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Reverse-engineered scalar fields in a D-dimensional framework

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Abstract: We are motivated to investigate the dynamics of scalar field dark energy proposals and focus on the reconstruction of a scalar field problem in a D-dimensional framework. With this purpose, we implement a correspondence between the tachyonic scalar field formulation and the additional dimensional cosmological scenario. Such connections help us to redefine the extra dimensional dynamics of the selected scalar field prescription.

Key words: Dark energy, scalar field, D-dimensional cosmology

1. Introduction

The current stage of our universe has been investigated by many astrophysical observations [1–6]. Subsequently, it was proven that an enigmatic content, which is usually dubbed "dark energy", is responsible for the current exotic nature of the universe. It is significant to note here that many ideas have been introduced to identify dark energy theoretically, but its dynamical nature still remains unclear [7–9]. The cosmological constant [10], scalar field definitions [9,11–14], unified energy density proposals [15–18], modified gravity theories [19–25], and even making use of additional dimensions [26–30] are possible theoretical ideas given in the literature to interpret the dynamical evolution of dark energy. Li et al. [31] and Cai et al. [32] prepared very useful briefs about the dark side of the universe, including interesting surveys of some theoretical models. The time-independent cosmological constant model, which was introduced by Einstein with the equation-of-state (EoS) $\omega = -1$, is the primordial idea of the mysterious dark energy [7,8,33,34]. For a perspective including a minimally coupled scalar field, it was shown [35–37] that the effective EoS parameter cannot take values that are crossing the phantom line ($\omega = -1$). Additionally, making use of a general scalar field Lagrangian density $L(\phi, \partial_\mu \phi)$, Vikman [38] concluded that there is no possible transition from the sector $\omega < -1$ to the other side $\omega > -1$ (or vice versa). Consequently, it is understood that the dark energy model is described via a nonlinear Lagrangian density in order to investigate the phantom transition via the shadow of minimal conjectures of nonkinetic interactions among the dark energy and all other constituents. Scalar field proposals including a nonlinear kinetic term [39–43] or a nonlinear higher derivative term [38], braneworld ideas [44,45], string theories [46], and modified gravity have also been introduced to discuss phantom crossing.

While focusing on the dynamical evolution of a scalar field model, an interesting idea, which is called the reverse engineering technique and also known as the reconstruction of the self-interacting potential, can be applied in the absence of a useful theoretical proposal [47–50]. Checking the given speedy expansion history, it can be seen that a potential reproducing the present evolution of our universe can be reconstructed. Ellis

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and Madsen [51] and Starobinsky [52] prepared noteworthy papers in this direction. Subsequently, Huterer and Turner [53,54] investigated the reconstruction mechanism by making use of the cosmological distance observation. Moreover, Rubano and Barrow [55] calculated an expression for a scalar field. In addition to all that is mentioned here, it is interesting to study the dynamics of a scalar field model in the framework of a D-dimensional theory. In light of the above works, we reestablish the dynamics of the tachyonic field from the D-dimensional cosmological perspective.

The outline of the present study is as follows: in the next section, we introduce some preliminary results briefly in order to give theoretical materials and the method of our investigation. Subsequently, in the third section, we obtain new expressions for the tachyonic scalar field function and its self-interacting potential. Then, in the fourth section of the work, we discuss our calculations graphically. Finally, in the last section, we give the closing remarks.

2. Preliminaries: materials and methods

In this section of our investigation, we introduce preliminary expressions and results given in the literature in order to outline the theoretical materials that we will use in further parts of the work. We shall start by writing an extra dimensional, homogeneous, and isotropic spacetime model represented by the following Friedmann–Robertson–Walker (FRW) type line-element [56]:

$$ds^2 = dt^2 - a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 dx_n^2\right\} \tag{1}$$

where

$$dx_n^2 = dy_1^2 + \sin^2 y_2 dy_2^2 + \ldots + \sin^2 y_1 \sin^2 y_2 \ldots \sin^2 y_{n-1} dy_{n-1}^2 \tag{2}$$

and $D = n+2$, $k = (-1,0,1)$, and $a(t)$ describe the total number of spacetime dimensions, the curvature parameter, and the scale factor, respectively.

