Research Article

Relativistic quantum mechanical spin-1 wave equation in 2 + 1 dimensional spacetime

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Abstract: In this study, we introduce a relativistic quantum mechanical wave equation of the spin-1 particle as an excited state of the zitterbewegung and show that it is consistent with the 2 + 1 dimensional Proca theory. At the same time, we see that in the rest frame, this equation has two eigenstates; particle and antiparticle states or negative and positive energy eigenstates, respectively, and satisfy SO(2,1) spin algebra. As practical applications, we derive the exact solutions of the equation in the presence of a constant magnetic field and a curved spacetime background. From these solutions, we find Noether charge by integrating the zeroth component of the spin-1 particle current, called charge density, on hyper surface and discuss pair production from this charge. And, we see that the discussion on the Noether charge is a useful tool for understanding the pair production phenomenon because it is derived from a probabilistic particle current.

Key words: Spin-1 particle wave equation, Gravity, QED_{2+1}, Pair production, Noether charge

1. Introduction

Physics in 2 + 1 dimensional spacetime presents many interesting and surprising results, both experimentally and theoretically. Therefore, in recent years, 2 + 1 dimensional theories have been widely studied in physical areas, such as gravity, high-energy particle theory, condensed matter physics (e.g., monolayer structures), topological field theory, and string theory [1–11]. In the view of general relativity, 2 + 1 dimensional gravity has a number of solutions such as black hole [12], wormhole [13], and propagating gravitational wave [14,15]. On the other hand, in the view of the 2 + 1 quantum electrodynamics (QED_{2+1}), the properties of an electron in graphene can be described by the 2 + 1 dimensional Dirac equation [16–19]. Also, the interquark potential in quantum chromodynamics is studied in these dimensions and it has a rich structure, as in the 3 + 1 dimensional spacetime [20,21]. Additionally, in the QED_{2+1} context, massive or massless Dirac equation has provided important physical results in flat [22–26] and curved backgrounds [27–31]. Although the physical properties of the Dirac particle has been widely investigated in the context, the physical behavior of the massive or massless spin-1 particle has almost never been considered quantum mechanically so far in this framework.

Duffin–Kemmer–Petiau (DKP) equation as a relativistic quantum mechanical wave equation for the spin-1 and spin-0 particles has a long history [32–38]. The equation has been discussed in the 3 + 1 spacetime dimensions to see whether or not there exists a classical correspondence in the context of the Maxwell theory [39]. Also, the relativistic quantum mechanical wave equation for the spin-1 particles is derived as an excited

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state of zitterbewegung in the same dimensions, but it does not include spin-0 part [40–42]. In this connection, the massless case of the equation was derived as a simple model of the zitterbewegung [43] and its equivalence with the Maxwell equations in a flat and curved background were discussed [43,44]. The spin-1 equation was also solved in the exponentially expanding universe and showed that its solutions are identical with complexified Maxwell equations in the limit $m^2 \to 0$ [45].

In 2 + 1 dimensional spacetime, the electrodynamical events are, classically, described by the 2 + 1 dimensional massive Proca gauge theory and Maxwell–Chern–Simons theory, and they are completely different from the Maxwell theory. Also, contrary to even dimensional cases, in these massive gauge theories; the gauge fields, $F^{\nu\rho}$, and potentials, A_μ, are related to each other as follows:

$$A_\mu = \frac{1}{2m}\varepsilon_{\mu\nu\rho}F^{\nu\rho}, \tag{1}$$

[46–51] and, at the same time, the spin-1 particle wave equation was discussed in this context. Also, with the pseudoclassical approach, the spin-1 particle wave equation in the 2 + 1 dimensions was studied by canonical quantization [52–54]. Nevertheless, discussions still continue on such an equation in the 2+1 dimensional spacetime structure, physically and mathematically [55–58]. From all these points of view, a relativistic quantum mechanical wave equation for a spin-1 particle in the 2 + 1 dimensional spacetime should be expected to satisfy the Proca equation and the relation between vector potentials, A_μ, and tensor fields, $F^{\mu\nu}$, in classical limit. On the other hand, to analyze the physical and mathematical behavior of a spin-1 particle interacting with a 2 + 1 (or 1 + 1) dimensional potential, it has been solved by using the 3 + 1 dimensional DKP equation, but solutions performed this way bring about a number of problems [59,60]. With these motivations, we discuss, in detail, a relativistic quantum mechanical wave equation for a spin-1 particle directly derived from the 2 + 1 dimensional Barut–Zanghi model [40] and find a relation between this equation and the 2 + 1 dimensional Proca theory. As a practical application, we discuss the exact solutions of the wave equation in a constant magnetic field and a curved background. In addition, we write a current expression of the spin-1 particle in this context and calculate it from these solutions. And finally, we find the Noether charge derived from the zeroth component of the current, called charge density, and discuss pair production in terms of this charge. And, we see that such a discussion is a useful tool for understanding the pair production phenomenon since the charge is derived from a probabilistic particle current.

The outline of this study is as follows: In Section 2, in the 2 + 1 dimensional spacetime, we introduce a relativistic quantum mechanical wave equation for a spin-1 particle as an excited state of the classical zitterbewegung model and construct its free particle solutions. We also discuss the particle and antiparticle solutions of the equation in the rest frame and show that the equation is equivalent to the Proca equation by defining vector potentials and electromagnetic fields in terms of the components of the spin-1 particle spinor. In Section 3, we obtain the exact solutions of the spin-1 particle equation in a constant magnetic field. From these solutions, we derive the energy eigenvalues of the particle and write the current densities and Noether charge. In Section 4, we find the exact solutions of the equation in the contracting and expanding 2 + 1 dimensional curved background and we derive the current and Noether charge from these solutions. In Section 5, we evaluate the results of the study.

2. The spin-1 particle in the 2+1 dimensional flat spacetime

A relativistic quantum mechanical wave equation for the spin-1 particle introduced in the $3 + 1$ dimensions was discussed as an excited state of the classical zitterbewegung model [40–42]. As the classical model of the zitterbewegung [40] in the $2 + 1$ dimensional spacetime, for which its symmetry and integrability properties are investigated [61], is quantized, it directly gives us the following relativistic quantum mechanical wave equation for the evolution of the free spin-1 particle [41]:

$$\{(\bar{\sigma}^\mu \otimes 1 + 1 \otimes \bar{\sigma}^\mu) P_\mu - (1 \otimes 1) 2M\} \Psi_{\alpha\beta}(x) = 0, \tag{2}$$

where M and P_μ are the mass and energy-momentum of the particle, respectively, and $\bar{\sigma}^\mu$ is defined in terms of the Pauli matrices as $\bar{\sigma}^\mu = (\sigma^3, i\sigma^1, i\sigma^2)$, and 1 is unit matrix. The matrices $\bar{\sigma}^\mu$ satisfy anticommutation relations in $2 + 1$ dimensional spacetime:

