_i^{(j,\upsilon)}, i,j = 1,2,...,n, \nu > 0\end{aligned}\right], \tag{30}$$

and we can obtain the reactance matrix, R^{υ}, using the following relation:

$$R^{\upsilon} = b^{\upsilon}(a^{\upsilon})^{-1}, \quad \nu > 0. \tag{31}$$

The partial cross-sections in the present work are determined (in πa_0^2) by:

$$\sigma_{ij}^{(\ell,\upsilon)} = \frac{4(2\ell+1)}{k_1^2}\left|T_{ij}^{\nu}\right|^2, \quad i,j = 1,2,3,...,n, \quad \nu > 0 \tag{32}$$

where k_1 is the momentum of the incident protons, ν is the number of iterations, and T_{ij}^{ν} is the elements of the $n \times n$ transition matrix T^{ν}, which is given by:

$$T^{\nu} = R^{\nu}\left(I - \tilde{i}R^{\nu}\right)^{-1}, \quad \nu > 0, \tag{33}$$

where R^{ν} is the reactance matrix and I is an $n \times n$ unit matrix and $\tilde{i} = \sqrt{-1}$.

The total cross-sections (in πa_0^2 units) can be obtained (in the νth iteration) by:

$$\sigma_{ij}^{\nu} = \sum_{\ell=0}^{\infty} \sigma_{ij}^{(\ell\upsilon)}, \quad i,j = 1,2,3,...,n, \quad \nu > 0 \tag{34}$$

3. Proton-lithium scattering

As an application of our MCSA, we are going to apply the above method in the case of $n = 4$ (four-channels CSA) to the scattering of p-Li. Our problem can be written in the following form:

$$p + Li(2s) = \begin{bmatrix} p + Li(2s) & \text{Elastic channel (first channel)} \\ H(1s) + Li^{+} & H(1s) \text{ formation channel (second channel)} \\ H(2s) + Li^{+} & H(2s) \text{ formation channel (third channel)} \\ H(2p) + Li^{+} & H(2p) \text{ formation channel (fourth channel)} \end{bmatrix} \tag{35}$$

$\Phi_1(r_1)$ is the valence electron wave function of the target (lithium) atom, which is calculated using Clementi's tables [12], and $\Phi_i(\rho_i)$, $i = 2,3,4$ are the wave functions of the hydrogen formation, which are given by:

$$\Phi_2 = \frac{1}{\sqrt{\pi}}\exp(-\rho_2), \quad \Phi_3 = \frac{1}{\sqrt{32\pi}}(2-\rho_3)\exp(-\rho_3/2) \text{ and } \Phi_4 = \frac{1}{\sqrt{32\pi}}\rho_4\cos\theta_{\rho_4,\sigma_4}\exp(-\rho_4/2). \tag{36}$$

4. Results and discussion

We start our calculations on p-Li scattering by testing the variation of the static potentials $U_{st}^{(1)}(x_1)$ and $U_{st}^{(i)}(\sigma_i)$, $i = 2,3,4$, of the considered channels with the increase of $x_1, \sigma_i(\ i = 2,3,4)$. In the second step, we consider the integration range, IR, to be $32a_0$ with Simpson's interval of 0.0625 to obtain the considered integration. It is found that excellent convergence can be obtained with Simpson's interval of $h = 0.0625$, $n = 512$ points, and $\nu = 50$. We have calculated the total cross-sections of p-Li scattering corresponding to $0 \le \ell \le 6$ at incident energies between 50 and 1000 keV. The Table shows the present total cross-sections of

Table. Present σ_{12}, σ_{13}, and σ_{14} (in πa_0^2) of p-Li scattering with the results of [1], [2], [3], and [7].