Einstein's general theory of relativity, which is defined by an action including a time-varying gravitational constant and a time-varying cosmological "constant", is identified by the following field equation:

$$R_{\alpha\beta} - \left[\frac{R}{2} + \Lambda(t)\right] g_{\alpha\beta} = 8\pi G(t)\left[(\rho + P)\, u_\alpha u_\beta - p g_{\alpha\beta}\right], \tag{3}$$

where ρ and P respectively stand for the energy density and pressure of the fluid while u_α is the higher-dimensional velocity vector. Also, $R_{\alpha\beta}$, $g_{\alpha\beta}$, and R represent the Ricci tensor, metric tensor, and curvature scalar, respectively. We assume here that the fluid is dominated by dark energy and we ignore dark matter contributions in order to compare the scenario with the tachyonic scalar field dark energy representation. After making use of the line-element (1) in the field equation (3), one can get the following independent field equations [56]:

$$\frac{n(n+1)}{2}\left[H^2 + \frac{k}{a^2}\right] - \Lambda = 8\pi G\rho, \tag{4}$$

$$n\left(\dot{H} + H^2\right) + \frac{n(n-1)}{2}\left[H^2 + \frac{k}{a^2}\right] - \Lambda = -8\pi G P, \tag{5}$$

where $H = \frac{\dot{a}}{a}$ shows the cosmic Hubble parameter and it estimates the expansion rate of the universe. Thus, considering the above results or the conservation relation $T_{\alpha\beta;\beta} = 0$ leads to the following continuity relation [56]:

$$\dot{\rho} + (n+1)\, H\,(\rho + P) = -\rho \frac{\dot{G}}{G} - \frac{\dot{\Lambda}}{8\pi G}. \tag{6}$$

One can observe from the above equation that the higher-dimensional energy density is not conserved due to the time-varying behaviors of G and Λ. It is known that the equivalence principle requires the following conservation relation:

$$\dot{\rho} + (n+1)\, H\,(\rho + P) = 0. \tag{7}$$

Subsequently, we should have [56]:

$$\dot{\Lambda} = -8\pi \dot{G} \rho, \tag{8}$$

which plays a significant role while investigating cosmological models.

We are now in a position to mention the method that we will proceed with in the next section of the work. In the first step, we introduce the energy density and pressure relations of the tachyonic scalar field dark energy representation in order to check the main features of the proposal. Considering those energy density and pressure relations, one can obtain new expressions for the tachyonic model. After introducing the required cosmological parameters, we equate them with higher-dimensional ones in order to find exact descriptions for the tachyonic scalar field function and its self-interacting potential.

3. Main calculations

First of all, we want to mention the EoS parameter, i.e. $\omega = P/\rho$, which is important while performing cosmological interpretations. Considering equations (4) and (5), we find that

$$\omega = \frac{\frac{n(n+1)}{2}\left[H^2 + \frac{k}{a^2}\right] - \Lambda}{\Lambda - n\left(\dot{H} + H^2\right) + \frac{n(n-1)}{2}\left[H^2 + \frac{k}{a^2}\right]}. \tag{9}$$

Now we focus on the tachyonic scalar field dark energy representation, which is given as one of the possible theoretical dark energy descriptions. The tachyonic condensate model is described by an effective scalar field with a Lagrangian density of the form [9] $L = -V(\varphi)\sqrt{1 + \partial_\mu \varphi \partial^\mu \varphi}$. Here, the tachyonic potential $V(\varphi)$ has a positive maximum at the origin (i.e. $V(\varphi) = V_0$ at $\varphi = 0$) and has a vanishing minimum where the potential vanishes (i.e. $V(\varphi) = 0$ at $\varphi \to \infty$) [57]. Considering this definition with Einstein's general theory of relativity, the following energy density and pressure expressions of the tachyonic model are calculated [9]:

$$\rho_T = \frac{V(\varphi)}{\sqrt{1 - \dot{\varphi}^2}}, \tag{10}$$

$$p_T = -V(\varphi)\sqrt{1 - \dot{\varphi}^2}. \tag{11}$$

This interesting representation of dark energy is described by a very significant EoS parameter, which takes values between –1 and 0 [49,58]. From this point of view, the tachyonic scalar field dark energy representation can be taken into account as a source of the dark energy as well as one of the possible theoretical models helping us to understand the inflation phase at a high energy level [59,60]. Thus, the tachyonic EoS parameter is written as below:

$$\omega = \frac{p_T}{\rho_T} = \dot{\varphi}^2 - 1. \tag{12}$$

We can now implement a correspondence between the higher dimensional scenario and the tachyonic scalar field dark energy model. Correspondence between these formulations can be defined by taking $\rho_T = \rho$, $p_T = p$ and $\omega_T = \omega$. Therefore, the set of these considerations leads to the following expressions:

$$\dot{\varphi}^2 = 1 + \omega_T = 1 + \omega, \tag{13}$$

$$V(\varphi) = \rho_T \sqrt{1 - \dot{\varphi}^2} = \rho\sqrt{-\omega}. \tag{14}$$