$$\{\bar{\sigma}_\mu, \bar{\sigma}_\nu\} = 2\eta_{\mu\nu},$$

where $\eta_{\mu\nu} = diag(1, -1, -1)$ is Minkowski metric in $2 + 1$ dimensional spacetime. On the other hand, the spinor of the spin-1 particle, $\Psi_{\alpha\beta}(x)$, is defined as

$$\Psi_{\alpha\beta}(r, t) = (\psi_+, \psi_0, \psi_0, \psi_-)^T, \tag{3}$$

where the T means transpose of the row matrix. In this model, the wave function, $\Psi_{\alpha\beta}(x)$, is the symmetric spinor with rank 2 and it is represented as a direct product of two Dirac spinors in the $2 + 1$ dimensions and the quantization of the classical system requires that $\Psi_{\alpha\beta}(x)$ should be symmetric with respect to the indices α, β, where the first (second) indices correspond to the first (second) set of the Dirac spinors. For this reason, $\Psi_{\alpha\beta}(x)$ has three linear independent components: ψ_+, ψ_0, ψ_-.

If we choose a plane wave solution as follows:

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \phi_+ \\ \phi_0 \\ \phi_0 \\ \phi_- \end{pmatrix} e^{-ip_\mu x^\mu}, \tag{4}$$

then the explicit form of the spin-1 particle equation is written in terms of the spinor components, ϕ_+, ϕ_0, ϕ_-:

$$(P_0 - M)\phi_+ + (iP_1 + P_2)\phi_0 = 0,$$

$$(iP_1 - P_2)\phi_+ + (iP_1 + P_2)\phi_- - 2M\phi_0 = 0, \tag{5}$$

$$(P_0 + M)\phi_- - (iP_1 - P_2)\phi_0 = 0.$$

If we add and subtract the first and third rows and organize the second row of Eq. (5), the following equations are obtained, respectively,

$$P_0(\phi_+ - \phi_-) + iP_1(2\phi_0) = M(\phi_+ + \phi_-),$$

$$P_0(\phi_+ + \phi_-) + P_2(2\phi_0) = M(\phi_+ - \phi_-), \tag{6}$$

$$iP_1(\phi_+ + \phi_-) - P_2(\phi_+ - \phi_-) = M(2\phi_0).$$

Now, we reorganize Eq. (6) as a first order partial differential equation system:

$$\partial^0 \left(i\frac{\phi_+ - \phi_-}{\sqrt{M}} \right) - \partial^1 \left(\frac{2\phi_0}{\sqrt{M}} \right) = \sqrt{M} \left(\phi_+ + \phi_- \right),$$

$$\partial^0 \left(-\frac{\phi_+ + \phi_-}{\sqrt{M}} \right) - \partial^2 \left(\frac{2\phi_0}{\sqrt{M}} \right) = i\sqrt{M} \left(\phi_+ - \phi_- \right), \tag{7}$$

$$\partial^1 \left(-\frac{\phi_+ + \phi_-}{\sqrt{M}} \right) - \partial^2 \left(i\frac{\phi_+ - \phi_-}{\sqrt{M}} \right) = \sqrt{M} \left(2\phi_0 \right).$$

From this equation system, if we define the complex gauge potentials and fields in terms of the spinor components as spinor valued gauge-one forms and two forms respectively as follows:

$$A^0 = \frac{2\phi_0}{\sqrt{M}}, \quad A^1 = i\frac{\phi_+ - \phi_-}{\sqrt{M}}, \quad A^2 = -\frac{\phi_+ + \phi_-}{\sqrt{M}} \tag{8}$$

and

$$F^{01} = \sqrt{M} \left(\phi_+ + \phi_- \right), \quad F^{02} = i\sqrt{M} \left(\phi_+ - \phi_- \right), \quad F^{12} = \sqrt{M} \left(2\phi_0 \right), \tag{9}$$

then using the relations given in Eqs. (8) and (9), we see that Eq. (7) satisfies the well-known relations between the gauge fields and potentials as follows:

$$\partial^\mu A^\nu - \partial^\nu A^\mu = F^{\mu\nu}. \tag{10}$$

According to the definitions in Eqs. (8) and (9), Eq. (7) becomes:

$$i\sqrt{M}\partial_0 \left(\phi_+ - \phi_- \right) + \sqrt{M}\partial_1 \left(2\phi_0 \right) = M^2 \left(\frac{\phi_+ + \phi_-}{\sqrt{M}} \right),$$

$$i\sqrt{M}\partial_0 \left(\phi_+ + \phi_- \right) - i\sqrt{M}\partial_2 \left(2\phi_0 \right) = M^2 \left(\frac{\phi_+ - \phi_-}{\sqrt{M}} \right), \tag{11}$$

$$\sqrt{M}\partial_1 \left(\phi_+ + \phi_- \right) + i\sqrt{M}\partial_2 \left(\phi_+ - \phi_- \right) = M^2 \left(\frac{2\phi_0}{\sqrt{M}} \right).$$

and these equations are in the form of the massive Proca equation in the 2 + 1 dimensional spacetimes:

$$\partial_\mu F^{\mu\nu} + M^2 A^\nu = 0. \tag{12}$$

In particular, from the definitions in Eqs. (8) and (9), we notice that the expressions of the vector potentials and fields in Eqs. (8) and (9), respectively, are related to each other by the following relations:

$$A^\mu = \frac{1}{2M} \varepsilon^{\mu\nu\rho} F_{\nu\rho}, \tag{13}$$

where $\varepsilon^{\mu\nu\rho}$ is the Levi-Civita symbol. These results are consistent with [51], and also if we eliminate the potentials by using Eq. (13) in the Proca equation, Eq. (12), then we derive the following equation:

$$\partial_\mu F^{\mu\nu} + \frac{M}{2} \varepsilon^{\nu\alpha\beta} F_{\alpha\beta} = 0. \tag{14}$$

Therefore, we say that these equations are compatible with the results of the topologically massive gauge theories [48–51].