k^2 keV	Present *H(1s)* σ_{12}	Banyard and Shirtcliffe [1] (1s) (CDW)	Ferrante and [2] (1s) (OBK)	Daniele et al. [3] (1s) (eikonal)	Present *H(2s)* σ_{13}	Banyard and Shirtcliffe [1] (2s) (CDW)	Ferrante and Fiordilino [2] (2s) (OBK)	Tiwari [7] (2s) (CPB)	Present *H(2p)* σ_{14}	Banyard and Shirtcliffe [1] (2p)
50	4.6753E-2				9.0561E-3		6.5E-1	3.596E-3	8.2737E-4	
100	1.9672E-2		6.84E-3	4.8947E-3	4.8653E-3		1.0E-3	6.746E-4	3.5363E-4	
150	1.1757E-2				2.2135E-3		4.9E-3	1.294E-4	1.5765E-4	
200	7.5246E-3	8.166E-3	1.873E-3	5.6149E-4	8.9237E-4	1.048E-3	2.8E-3	3.294E-5	1.2033E-4	1.467E-04
250	5.4237E-3				6.0929E-4			1.327E-5	9.0672E-5	
300	3.5916E-3				3.5432E-4			8.489E-6	6.5326E-5	
350	2.3321E-3				2.3564E-4				4.6673E-5	
400	1.3622E-3				1.5465E-4				2.3917E-5	
450	5.7923E-4				8.6726E-5				8.7861E-6	
500	3.5588E-4	3.878E-4			4.5163E-5	5.212E-5		9.828E-7	4.7005E-6	5.751E-06
550	2.6345E-4				4.0872E-5				3.2793E-6	
600	2.0526E-4				3.2844E-5				2.2395E-6	
650	1.6767E-4				2.3643E-5				1.5457E-6	
700	1.2358E-4				1.5971E-5				1.0823E-6	
750	8.6739E-5				9.8174E-6				8.8835E-7	
800	5.3231E-5	5.778E-5			6.8517E-6	7.711E-6		2.278E-7	6.5791E-7	7.604E-07
850	4.0683E-5				5.3567E-6				5.6372E-7	
900	3.0095E-5				4.3973E-6				4.5517E-7	
950	2.5657E-5				3.0035E-6				3.3423E-7	
1000	2.0139E-5	2.189E-5			2.4529E-6	2.906E-6		9.366E-8	2.7116E-7	2.716E-07

p-Li scattering with those of Banyard and Shirtcliffe [1], Ferrante and Fiordilino [2], Daniele et al. [3], and Tiwari [7] in the energy range of 50–1000 keV. Our results and the available compared results in the range of energy of 500–1000 keV are also displayed in Figures 2–4. In Figure 5 we also show the present results of the total cross-sections of the four channels (elastic and the hydrogen formation (*H (1s)*, *H (2s)*, *H (2p)*) in the same range of energy (50–1000 keV). The present values of the total cross-sections of the four channels have trends similar to the comparison results. Our values of the total cross-sections of the four channels decrease with the incident energies. The calculated total cross-sections σ_{12} of *H (1s)* are about 7.85%–8% lower than the results of Banyard and Shirtcliffe [1]. The total cross-sections σ_{13} of *H (2s)* are about 11.1%–15.6% lower than those of Banyard and Shirtcliffe [1]. Our results of the total cross-sections σ_{14} of *H(2p)* are about 13.5%–18.3% lower than the available values of Banyard and Shirtcliffe [1]. We also noticed that the available compared results of Ferrante and Fiordilino [2], Daniele et al. [3], and Tiwari [7] are higher than our results. The present calculations show that we have more H-formation if we open more excited channels of hydrogen formation in the collision of protons with lithium atoms. The present calculated total cross-sections have the same trend as

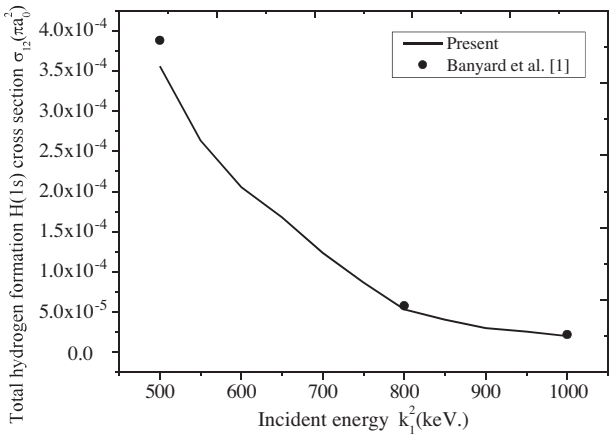

Figure 2. σ_{12}(in πa_0^2) of p-Li scattering with those of Banyard and Shirtcliffe [1].