It can be seen here that the kinetic term $\dot{\varphi}^2$ and the tachyonic self-interacting potential V(φ) may exist if $-1 \leq \omega \leq 0$. This result indicates that the phantom line cannot be crossed with a speedy expansion. Using relations (4), (5), and (9) in the above expressions, it can be calculated that

$$\varphi = \varphi_0 + \int_{t_0}^{t} \sqrt{1 + \frac{\frac{n(n+1)}{2}\left[H^2 + \frac{k}{a^2}\right] - \Lambda}{\Lambda - n\left(\dot{H} + H^2\right) + \frac{n(n-1)}{2}\left[H^2 + \frac{k}{a^2}\right]}}\, dt, \tag{15}$$

$$V(\varphi) = \frac{1}{8\pi G}\left[\frac{n(n+1)}{2}\left[H^2 + \frac{k}{a^2}\right] - \Lambda\right]\sqrt{\frac{\Lambda - \frac{n(n+1)}{2}\left[H^2 + \frac{k}{a^2}\right]}{\Lambda - n\left(\dot{H} + H^2\right) + \frac{n(n-1)}{2}\left[H^2 + \frac{k}{a^2}\right]}}. \tag{16}$$

4. Viable cases including graphical analyses

In the previous section, we computed two significant expressions for the D-dimensional form of the tachyonic scalar field dark energy representation. As we mentioned in the first section of this work, in the literature, there are various scalar field formulations, but obtaining exact expressions for the corresponding scalar field functions and their self-interacting potentials is very difficult. In addition to this, there is no definite reason for selecting one of those models in order to explain the recent astrophysical dataset successfully. That is why our conclusions may be useful for further investigations. Here, we focus on the tachyonic proposal only, but one can also redefine some other scalar field proposals (quintessence, dilaton, k-essence or the DBI-essence. etc.) by making use of our method and the preliminary calculations we give in this work.

We can consider some viable cases of the D-dimensional framework in order to find more specific results. Below, we introduce three representations including specific expressions of some cosmological quantities:

- Case I: Making use of various assumptions, some theoretical cosmologists proposed [61,62] that $\Lambda \sim \frac{1}{a^2}$ [56]. Subsequently, Chen and Wu [60] assumed that $\Lambda(t) = \frac{\zeta_1}{a^2}$ where $\zeta_1 = nk\left[\frac{n+1}{2} - \frac{1}{\gamma}\right]$. Here, γ denotes an auxiliary parameter. Considering this case, Singh et al. [56] found that $a(t) = \left\{\frac{(n+1)\gamma\zeta_2 t}{2}\right\}^{\frac{2}{(n+1)\gamma}}$ and $G(t) = \zeta_3 + \frac{nk}{8\pi\gamma\zeta_4}\left\{\frac{(n+1)\gamma\zeta_2 t}{2}\right\}^{2-\frac{4}{(n+1)\gamma}}$ where ζ_2 and ζ_3 are integration constants. Also, $\zeta_4 = \rho_0 a_0^{(n+1)\gamma}$, where the suffix 0 indicates the current values of that parameter.

- Case II: Focusing on the spirit of quantum cosmology, Chen and Wu [60] assumed that $\Lambda(t) \sim \frac{L_{pl}^{n-2}}{a^n}$ with $\hbar = c = 1$, where L_{pl} shows the Planck length. Later, in a subsequent paper, Carvalho et al. [63] generalized this case and introduced that $\Lambda(t) = \frac{\eta_1}{a^2} + \frac{\eta_2}{H^{-2}}$ where η_1 and η_2 are two adjustable dimensionless parameters. Moreover, Singh et al. [56] discussed this case in a different problem and calculated $a(t)$ and $G(t)$. They found that $a(t) = \sqrt{\beta_n} t$ and $G(t) = G_0 + \left\{ \frac{(\eta_1 + \beta_n \eta_2)\beta_n^{\frac{(n+1)\gamma - 2}{2}}}{4\pi\zeta_4[(n+1)\gamma - 2]} \right\}$, where $\beta_n = \frac{2\eta_1\gamma + nk[2-(n+1)\gamma]}{n(n+1)\gamma - 2\eta_2\gamma - 2n}$ and G_0 indicates a constant.

- Case III: Sistero [64] suggested a cosmological scenario including varying cosmological parameters. Subsequently, Singh et al. [56], by making use of Sistero's work, assumed that $G(t) = \zeta_5 a^{\varepsilon}(t)$, where $\varepsilon \geq 0$ and ζ_5 represents the proportionality constant. Consequently, considering the flat case $(k = 0)$, Singh et al. [56] computed the cosmic scale factor $a(t)$ and the time-varying gravitational parameter $G(t)$. Thus, they found that $a(t) = D_n t^{\frac{2}{(n+1)\gamma - \varepsilon}}$ and $\Lambda(t) = \zeta_5 D_n^{\varepsilon} t^{\frac{2\varepsilon}{(n+1)\gamma - \varepsilon}}$ where $D_n = \left[\frac{E_n}{2[(n+1)\gamma - \varepsilon]^{-1}}\right]^{\frac{2}{(n+1)\gamma - \varepsilon}}$ and $E_n = \sqrt{\frac{16\pi\zeta_4\zeta_5\gamma}{n[(n+1)\gamma - \varepsilon]}}$.