To discuss the free particle solutions of the spin-1 particle from Eq. (5), at first, the components ϕ_+ and ϕ_- are written as

$$\phi_\pm = \frac{P}{P_0 \mp M} e^{\mp i\left(\varphi + \frac{\pi}{2}\right)} \phi_0, \tag{15}$$

where $tan\varphi = P_2/P_1$ and P is the magnitude of the momentum vector, $\mathbf{P} = (P_1, P_2)$. Then, substituting ϕ_+ and ϕ_- into the second row of Eq. (5), we obtain the following free particle relativistic wave equation for ϕ_0:

$$\left(P_0^2 - P_1^2 - P_2^2 - M^2\right) \phi_0 = 0. \tag{16}$$

This is the Proca equation for ϕ_0 in the 2 + 1 dimensions and it has the correct energy-momentum condition. Then, the normalized wave function for Eq. (4) is obtained as

$$\Psi(\mathbf{r}, t) = \frac{1}{2\sqrt{|P_0|M}} \begin{pmatrix} (M + P_0) e^{-i\left(\varphi + \frac{\pi}{2}\right)} \\ P \\ P \\ (M - P_0) e^{i\left(\varphi + \frac{\pi}{2}\right)} \end{pmatrix} e^{-ip_\mu x^\mu}, \tag{17}$$

where the normalization condition is given by

$$\Psi(\mathbf{r}, t)^{*T} \Psi(\mathbf{r}, t) = \frac{|P_0|}{M}. \tag{18}$$

In the rest frame, the particle has only two states which are

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-iMt} \quad \text{for } P_0 = M \tag{19}$$

and

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{iMt} \quad \text{for } P_0 = -M. \tag{20}$$

They are the particle and antiparticle solutions, and at same time, they are eigenstates of spin operator, S_{12}, with eigenvalues $+1$ and -1, respectively.

On the other hand, Eq. (5) can also be rewritten as matrix equation in the following form:

$$\left(\beta^\mu P_\mu - M\right) \Psi = 0,$$

where the complex spinor Ψ has the three linear independent components in this representation:

$$\Psi(x_1, x_2, t) = \left(\psi_+, \ \sqrt{2}\psi_0, \ \psi_-\right)^T$$

and β matrices are Hermitian spin-1 matrices in the $2 + 1$ dimensional spacetime and they can be defined as

$$\beta^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \beta^1 = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \beta^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \tag{21}$$

and, it is clear that they satisfy the following SO(2,1) spin-1 algebra:

$$[\beta^\mu, \beta^\nu] = -i\epsilon^{\mu\nu\rho}\beta_\rho.$$

These matrices also satisfy the following relation:

$$\eta_{\mu\nu}\beta^\mu\beta^\nu = 2I.$$

To discuss the conserved current for the spin-1 particle, we also point out that the Hermitian conjugate of the β^μ matrices obeys the following transformation:

$$(\beta^\mu) = \gamma (\beta^\mu)^{T*} \gamma,$$

where γ is

$$\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{22}$$

and the Hermitian conjugate of the wave function is

$$\bar{\psi} = (\psi)^{T*} \gamma. \tag{23}$$

Then, in this context, the conserved current for the spin-1 particle becomes

$$J^\mu = \bar{\psi}\beta^\mu\psi \tag{24}$$

and the current components in terms of the spin-1 particle spinor components are explicitly given as follows:

$$J^0 = |\phi_+|^2 - |\phi_-|^2,$$
$$J^1 = i (\phi_+ + \phi_-)^* \sqrt{2}\phi_0 - i\sqrt{2}\phi_0^* (\phi_+ + \phi_-), \tag{25}$$
$$J^2 = (\phi_+ - \phi_-)^* \sqrt{2}\phi_0 + \sqrt{2}\phi_0^* (\phi_+ - \phi_-).$$

By means of the definitions in Eqs. (8) and (9), the components of the current may also be written in terms of the Proca fields as

$$J^0 = \frac{1}{4M} \left[|F^{01} - iF^{02}|^2 - |F^{01} + iF^{02}|^2 \right],$$
$$J^1 + iJ^2 = \frac{i}{4M} \left[\left(F^{01} - iF^{02} \right)^* F^{12} \right] - \frac{i}{4M} \left[\left(F^{12} \right)^* \left(F^{01} + iF^{02} \right) \right], \tag{26}$$

where J^0 corresponds to the difference between the right-handed and left-handed electric fields and J^1 and J^2 are the generalization of the Poynting vectors into the $2 + 1$ dimensional complex fields. Furthermore, the current components can be expressed in terms of the gauge potentials and fields as follows:

$$J^0 = \frac{i}{4} \left[\left(A_1^* F^{10} - A_1 \left(F^{10} \right)^* \right) + \left(A_2^* F^{20} - A_2 \left(F^{20} \right)^* \right) \right], \tag{27}$$

$$J^1 = -\frac{i}{4}\left[\left(A_0^* F^{01} - A_0\left(F^{01}\right)^*\right) + \left(A_2^* F^{21} - A_2\left(F^{21}\right)^*\right)\right],\tag{28}$$

$$J^2 = -\frac{i}{4}\left[\left(A_0^* F^{02} - A_0\left(F^{02}\right)^*\right) + \left(A_1^* F^{12} - A_1\left(F^{12}\right)^*\right)\right].\tag{29}$$

It is also important that these results are, formally, consistent with the following expression for the conserved current of the Proca Fields in $3+1$ dimensions [62]:

$$J_\mu = \varphi^\nu G_{\mu\nu}^* - G_\mu^\nu \varphi_\nu^*,$$

where φ^ν is four vector potential, $G_{\mu\nu}$ is the antisymmetric field-strength tensor.

3. The spin-1 particle in a constant magnetic field

The motion of charged particles in a magnetic field is an important problem in classical and quantum electrodynamics. This problem is a 2+1 dimensional problem because of the axial symmetry [63,64]. Therefore, to discuss the relativistic quantum mechanical behavior of the charged spin-1 particle in a constant magnetic field, we write the relativistic wave equation for the spin-1 particle in the presence of electromagnetic fields as

$$\{(\sigma^\mu \otimes 1 + 1 \otimes \sigma^\mu)\pi_\mu - 2\,(1 \otimes 1)\,M\}\Psi = 0,\tag{30}$$

where π_μ is the generalized momentum of the particle and the explicit form of it in terms of an electromagnetic potential, A_μ, is $\pi_\mu = p_\mu - eA_\mu$, where e is the charge of the particle. Then, Eq. (30) becomes

$$\begin{bmatrix} 2\left(\pi_0 - M\right) & i\left(\pi_1 - i\pi_2\right) & i\left(\pi_1 - i\pi_2\right) & 0 \\ i\left(\pi_1 + i\pi_2\right) & -2M & 0 & i\left(\pi_1 - i\pi_2\right) \\ i\left(\pi_1 + i\pi_2\right) & 0 & -2M & i\left(\pi_1 - i\pi_2\right) \\ 0 & i\left(\pi_1 + i\pi_2\right) & i\left(\pi_1 + i\pi_2\right) & -2\left(\pi_0 + M\right) \end{bmatrix}\begin{bmatrix} \psi_+ \\ \psi_0 \\ \psi_0 \\ \psi_- \end{bmatrix} = 0.\tag{31}$$