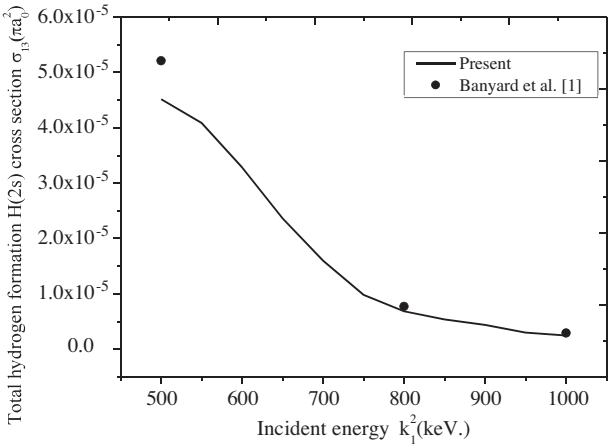

Figure 3. σ_{13}(in πa_0^2) of p-Li scattering with those of Banyard and Shirtcliffe [1].

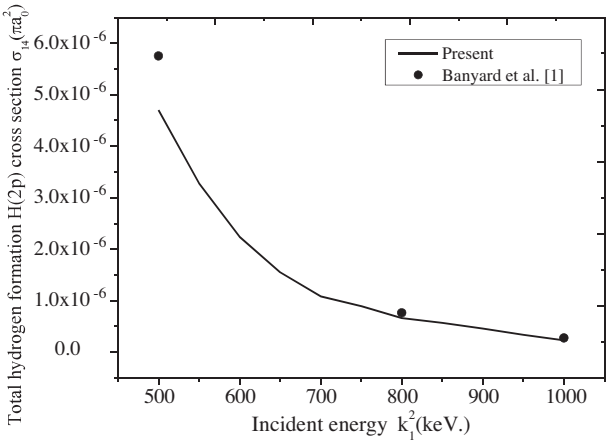

Figure 4. σ_{14}(in πa_0^2) of p-Li scattering with those of Banyard and Shirtcliffe [1].

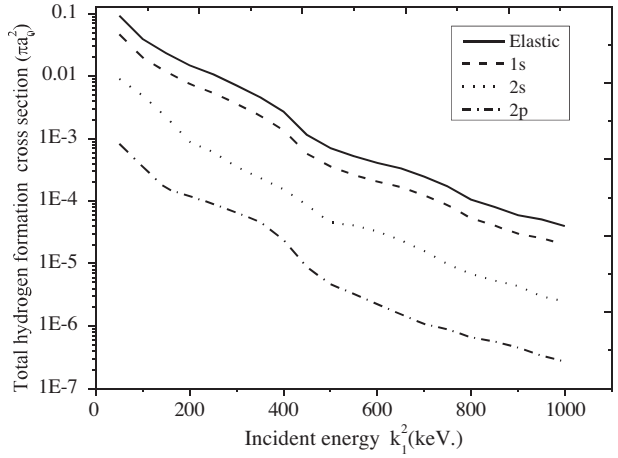

Figure 5. $H(1s), H(2s)$, and $H(2p)$ cross-sections (in πa_0^2) of p-Li scattering.

the comparison results and give good agreement with the available previous results of Banyard and Shirtcliffe [1].

5. Conclusions

p-Li scattering was studied using MCSA as a four-channel problem (elastic, *H(1s)*, *H(2s)*, and *H(2p)*). Our interest was focused on the formation of ground, $H(1s)$, and excited hydrogen, $H(2s)$, and $H(2p)$ in p-Li scattering. The difference between the four-channel problem and the three- or two-channel problems is in improving the total cross-sections of the considered channel by adding the effect of more kernels of the other three channels (in the two-channel problem, we have only one kernel, and in three channels, we have two kernels), which give more H-formation in the considered states. We expect that we can obtain more hydrogen formation if we open more channels in our calculation, which we will consider in future work.

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