In further calculations, we also need the expression of the Hubble parameter. According to the specific case given above, one can reach the results given in Table 1.

Table 1. Specific expressions of the cosmic Hubble parameter.

Specific case	Case I	Case II	Case III
Hubble parameter $H(t)$	$\frac{2}{(n+1)\gamma t}$	$\frac{1}{t}$	$\frac{2}{[(n+1)\gamma - \varepsilon]t}$

Now we want to turn back to the EoS parameter given in equation (9). Considering specific cases with Table 1, we can find more specific expressions for the EoS parameter given in Table 2.

Table 2. Specific expressions of the cosmic EoS parameter.

Specific case	EoS parameter ω
Case I	$-1 + \frac{n^2\left[\frac{4}{(n+1)^2\gamma^2 t^2} + k\left\{\frac{2}{(n+1)\gamma\zeta_2 t}\right\}^{\frac{4}{(n+1)\gamma}}\right]}{\left[\frac{kn(n-1)}{2} + \zeta_1\right]\left\{\frac{2}{(n+1)\gamma\zeta_2 t}\right\}^{\frac{4}{(n+1)\gamma}} + \frac{2n(n-1)}{(n+1)^2\gamma^2 t^2}}$
Case II	$-1 + \frac{n^2\left[1 + \frac{k}{\beta_n}\right]}{\left(\frac{\eta_1}{\beta_n} + \eta_2\right) + \frac{n(n-1)}{2}\left[\frac{k}{\beta_n} + 1\right]}$
Case III	$-1 + \frac{\left[\frac{2n}{[(n+1)\gamma - \varepsilon]t}\right]^2}{\zeta_5 D_n^{\varepsilon} t^{\frac{2\varepsilon}{(n+1)\gamma - \varepsilon}} + \frac{n(n-1)}{2}\left[\frac{2}{[(n+1)\gamma - \varepsilon]t}\right]^2}$

For the second case, it is seen that the EoS parameter is not evaluated in time. Moreover, the EoS parameter depends on the number of dimensions in all three specific cases. One can plot the corresponding EoS parameters for different numbers of spacetime dimensions and determine the type of dark energy (phantom or quintessence types) by focusing on their values.

Moreover, one can also discuss the higher dimensional form of the tachyonic scalar field dark energy prescription by making use of equations (15) and (16) together with the relations given in Table 1 and the expressions given in specific cases I, II, and III. The recent astronomical dataset introduced by the SNe-Ia [1], WMAP [2], SDSS [3], X-ray [4], and Planck [6,7] collaborations strongly indicated a spatially flat geometry of the universe at large scales. From this point of view, we assume that $k = 0$ in further analysis. Thus, we reach the following results:

$$\varphi_I = \sqrt{1 + \frac{1}{n - 3 + (n+1)\gamma} t}, \tag{17}$$

$$\varphi_{II} = [n(n+1)\gamma - 2(n+\gamma)] \sqrt{\frac{2n}{(n-1)[n(n+1)\gamma - 2(n+\gamma)]^2 + 4\gamma[(n+1)\gamma - 2]} t}, \tag{18}$$

$$\varphi_{III} = \frac{[3 - (n+1)\gamma] - 2n}{[3 - (n+1)\gamma] - (n-1)_2} F_1 \left\{1, \Psi; 1 + \Psi; \Omega t^{\frac{2}{(n+1)\gamma - 1} + 2}\right\}, \tag{19}$$

$$V_I = \frac{1}{8\pi\gamma} \left[\frac{2n}{(n+1)\gamma^2 t^2}\right] \sqrt{-(1+\gamma)}, \tag{20}$$

$$V_{II} = \frac{nt^{-2}\left[(n+1)\gamma - \frac{[n(n+1)\gamma - 2(n+\gamma)]}{[(n+1)\gamma - 2]^{-1}}\right]}{16\pi\gamma + \frac{\gamma}{4}\left[\frac{2\gamma}{[n(n+1)\gamma - 2(n+\gamma)]}\right]^{\frac{(n+1)\gamma - 2}{2}}} \sqrt{\frac{[(n+1)\gamma - 2] - \frac{\gamma(n+1)}{[n(n+1)\gamma - 2(n+\gamma)]}}{[(n+1)\gamma - 2] + \frac{\gamma(n-1)}{[n(n+1)\gamma - 2(n+\gamma)]}}}, \tag{21}$$