Letting $\pi_\pm = \pi_1 \pm i\pi_2$, and writing π_0 and $\pi_\pm$ in polar coordinates, (r,θ) for a constant magnetic field, i.e. $A_0 = 0$, $A_1 = 0$, $A_2 = Br/2$ as follows:

$$\pi_0 = iE,$$

$$\pi_\pm = M\zeta e^{-i\theta}\left(-i\frac{\partial}{\partial\sqrt{\rho}} \pm \frac{1}{\sqrt{\rho}}\frac{\partial}{\partial\theta} \mp i\sqrt{\rho}\right),$$

where B is magnitude of the constant magnetic field, $\zeta = \sqrt{\frac{eB}{2M^2}}$, $\epsilon = \frac{E}{M}$ and $\rho = \frac{eBr^2}{2}$. Using separation of variables method, the components of the general wave function can be written as follows:

$$\psi_+\left(\rho,\theta\right) = e^{-iEt+i(k+1)\theta}\varphi_1\left(\rho\right),$$

$$\psi_-\left(\rho,\theta\right) = e^{-iEt+i(k-1)\theta}\varphi_{-1}\left(\rho\right),\tag{32}$$

$$\psi_0\left(\rho,\theta\right) = e^{-iEt+ik\theta}\varphi_0\left(\rho\right).$$

And substituting the expressions of π_0 and $\pi_\pm$ and the components of the general wave function into Eq. (31), it becomes

$$\frac{(\epsilon - 1)}{\zeta} \varphi_1(\rho) + 2\sqrt{\rho}\left(\frac{d}{d\rho} + \frac{k}{2\rho} - \frac{1}{2}\right)\varphi_0(\rho) = 0,$$

$$\frac{(\epsilon + 1)}{\zeta} \varphi_{-1}(\rho) - 2\sqrt{\rho}\left(\frac{d}{d\rho} - \frac{k}{2\rho} + \frac{1}{2}\right)\varphi_0(\rho) = 0, \tag{33}$$

$$\zeta\sqrt{\rho}\left(\frac{d}{d\rho} - \frac{(k-1)}{2\rho} + \frac{1}{2}\right)\varphi_1(\rho) + \zeta\sqrt{\rho}\left(\frac{d}{d\rho} + \frac{(k+1)}{2\rho} - \frac{1}{2}\right)\varphi_{-1}(\rho) = \varphi_0(\rho).$$

Letting $\varphi_0(\rho) = \frac{\chi(\rho)}{\sqrt{\rho}}$, we get the Whittaker differential equation:

$$\frac{d^2\chi(\rho)}{d\rho^2} + \left(-\frac{1}{4} + \frac{\lambda}{\rho} + \frac{\frac{1}{4} - \frac{k^2}{4}}{\rho^2}\right)\chi(\rho) = 0. \tag{34}$$

Then, we can construct the general solution in terms of the Whittaker functions, $M_{\lambda,\frac{k}{2}}(\rho)$, as follows;

$$\begin{pmatrix} \psi_+ \\ \psi_0 \\ \psi_- \end{pmatrix} = Ne^{i(k\theta - Et)} \begin{pmatrix} \frac{-2\zeta e^{i\theta}}{\epsilon - 1}\left[\left(\frac{k}{\rho} - 1\right)M_{\lambda,\frac{k}{2}}(\rho) + \frac{\left(\frac{k+1}{2} - \lambda\right)}{k+1}M_{\lambda,\frac{k}{2}+1}(\rho)\right] \\ \frac{M_{\lambda,\frac{k}{2}}(\rho)}{\sqrt{\rho}} \\ \frac{2\zeta e^{-i\theta}}{\epsilon + 1}\frac{\left(\frac{k+1}{2} - \lambda\right)}{k+1}M_{\lambda,\frac{k}{2}+1}(\rho) \end{pmatrix},$$

where $\lambda - \frac{k}{2} = \frac{\epsilon^2 - 1}{4\zeta^2} - \frac{\epsilon}{2} = n + \frac{1}{2}$ and N is the normalization constant. From this solution, the energy eigenvalues are obtained as

$$E_\pm = M\left(\zeta^2 \pm \sqrt{\zeta^4 + 1 + 2\zeta^2(2n+1)}\right). \tag{35}$$

On the other hand, taking the asymptotic form of the solutions in Eq. (32) [65], we get the components of the current in Eq. (24) as follows:

$$J^0 = |N|^2 \frac{\Gamma(k+1)^2}{\Gamma(n+k+1)^2}\frac{4}{M}\zeta^2\rho^{2n+k-1}e^{-\rho}\left[\left(\frac{n+1+\rho}{\epsilon - 1}\right)^2 - \left(\frac{n}{\epsilon + 1}\right)^2\right], \tag{36}$$

$$J^1 + iJ^2 = |N|^2 \frac{\Gamma(k+1)^2}{\Gamma(n+k+1)^2}4M\zeta\rho^{2n+k-\frac{1}{2}}e^{-i\theta}e^{-\rho}\left[2\epsilon n + (\epsilon + 1)(1+\rho)\right], \tag{37}$$

where the zeroth component of the current, J^0, is called charge density and the rest component of the current, J^1 and J^2, are called current densities, [66,67], and

$$|N| = \frac{\Gamma(\alpha)M(\epsilon^2 - 1)}{\Gamma(2n+k)^{\frac{1}{2}}\Gamma(k+1)\left((\epsilon + 1)^2\left[(k+1)^2 - 4 + (\alpha + n)\beta\right] - (\epsilon - 1)^2 4\alpha^2\right)^{\frac{1}{2}}},$$

where $\alpha = n+k+1$ and $\beta = 18n+3k+7$. Under the strong magnetic field, i.e. $\xi^4 \gg 1$, the energy eigenvalues in Eq. (35) and the zeroth component of the current in Eq. (36), i.e. the charge density, become respectively,

$$E_+ \approx 2M\zeta^2 + M\left(2n+1\right) + O\left(\zeta^{-2}\right), \tag{38}$$

$$E_- \approx -M\left(2n+1\right) + O\left(\zeta^{-2}\right), \tag{39}$$

and

$$J_+^0 = \frac{2M\zeta^2\left[(n+1+\rho)^2 - n^2\right]e^{-\rho}\rho^{2n+k-1}}{\pi\Gamma\left(2n+k\right)\left[(k+1)^2 - 4 + (\alpha+n)\beta - 4\alpha^2\right]}, \tag{40}$$

$$J_-^0 = \frac{2M\zeta^2 n^2\left[\rho + 2(n+1)\right]e^{-\rho}\rho^{2n+k}}{\pi\Gamma(2n+k)\left[n^2\left(k+1\right)^2 - 4 + (\alpha+n)\beta - 4(n+1)^2\alpha^2\right]}. \tag{41}$$