$$V_{III} = \frac{\left[\frac{2n(n+1)}{[(n+1)\gamma - 1]^2 t^2} - \Theta t^{\frac{2}{(n+1)\gamma - 1}}\right]}{8\pi\Theta t^{\frac{2}{(n+1)\gamma - 1}}} \sqrt{\frac{\Theta t^{\frac{2}{(n+1)\gamma - 1} + 2} - \frac{2n(n+1)}{2[(n+1)\gamma - 1]^2}}{\Theta t^{\frac{2}{(n+1)\gamma - 1} + 2} - \frac{2n[3 - (n+1)\gamma + (n-1)]}{[(n+1)\gamma - 1]^2}}}, \tag{22}$$

$$\omega_I = -1 + \frac{2n}{n-1}, \tag{23}$$

$$\omega_{II} = -1 + \frac{\gamma n}{\gamma n - 1}, \tag{24}$$

$$\omega_{III} = -1 + \frac{4n^2}{\left\{\frac{4\pi\gamma}{n}[(n+1)\gamma - 1]\right\}^{\frac{1}{(n+1)\gamma - 1}} [(n+1)\gamma - 1]^2 t^{\frac{2}{(n+1)\gamma - 1}} + 2n(n-1)}, \tag{25}$$

where

$$\Psi = \frac{(n+1)\gamma - 1}{2(n+1)\gamma}, \tag{26}$$

$$\Omega = \frac{[(n+1)\gamma - 1]^{3 + \frac{1}{(n+1)\gamma - 1}}}{4n(n+1)\gamma} \left[\frac{4\pi\gamma}{n}\right]^{\frac{1}{(n+1)\gamma - 1}}, \tag{27}$$

$$\Theta = \left\{\frac{4\pi\gamma}{n}[(n+1)\gamma - 1]\right\}^{\frac{1}{(n+1)\gamma - 1}}. \tag{28}$$

In the above results, we assume $\varphi_I^0 = \varphi_{II}^0 = \varphi_{III}^0 = 0$, $\zeta_2 = \zeta_3 = \zeta_4 = \zeta_5 = 1$, $\varepsilon = 1$, and $\eta_1 = \eta_2 = 1$ for the sake of simplicity. Note that φ_I^0, φ_{II}^0, and φ_{III}^0 represent the present values of the scalar field function for the corresponding viable cases and ${}_2F_1$ is the Gauss hypergeometric function of the second kind, which is written as

$$ {}_2F_1\{A, B; C; y\} = \sum \frac{A_d B_d}{C_d d!} y^d, \tag{29}$$

where the Pochhammer symbol M_d (generalizing coefficients A_d, B_d, and C_d) is given by $M_d = M(M+1)...(M+d-1)$ with $M_0 = 1$.

Now we can discuss the above results graphically. In Figures 1–6, we plot evolutions of the tachyonic scalar field function and its self-interacting potential for cases I, II, and III, respectively.

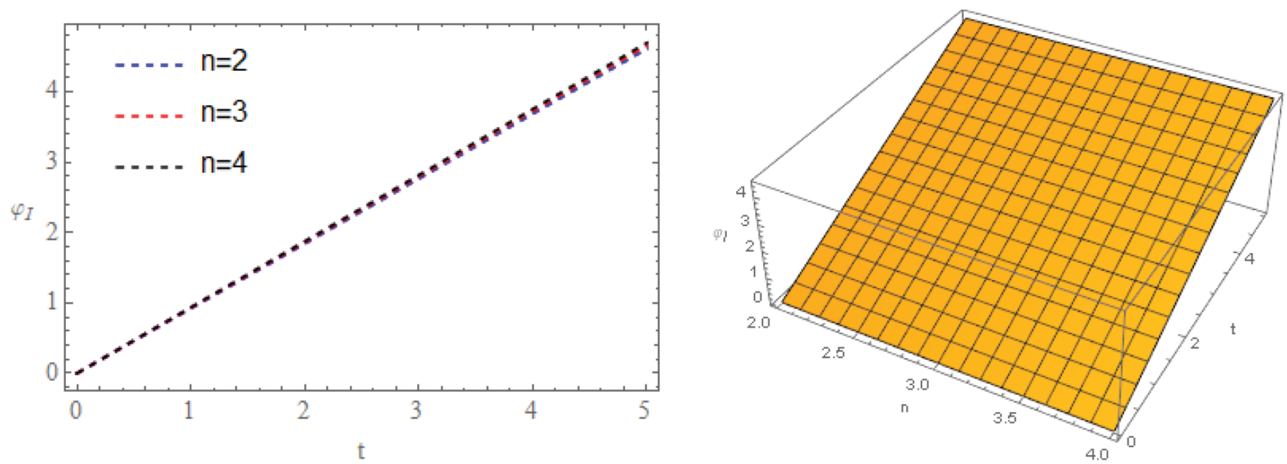

Figure 1. Evolution of the tachyonic scalar field function for case I. Here, we assumed $\gamma = -1.9$.