On the other hand, under the weak magnetic field condition, i.e. $\xi^4 \ll 1$, the positive and negative energy eigenvalues become

$$\tilde{E}_+ \approx M\left(1 + 2\left(n+1\right)\zeta^2\right) + O\left(\zeta^4\right), \tag{42}$$

$$\tilde{E}_- \approx -M\left(1 + 2n\zeta^2\right) + O\left(\zeta^4\right) \tag{43}$$

and then the charge densities for the $\pm$ energy eigenvalues are calculated as

$$\tilde{J}_+^0 = \frac{2M\zeta^2 2n+k-1 e^{-}\left(n+1+\right)^2}{\pi\left(2n+k\right)\left[(k+1)^2 - 4 + (\alpha+n)\beta\right]}, \tag{44}$$

$$\tilde{J}_-^0 = \frac{2M\zeta^2 2n+k-1 e^{-}n^2}{\pi(2n+k)4\alpha^2}, \tag{45}$$

respectively. Taking the charge densities, J_+^0 and $\tilde{J}_+^0$ the Noether charges of these densities are computed by the following relations:

$$Q_\pm = \int \left(J^0\right)_\pm dS_0 \tag{46}$$

where the $Q_\pm$ are Noether charges and the dS_0 is a hypersurface [67–70]. If the currents, $J_\pm^0$ and $\tilde{J}_\pm^0$, are integrated on the hypersurface, $dS_0 = \rho d\rho d\theta$, we get the Noether charges, respectively, as follows;

$$\frac{Q_+}{M} = \frac{2\zeta^2\left[(2n+1) + (2n+k)\left(4n+k+3\right)\right]}{\pi\left[(k+1)^2 - 4 + (\alpha+n)\beta - 4\alpha^2\right]}, \tag{47}$$

$$\frac{Q_-}{M} = \frac{2\zeta^2 n^2(2n+k)(4n+k+3)}{\pi n^2\left[(k+1)^2 - 4 + (\alpha+n)\beta\right] - 4(n+1)^2\alpha^2}, \tag{48}$$

$$\frac{\tilde{Q}_+}{M} = \frac{2\zeta^2 \left[(n+1)^2 + (2n+k)(4n+k+3)\right]}{\pi \left[(k+1)^2 - 4 + (\alpha+n)\beta\right]}, \tag{49}$$

$$\frac{\tilde{Q}_-}{M} = \frac{\zeta^2 n^2}{2\pi\alpha^2}. \tag{50}$$

We consider the behavior of the proportion $\frac{Q}{M}$ according to the strong and weak magnetic fields, see Figures 1 and 2. From these figures, we see that the $\frac{Q_+}{M}$ values coincide with the $\frac{Q_-}{M}$ values as from $n \geq 10$ under the strong magnetic field conditions while the $\frac{\tilde{Q}_+}{M}$ values coincide with the $\frac{\tilde{Q}_-}{M}$ values as from $n \geq 750$ under the weak magnetic field conditions; however, under both conditions, the difference between the $\frac{Q_+}{M}$ values and the $\frac{Q_-}{M}$ values increase as the n values get smaller. Also, given the Noether charge derived from the probabilistic particle current, we can say that the magnetic field on the particle production is more effective in the small n values with respect to the antiparticle production, but for the large n values it is same.
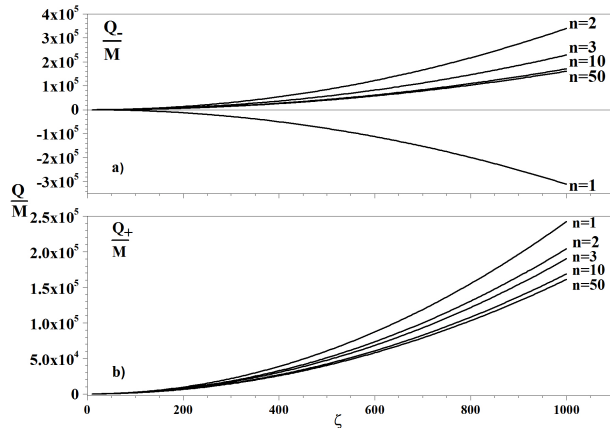
Figure 1. The graph presents $\frac{Q_\pm}{M}$ as a function of $\zeta (\zeta^4 \gg 1, \ k = \frac{3}{2})$. a) The curve for $\frac{Q_-}{M}$ presents a negative increase for $n = 1$, and it has a positive maximum value for $n = 2$. It also decreases with increasing values of n. b) The $\frac{Q_+}{M}$ curve reaches to a maximum value for $n = 1$. Increasing n shows a similar behavior to $\frac{Q_-}{M}$.
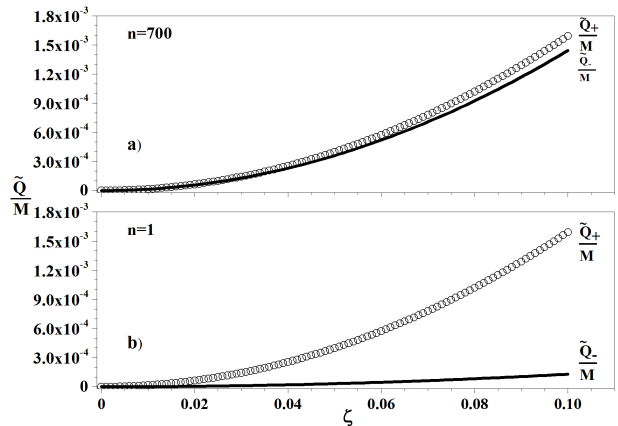
Figure 2. The $\frac{\tilde{Q}_\pm}{M}$ is plotted as a function of $\zeta (\zeta^4 \ll 1, \ k = \frac{3}{2})$. The $\frac{\tilde{Q}_+}{M}$ curves for all of n values get the same values. $\frac{\tilde{Q}_-}{M}$ curves change with increasing n values. Then curves approach to $\frac{\tilde{Q}_+}{M}$ curves. The curves merge for $n \geq 750$. The $\frac{\tilde{Q}_+}{M}$ is plotted as a function of $\zeta (\zeta^4 \ll 1, \ k = \frac{3}{2})$. For $n = 1$ and $n = 700$ in respective panels.