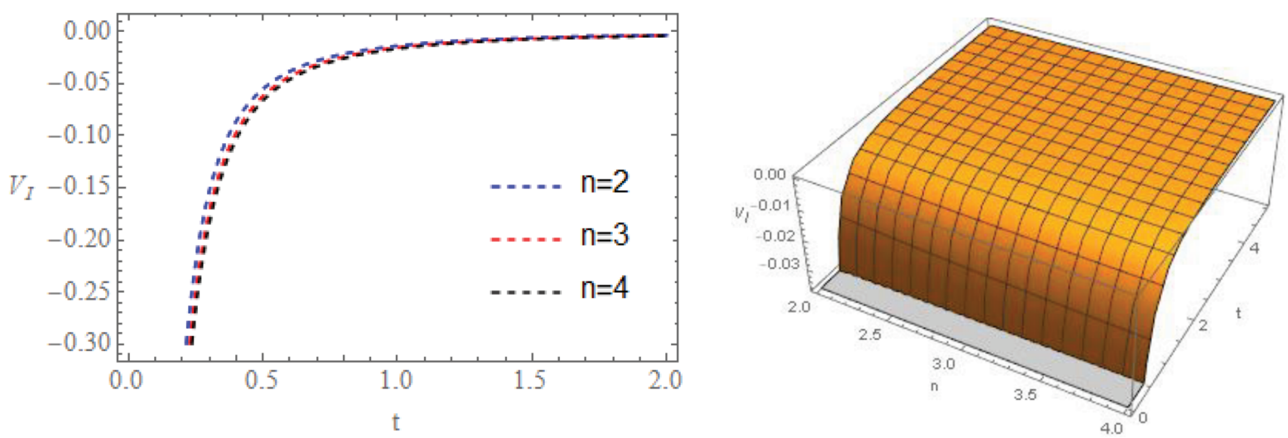

Figure 2. Evolution of the tachyonic self-interacting potential for case I. Here, we assumed $\gamma = -1.9$.

5. Discussion and perspectives

In the present work, we have mainly focused on the dynamics of scalar field dark energy models in a D-dimensional framework. As we mentioned in the first section, both the scalar field and the extra dimensional idea are two possible theoretical candidates proposed to explain the nature of cosmic dark energy. Fundamental theories such as the string/M theory give various scalar field definitions, but they cannot yield exact expressions

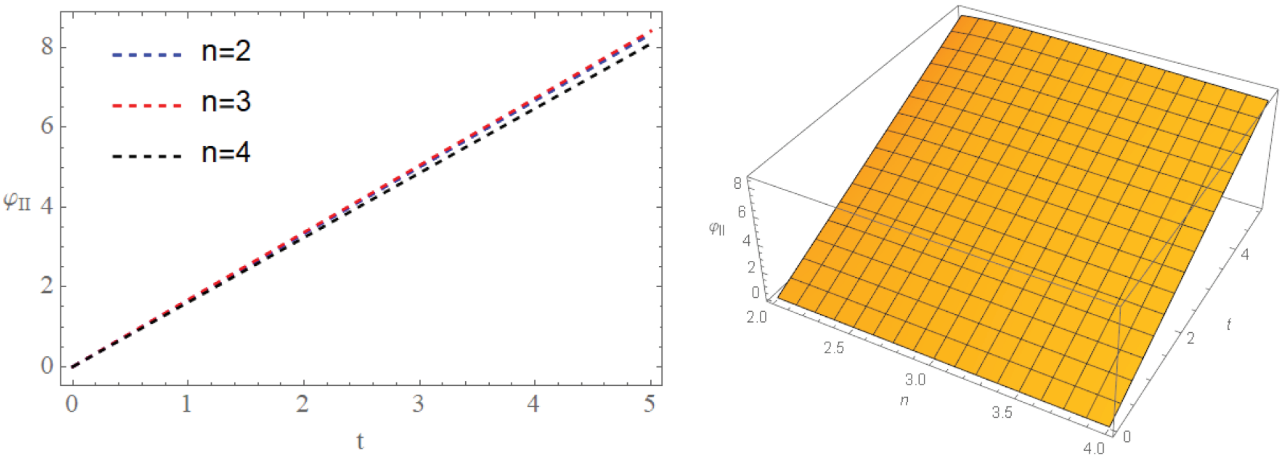

Figure 3. Evolution of the tachyonic scalar field function for case II. Here, we assumed $\gamma = -1.9$.