4. The spin-1 particle in the (2 + 1) dimensional curved spacetime

To discuss the physical and mathematical features of the spin-1 particle in a $2 + 1$ dimensional spacetime curved background, the relativistic quantum mechanical wave equation of the spin-1 particle is easily generalized to a curved spacetime as follows:

$$\left[\left(\bar{\sigma}^\mu \otimes 1 + 1 \otimes \bar{\sigma}^\mu\right)\left(P_\mu - i\Omega_\mu\right) - \left(1 \otimes 1\right) 2M\right]\Psi = 0, \tag{51}$$

where $\bar{\sigma}^\mu(x)$ are the spacetime dependent Dirac matrices and Ω_μ is the spin connection of the spin-1 particle in $2 + 1$ dimensions and its expression in terms of the spin$-\frac{1}{2}$ connection of the spin particles, $\Gamma_\mu(x)$, is written

as

$$\Omega_\mu = \Gamma_\mu(x) \otimes 1 + 1 \otimes \Gamma_\mu(x).$$ (52)

To derive solutions of the spin-1 particle wave equation in a curved spacetime, as an example, we consider the following 2 + 1 dimensional gravitational background [71]:

$$ds^2 = l^2 \left[d\tau^2 - cosh^2\tau \left(d\theta^2 + sin^2\theta d\phi^2 \right) \right],$$ (53)

where $\tau = t/l$ and $\tau \in (-\infty, \infty)$, $\theta \in [0, \pi)$, $\phi \in [0, 2\pi)$ and l is the radius of the universe and related to the cosmological constant, Λ, as $l = \frac{1}{|\Lambda|}$. Also, the universe is two spheres. It contracts to its minimum area of $4\pi l^2$ at $\tau = 0$ and expands again [71]. Then, the metric tensor of the gravitational background and its inverse are written in the following way:

$$g_{\mu\nu} = diag \left(l^2, -l^2 cosh^2\tau, -l^2 cosh^2\tau \, sin^2\theta \right),$$

$$g^{\mu\nu} = diag \left(1/l^2, -1/l^2 cosh^2\tau, -1/l^2 cosh^2\tau \, sin^2\theta \right),$$ (54)

respectively, and the $g^{\mu\nu}$ is defined in terms of the triad fields, $e_i^\mu(x)$, as follows:

$$g^{\mu\nu} = e_i^\mu(x) \, e_j^\nu(x) \, \eta^{ij}.$$ (55)

The triads of the curved background are explicitly written as

$$e_i^\mu(x) = diag \left(1/l, \, 1/lcosh\tau, 1/lcosh\tau \, sin\theta \right).$$ (56)

The spacetime dependent Dirac matrices, $\bar{\sigma}^\mu(x)$, are presented in terms of the triads and the constant Dirac matrices, $\bar{\sigma}^i$, as follows:

$$\bar{\sigma}^\mu = e_i^\mu(x) \, \bar{\sigma}^i.$$ (57)

And, the explicit form of the $\Gamma_\mu(x)$ is

$$\Gamma_\mu(x) = -\frac{1}{8} g_{\alpha\nu} \Gamma_{\beta\mu}^\nu \left[\bar{\sigma}^\alpha(x), \bar{\sigma}^\beta(x) \right],$$ (58)

where $\Gamma_{\beta\mu}^\nu$ are the Christoffel symbols, and also its components in the curved background [27] are written as

$$\Gamma_0 = 0, \; \Gamma_1 = -\frac{1}{2} \bar{\sigma}^0 \bar{\sigma}^1 sinh\tau,$$

$$\Gamma_2 = -\frac{1}{2} \left(\bar{\sigma}^0 \bar{\sigma}^2 sinh\tau \, sin\theta + \bar{\sigma}^1 \bar{\sigma}^2 \, cos\theta \right).$$ (59)

Then, Eq. (51) in the curved background is as follows:

$$\left(\partial_\tau + tanh\tau + iMl \right) \psi_+ - \frac{i}{cosh\tau} \left(\partial_\theta - i\frac{\partial_\phi}{sin\theta} \right) \psi_0 = 0,$$

$$\frac{i}{cosh\tau}\left(\partial_\theta + i\frac{\partial_\phi}{sin\theta}+cot\theta\right)\psi_+ + \frac{i}{cosh\tau}\left(\partial_\theta - i\frac{\partial_\phi}{sin\theta}+cot\theta\right)\psi_- = -2iMl\psi_0, \tag{60}$$

$$\frac{i}{cosh\tau}\left(\partial_\theta + i\frac{\partial_\phi}{sin\theta}\right)\psi_0 - \left(\partial_\tau + tanh\tau - iMl\right)\psi_- = 0.$$

To find the general solution, we use the separation of variable method. In this connection, the rising and lowering operators of the spin-1 particle, $\partial_\pm$, are defined as

$$\partial_\pm = \left(\mp\partial_\theta + i\frac{\partial_\phi}{sin\theta}+\frac{1}{2}\left(\bar{\sigma}^0 \otimes 1 + 1 \otimes \bar{\sigma}^0\right)cot\theta\right) \tag{61}$$

and their eigenfunctions in terms of the rotation group $d_{\lambda,m}^j(\theta)$ are expressed as

$$D_{\lambda,m}^j(\theta,\phi) = \langle\lambda|R(\theta,\phi)|jm\rangle = e^{i\lambda\phi}d_{\lambda,m}^j(\theta). \tag{62}$$

And, the irreducible representations of the rotation group $d_{\lambda,m}^j(\theta)$ are given by

$$d_{\lambda,m}^j(\theta) = \frac{(-1)^{j-m}}{(\lambda+m)!}\sqrt{\frac{(j+\lambda)!\,(j+m)!}{(j-\lambda)!\,(j-m)!}}sin(\theta/2)^{2j}cot(\theta/2)^{\lambda+m}$$

$$\times {}_2F_1\left(\lambda - j, m - j; \lambda + m + 1; -cot^2(\theta/2)\right) \tag{63}$$

[72]. From the separation of variable method, $\Psi(\tau,\theta,\phi)$ can be expanded in terms of the angular momentum eigenfunctions as

$$\Psi(\tau,\theta,\phi) = 4\pi\sum_{jm}\frac{(2j+1)}{cosh\tau}\begin{pmatrix} F_+(\tau)D_{+1,m}^j(\theta,\phi) \\ F_0(\tau)D_{0,m}^j(\theta,\phi) \\ F_-(\tau)D_{-1,m}^j(\theta,\phi) \end{pmatrix} \tag{64}$$

and the $\partial_\pm$ operators act on $D_{\lambda,m}^j(\theta,\phi)$ as follows

$$\partial_\pm D_{\lambda,m}^j(\theta,\phi) = \sqrt{(j\pm\lambda+1)(j\mp\lambda)}D_{\lambda\pm1,m}^j(\theta,\phi). \tag{65}$$

Then, Eq. (60) is rewritten as

$$(iMl + \partial_\tau)F_+(\tau) - \frac{i}{cosh\tau}\sqrt{j(j+1)}F_0(\tau) = 0,$$

$$(iMl - \partial_\tau)F_-(\tau) + \frac{i}{cosh\tau}\sqrt{j(j+1)}F_0(\tau) = 0,$$

$$MlF_0(\tau) - \frac{1}{2cosh\tau}\sqrt{j(j+1)}\left(F_-(\tau) - F_+(\tau)\right) = 0. \tag{66}$$

From these equations, the $F_0(\tau)$ can be defined as

$$F_0(\tau) = \frac{1}{2Mlcosh\tau}\sqrt{j(j+1)}\left(F_-(\tau) - F_+(\tau)\right) = 0, \tag{67}$$

and substituting it into the first and second rows of Eq. (66), we get the following equations for the $F_\pm(\tau)$:

$$\left[\partial_\tau^2 + 2\left(tanh\tau\right)\partial_\tau \pm 2iMltanh\tau + M^2l^2 + \frac{j(j+1)}{cosh^2\tau}\right]F_\pm(\tau) = 0. \tag{68}$$

These differential equations are satisfied by the following solutions [73]:

$$F_+(\tau) = \left(\frac{1 - tanh\tau}{2}\right)^{j+1}\left[C_+\xi(\tau)^{-\frac{iMl}{2}}F_1(\tau) + D_-\xi(\tau)^{\frac{iMl}{2}+1}F_2(\tau)\right], \tag{69}$$

$$F_-(\tau) = -\left(\frac{1 - tanh\tau}{2}\right)^{j+1}\left[D_+\xi(\tau)^{-\frac{iMl}{2}+1}F_3(\tau) + C_-\xi(\tau)^{\frac{iMl}{2}}F_4(\tau)\right], \tag{70}$$

where the $F_i(\tau)$ functions are abbreviations of the Hypergeometric functions as follows;

$$F_1(\tau) = {}_2F_1\left(-j - iMl, -j - 1; -iMl; -\xi(\tau)\right),$$

$$F_2(\tau) = {}_2F_1\left(-j + iMl, -j + 1; 2 + iMl; -\xi(\tau)\right),$$

$$F_3(\tau) = {}_2F_1\left(-j - iMl, -j + 1; 2 - iMl; -\xi(\tau)\right),$$

$$F_4(\tau) = {}_2F_1\left(-j + iMl, -j - 1; iMl; -\xi(\tau)\right)$$

and $\xi(\tau) = \frac{1+tanh\tau}{1-tanh\tau}$. Also, the coefficients $C_\pm$ and $D_\pm$ are related each other by the following relations:

$$C_\pm = \frac{\left(Ml \pm \frac{i}{2}\right)^2 + \frac{1}{4}}{\left(j + \frac{1}{2}\right)^2 - \frac{1}{4}}D_\pm. \tag{71}$$

To construct the wave function when τ goes to $-\infty$, we consider the asymptotic forms of $F_+(\tau), F_0(\tau)$ and $F_-(\tau)$ in Eqs. (67), (69), and (70), then the wave function is written as

$$\Psi(\tau, \theta, \phi) = \sum_{jm} 2\sqrt{2j+1}e^\tau \begin{pmatrix} (C_+e^{-iMl\tau} + e^{2\tau}D_-e^{iMl\tau})D_{+1,m}^j(\theta, \phi) \\ -\frac{\sqrt{j(j+1)}}{Ml}e^\tau(F_+e^{-iMl\tau} + F_-e^{iMl\tau})D_{0,m}^j(\theta, \phi) \\ -(e^{2\tau}D_+e^{-iMl\tau} + C_-e^{iMl\tau})D_{-1,m}^j(\theta, \phi) \end{pmatrix}, \tag{72}$$

where $F_+ = C_+ + e^{2\tau}D_+$ and $F_- = C_- + e^{2\tau}D_-$ The asymptotic expressions of the particle Ψ^+ and antiparticle Ψ^- solutions can be written separately as the following

$$\Psi^+ = 2\sqrt{2j+1}e^\tau e^{-iMl\tau} \begin{pmatrix} C_+D_{+1,m}^j(\theta, \phi) \\ -\frac{\sqrt{j(j+1)}}{Ml}e^\tau\left(C_+ + e^{2\tau}D_+\right)D_{0,m}^j(\theta, \phi) \\ -D_+e^{2\tau}D_{-1,m}^j(\theta, \phi) \end{pmatrix}, \tag{73}$$

$$\Psi^- = 2\sqrt{2j+1}e^\tau e^{iMl\tau} \begin{pmatrix} D_- e^{2\tau} D^j_{+1,m}(\theta,\phi) \\ -\frac{\sqrt{j(j+1)}}{Ml} e^\tau (C_- + e^{2\tau} D_-) D^j_{0,m}(\theta,\phi) \\ -C_- D^j_{-1,m}(\theta,\phi) \end{pmatrix}, \tag{74}$$

where C_+ and C_- are normalization coefficients, and they are calculated as

$$|C_+| = |C_-| = \frac{1}{l} \left[1 - c_1 e^{2\tau} + 3c_2 e^{4\tau} - c_3 e^{6\tau} \right]^{-\frac{1}{2}}, \tag{75}$$

where c_1, c_2, and c_3 are

$$c_1 = \frac{j(j+1)}{M^2 l^2},$$

$$c_2 = \frac{j^2(j+1)^2}{M^2 l^2 (1 + M^2 l^2)},$$

$$c_3 = \frac{j^3(j+1)^3}{M^4 l^4 (1 + M^2 l^2)}.$$

To discuss the particle creation in this background, we construct the particle and the antiparticle solutions as $\tau \to \infty$. As τ goes to ∞, the wave function is

$$\tilde{\Psi}(\tau,\theta,\phi) = \sum_{jm} 2\sqrt{2j+1}e^{-\tau} \begin{pmatrix} (\tilde{C}_+ e^{-iMl\tau} + e^{-2\tau}\tilde{D}_- e^{iMl\tau}) D^j_{+1,m}(\theta,\phi) \\ -\frac{1}{Ml}\sqrt{j(j+1)}e^{-\tau}(\tilde{F}_+ e^{-iMl\tau} + \tilde{F}_- e^{iMl\tau}) D^j_{0,m}(\theta,\phi) \\ -(e^{-2\tau}\tilde{D}_+ e^{-iMl\tau} + \tilde{C}_- e^{iMl\tau}) D^j_{-1,m}(\theta,\phi) \end{pmatrix}, \tag{76}$$

where $\tilde{F}_+$ and $\tilde{F}_-$ are $\tilde{F}_+ = \tilde{C}_+ + e^{-2\tau}\tilde{D}_+$ and $\tilde{F}_- = \tilde{C}_- + e^{-2\tau}\tilde{D}_-$. The asymptotic expressions of the particle $_+\tilde{\Psi}$ and antiparticle $_-\tilde{\Psi}$ solutions can be written separately as follows:

$$_+\tilde{\Psi} = 2\sqrt{2j+1}e^{-\tau}e^{-iMl\tau} \begin{pmatrix} \tilde{C}_+ D^j_{+1,m}(\theta,\phi) \\ -\frac{\sqrt{j(j+1)}}{Ml}e^{-\tau}\left(\tilde{C}_+ + e^{-2\tau}\tilde{D}_+\right) D^j_{0,m}(\theta,\phi) \\ -e^{-2\tau}\tilde{D}_+ D^j_{-1,m}(\theta,\phi) \end{pmatrix}, \tag{77}$$