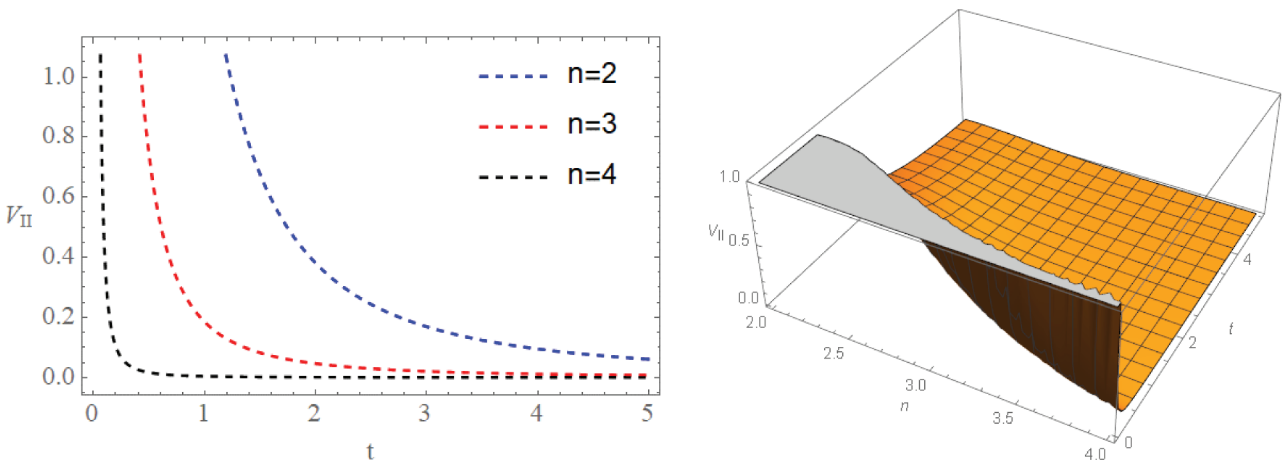

Figure 4. Evolution of the tachyonic self-interacting potential for case II. Here, we assumed $\gamma = -1.9$.

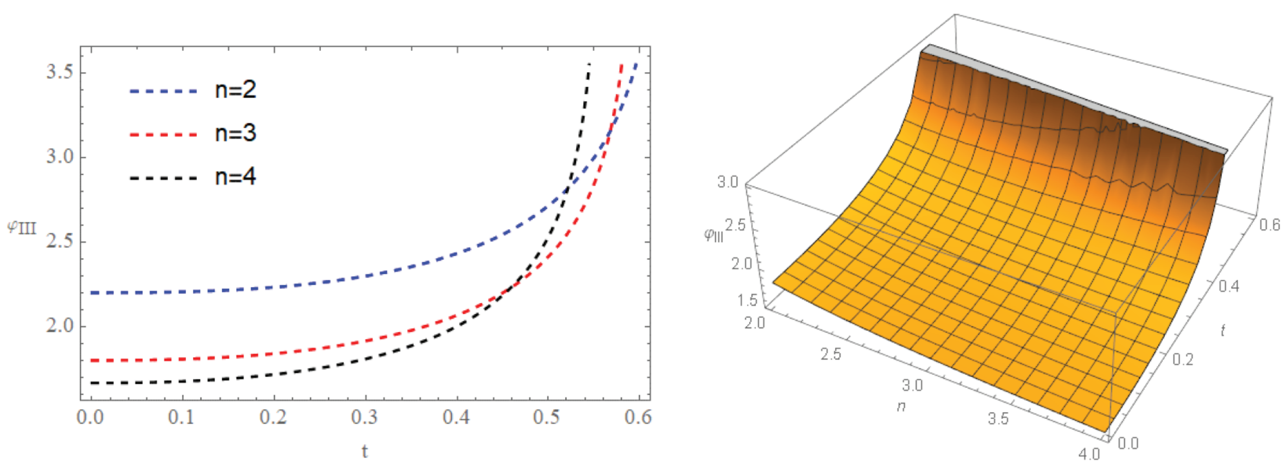

Figure 5. Evolution of the tachyonic scalar field function for case III. Here, we assumed $\gamma = 1.9$.

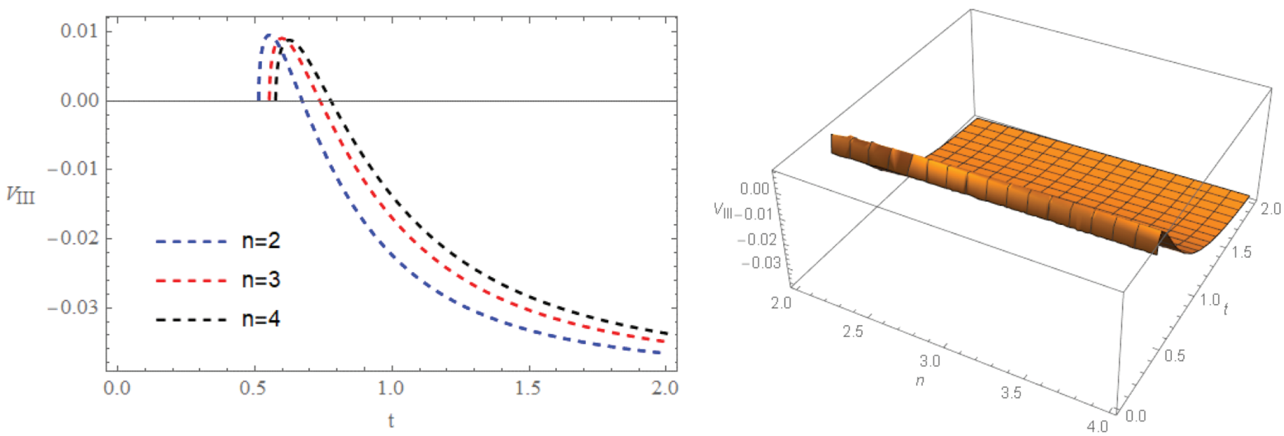

Figure 6. Evolution of the tachyonic self-interacting potential for case III. Here, we assumed $\gamma = 1.9$.

for the scalar field function $\boldsymbol{\varphi}$ and the corresponding self-interacting potential $\boldsymbol{V}(\boldsymbol{\varphi})$. From this point of view, we have applied the reverse engineering method by making use of a scalar field definition (tachyonic description) and a D-dimensional cosmological scenario. Thus, we have achieved exact relations for the tachyonic scalar field proposal. It is significant to emphasize here that our results can also be used to reconstruct other scalar field formulations such as quintessence, dilaton, k-essence, etc.

- The quintessence field is described by the following relations [65,66]:

$$\rho_Q = \frac{\dot{\phi}^2}{2} + V(\phi), \quad P_Q = \frac{\dot{\phi}^2}{2} - V(\phi), \quad \omega_Q = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}. \tag{30}$$

Making use of the above relations together with equations (4), (5), and (9), we obtain:

$$\dot{\phi}^2 = (1 + \omega_Q)\,\rho_Q = (1 + \omega)\rho, \tag{31}$$

$$V = \frac{1}{2}(1 - \omega_Q)\,\rho_Q = (1 - \omega)\rho. \tag{32}$$

Thus, using the same process that we performed for the tachyonic model, the above results can be easily written in exact forms.

- The dilaton field is defined by the following expressions [67,68]:

$$\rho_D = 3he^{\lambda\psi}\chi^2 - \chi, \quad P_D = he^{\lambda\psi}\chi^2 - \chi, \quad \omega_D = \frac{he^{\lambda\psi}\chi - 1}{3he^{\lambda\psi}\chi - 1}, \tag{33}$$

where $2\chi = \dot{\psi}^2$ and h and λ are positive constants. Now one can implement the correspondence between the dilaton type scalar field model and D-dimensional cosmological scenario. It can be written that:

$$h\chi e^{\lambda\psi} = \frac{\omega_D - 1}{3\omega_D - 1} = \frac{\omega - 1}{3\omega - 1}. \tag{34}$$

Thus, one obtains the following expression:

$$e^{\frac{\lambda\psi}{2}}\dot{\chi} = \sqrt{\frac{2}{h}\frac{\omega - 1}{3\omega - 1}}. \tag{35}$$

- The k-essence (kinetic quintessence) scalar field dark energy model is a generalization of canonical scalar field models such as quintessence. The model is given by the following definitions [69,70]:

$$\rho_K = f(\vartheta)(3\Omega^2 - \Omega), \quad P_K = f(\vartheta)(\Omega^2 - \Omega), \quad \omega_K = \frac{\Omega - 1}{3\Omega - 1}. \tag{36}$$

After equating the EoS parameter of the k-essence with equation (9), it can be found that:

$$f(\vartheta) = \frac{\rho_K}{3\Omega^2 - \Omega} = \frac{\rho}{3\Omega^2 - \Omega}, \tag{37}$$

where

$$\Omega = \frac{\omega_K - 1}{3\omega_K - 1} = \frac{\omega - 1}{3\omega - 1}. \tag{38}$$

As we wrote above, one can discuss some other scalar field models by focusing on our theoretical conclusions. We have shown that with this method, one can express a scalar field function and its self-interacting potential. With the help of these results, we can discuss further cosmological issues such as the thermodynamic features and their cosmological indications for the scalar field dark energy proposals. As a matter of fact, such correspondences have very important roles while investigating how different dark energy ideas are mutually related to each other. The scalar field proposals have very interesting features for discussing phantom crossing while the reengineered self-interacting potential has noteworthy physical results for modern theoretical physics.

Moreover, Vulcanov [71] also discussed the reverse engineered tachyonic scalar field model. Here, we have extended that investigation to a more general cosmological scenario including extra dimensions and time-dependent cosmological and gravitational “constants”.

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