$$_-\tilde{\Psi} = 2\sqrt{2j+1}e^{-\tau}e^{iMl\tau} \begin{pmatrix} \tilde{D}_- e^{-2\tau} D^j_{+1,m}(\theta,\phi) \\ -\frac{\sqrt{j(j+1)}}{Ml}e^{-\tau}(\tilde{C}_- + e^{-2\tau}\tilde{D}_-) D^j_{0,m}(\theta,\phi) \\ -\tilde{C}_- D^j_{-1,m}(\theta,\phi) \end{pmatrix}, \tag{78}$$

where $\tilde{C}_+ = a_1 C_+$, $\tilde{C}_- = b_2 C_-$, $\tilde{D}_+ = b_1 D_+$, and $\tilde{D}_- = a_2 D_-$. Here, $\tilde{C}_+$ and $\tilde{C}_-$ are normalization coefficients and the $a_1, b_1, a_2,$ and b_2 are

$$a_1 = \frac{\Gamma(-iMl)\Gamma(-iMl+1)}{\Gamma(-iMl-j)\Gamma(-iMl+j+1)},$$

$$a_2 = \frac{\Gamma\left(2+iMl\right)\Gamma(iMl-1)}{\Gamma\left(-j+iMl\right)\Gamma(j+iMl+1)},$$

$$a_3 = \frac{\Gamma\left(2-iMl\right)\Gamma(-iMl-1)}{\Gamma\left(-iMl-j\right)\Gamma(-iMl+j+1)},$$

$$a_4 = \frac{\Gamma\left(iMl\right)\Gamma(iMl+1)}{\Gamma\left(-j+iMl\right)\Gamma(j+iMl+1)}.$$

Taking Eq. (77) and Eq. (78), we obtain the normalization coefficients as follows:

$$\left|\tilde{C}_+\right| = \left|\tilde{C}_-\right| = \frac{1}{l}\left[1 - c_1 e^{-2\tau} + 3c_2 e^{-4\tau} - c_3 e^{-6\tau}\right]^{-\frac{1}{2}}. \tag{79}$$

The current for the spin-1 particle in $2+1$ dimensional curved spacetime can be computed by using Eqs. (21) and (24). Therefore, using Eqs. (73)–(75) and (24), the particle charge density $\left(J_+^0\right)$ and the antiparticle charge density $\left(J_-^0\right)$ are obtained in the following way:

$$(J^0)_\pm = \pm\frac{1}{l}4e^{2\tau}\left(2j+1\right)|C_+|^2 D_{\pm1,m}^j(\theta,\phi)^* D_{\pm1,m}^j(\theta,\phi)$$
$$- |D_\pm|^2 e^{4\tau} D_{\mp1,m}^j(\theta,\phi)^* D_{\pm1,m}^j(\theta,\phi)\} \tag{80}$$

and using the orthogonality relation for the rotation group [72], the Noether charges from these densities are calculated as

$$Q_\pm \cong \pm\frac{1}{l}\left(1 + c_1 e^{2\tau} + O\left(e^{4\tau}\right)\right). \tag{81}$$

In the same way, from Eqs. (77), (78), and (24), the particle charge density, $\left(\tilde{J}_+^0\right)$, and the antiparticle charge density, $\left(\tilde{J}_-^0\right)$, as τ goes to ∞, are obtained in the following way:

$$(\tilde{J}^0)_\pm = \pm\frac{1}{l}4e^{-2\tau}\left(2j+1\right)\{\left|\tilde{C}_+\right|^2 D_{\pm1,m}^j(\theta,\phi)^* D_{\pm1,m}^j(\theta,\phi)$$
$$- \left|\tilde{D}_\pm\right|^2 e^{-4\tau} D_{\mp1,m}^j(\theta,\phi)^* D_{\pm1,m}^j(\theta,\phi)\} \tag{82}$$

and, from these densities, the Noether charges for the particle and antiparticle are calculated as

$$\tilde{Q}_\pm \cong \pm\frac{1}{l}\left(1 + c_1 e^{-2\tau} + O\left(e^{4\tau}\right)\right). \tag{83}$$

These results show that as τ goes to $\pm\infty$, $\frac{\tilde{Q}_\pm}{|\Lambda|}$ and $\frac{Q_\pm}{|\Lambda|}$ respectively, converge to ±1. This means that the universe only is composed of the $\pm|\Lambda|$ vacuum energies in its beginning and ending time.

5. Concluding remarks

In this study, we have introduced a relativistic quantum mechanical wave equation of the spin-1 particle as an excited state of the zitterbewegung. This wave function has three independent components. And taking a complex vector potential, A_μ, and fields, $F^{\mu\nu}$, in terms of three components, we show that the equation

is consistent with the 2 + 1 dimensional Proca theory. At the same time, we see that this equation has two eigenstates, particle and antiparticle states or negative and positive energy eigenstates, respectively, in the rest frame and satisfy SO(2,1) spin algebra. Apart from the free particle solution of the equation, we have derived the exact solutions of it in presence of the constant magnetic field and the curved background. From these solutions, we have constructed the current components and observed a spin-1 particle current in presence of a constant magnetic field that decreases by $e^{-\rho}$, while the spin-1 particle current in the curved spacetime oscillates in time, which means that there are temporary particle creations. On the other hand, from the charge densities, we evaluate the Noether charges and we see that the $\frac{Q_+}{M}$ values coincide with the $\frac{Q_-}{M}$ values as from $n \geq 10$ under the strong magnetic field conditions, while the $\frac{\tilde{Q}_+}{M}$ values coincide with the $\frac{\tilde{Q}_-}{M}$ values as from $n \geq 750$ the weak magnetic field conditions. In this situation, the presence of magnetic field triggers the particle production more than antiparticle production in the small n values, but in the large values, the particle and antiparticle production are the same. Also, in the curved background, as τ goes to $\pm\infty$, the $\frac{\tilde{Q}_\pm}{|\Lambda|}$ and $\frac{Q_\pm}{|\Lambda|}$, respectively, converge to ± 1. So, it can be said that, in the beginning and ending of the time, the universe may have completely been composed of the particle and antiparticle with the positive and negative $|\Lambda|$ vacuum energies, respectively.

If we compare the interactions of a spin-1 particle with a constant magnetic field and a curved spacetime background, we see that the excitation energies are proportional to the zitterbewegung frequencies, $M + \frac{eB}{M}$ and $\frac{M}{|\Lambda|}$ respectively. So, $1 + \frac{eB}{M^2}$ corresponds to $\frac{1}{|\Lambda|}$, the inverse of the cosmological constant.

Finally, besides the results obtained in this study, we also see that the consistent spin-1 particle wave equation in the 2 + 1 dimensional spacetime is a usefull tool for discussing the Hawking radiation [73].

Acknowledgment

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