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The ground state of pentaquarks with cluster structure in the few-body system approach

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Abstract: In this study, an analytical model is presented based on the Nikiforov–Uvarov method for the Θ^+ exotic KN resonance as a kind of pentaquark. The radial Schrödinger equation for the central Yukawa and modified Yukawa potentials was solved for Θ^+ by use of the Nikiforov–Uvarov method. The energy eigenvalues and corresponding wave functions of Θ^+ were obtained in terms of potential coefficients and n, l quantum numbers. Finally, the potential coefficients were determined according to the binding energy of Θ^+ exotic KN resonance. The potentials and wave functions of Θ^+ exotic KN resonance for different sets of potential coefficients are also shown schematically.

Key words: Pentaquark, diquark-triquark model, modified Yukawa potential, Nikiforov–Uvarov method, radial Schrödinger equation

1. Introduction

Several experimental and theoretical reports have claimed the observation of Θ^+ pentaquarks with mass of 1540 MeV and small width of 20 MeV [1–5]. Pentaquarks are exotic hadrons consisting of four quarks and one antiquark and have been described by several models such as the quark model and bag models [1,2]. The exotic pentaquarks have an antiquark whose flavor is different from any of its four flavors. The $uudd\bar{s}$ pentaquark with strangeness quantum number of S $= +1$ was predicted by the chiral quark soliton model [4] and many other theoretical studies [4–10] before its experimental observation [11]. From the first day of observation, many experimental activities have been performed to prove and characterize these exotic states [12–17]. Various groups around the world have published reports on the existence of the pentaquark state. Five quarks bag [5], diamond [6], triquark-diquark [7], diquark-diquark-antiquark [8,9], and hadronic molecule [10] are the most famous structures proposed for the pentaquark. The chiral soliton model has predicted spin value of 1/2 and positive parity for Θ^+ pentaquarks, with a light mass of about 1540 MeV/c^2 and width of $\sim$20 MeV [1]. The hadronic molecule model is a kind of diquark-triquark model, which includes a baryon such as uud or udd and a meson like $d\bar{s}$ or $u\bar{s}$ [18]. The mass of Θ^+ as presented in experiments ranged from 1520 to 1560 Mev/c^2 [19].

In this work, we present an analytical method to obtain the binding energy of a diquark-triquark model of the Θ^+ pentaquark. In this regard, the radial Schrödinger equation was solved by Nikiforov–Uvarov (NU) method applying Yukawa and modified Yukawa potentials [20] to describe diquark–triquark clusters interaction.

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2. Cluster–cluster interaction with the Yukawa potential and modified Yukawa potentials

In nonrelativistic quantum physics, the solution of the radial Schrödinger equation with a physical potential is of crucial importance for the definition of a quantum system [21]. Various analytical methods such as super symmetry [22] and the NU method [23] have been developed in this context. For a two-cluster system, the Schrödinger equation for the central potential V(r) has the following form [24]:

$$\frac{-\hbar^2}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)\psi(r)+\left(V(r)+\frac{\hbar^2 l(l+1)}{2\mu r^2}\right)\psi(r)=E\psi(r), \tag{1}$$

where $\psi(r)$ is the wave function, $V(r)$ shows the potential energy between two clusters, E denotes the energy eigenvalues, $\hbar$ represents the Planck constant, and μ is the reduced mass. By introducing the reduced radial wave function $u(r)=r\psi(r)$, Eq. (1) is reduced to an equivalent one-dimensional problem, namely:

$$\frac{-\hbar^2}{2\mu}\frac{d^2u(r)}{dr^2}+\left(V(r)+\frac{\hbar^2 l(l+1)}{2\mu r^2}\right)u(r)=Eu(r), \tag{2}$$

with the boundary condition of $u(r)|_{r=0}=0$. The core–cluster interaction may be constructed from a nucleon–nucleon interaction [25]. The Yukawa potential [26] as a short-range nucleon–nucleon interaction may be used as the interacting potential between two clusters, which has the following form:

$$V(r)=-V_0\frac{e^{-\alpha r}}{r}\quad,\quad \alpha,\, V_0>0, \tag{3}$$

where V_0 is the depth of the potential well and σ is related to the range of the potential. Due to nuclear force saturation at lower distances, one may add a repulsive term to the Yukawa potential [27]:

$$V(r)=-V_0\frac{e^{-\alpha r}}{r}+V_1\frac{e^{-\alpha r}}{r^2},\qquad \alpha,\, V_0,\, V_1>0, \tag{4}$$

where V_1 is a positive constant for the repulsive part of the interaction potential. In the next section, we transform the equivalent radial Schrödinger equation of Eq. (2) for the potentials in Eqs. (3) and (4) into the form of the NU differential equation, which will be solved analytically.

3. Application of the NU method

The equivalent radial Schrödinger equation of Eq. (2) can be converted into a hypergeometric type second-order differential equation by suitable variable transformation. The NU method can be used to solve the hypergeometric differential equation with an appropriate coordinate transformation s = s(r) [23]:

$$\psi''(s)+\frac{\bar{\tau}(s)}{\sigma(s)}\psi'(s)+\frac{\bar{\sigma}(s)}{\sigma(s)^2}\psi(s)=0, \tag{5}$$

where $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials of at most second order, and $\bar{\tau}(s)$ is a first-order polynomial. Eq. (5) can be solved by separation of variables as:

$$\psi(s)=\phi(s)\,y(s). \tag{6}$$

By inserting Eq. (6) into Eq. (5), the hypergeometric differential equation will be:

$$\sigma(s)\, y''(s) + \tau(s)\, y'(s) + \lambda y(s) = 0, \tag{7}$$

$$\frac{\phi'(s)}{\phi(s)} = \frac{\pi(s)}{\sigma(s)}, \tag{8}$$

in which the function of $\pi(s)$, the parameter of λ, and the function of $\tau(s)$ have the following form:

$$\pi(s) = \frac{\sigma'(s) - \bar{\tau}(s)}{2} \pm \sqrt{\left(\frac{\sigma'(s) - \bar{\tau}(s)}{2}\right)^2 - \bar{\sigma}(s) + k\sigma(s)} \tag{9}$$

$$\lambda = k + \pi'(s) \tag{10}$$

$$\lambda = \lambda_n = -n\tau'(s) - \frac{n(n+1)}{2}, \quad n = 0, 1, 2, \ldots, \tag{11}$$

$$\tau(s) = 2\pi(s) + \bar{\tau}(s), \tag{12}$$

where $\tau'(s)$ has negative value and k can be found such that the square root in Eq. (9) must be the square of a polynomial that is at most a first-degree polynomial. $y(s)$ can be obtained using the Rodrigues relation as:

$$y_n(s) = \frac{B_n}{\rho[s]} \frac{d^n}{ds^n}[\sigma(s)^n \rho(s)], \tag{13}$$

where B_n is a normalization constant and the function $\rho(s)$ is obtained by:

$$(\sigma(s)\rho(s))' = \tau(s)\rho(s). \tag{14}$$

4. Solutions of the Schrödinger equation for the Yukawa and modified Yukawa potential

In this section, we employ the NU method for the equivalent radial Schrödinger equation with Yukawa and modified Yukawa potentials to obtain the eigenenergies and eigenfunctions of the pentaquark in the diquark-triquark cluster scheme. The exact solution of Eq. (2) with Yukawa and modified Yukawa potentials in Eqs. (3) and (4) is difficult; therefore, it is necessary to introduce a new variable and make some approximations. The approximate expansions of $1/r$ and $1/r^2$ using a Pekeris approximation scheme [28] are given as:

$$\frac{1}{r} = \frac{\alpha}{(1 - e^{-\alpha r})}, \qquad \frac{1}{r^2} = \frac{\alpha^2}{(1 - e^{-\alpha r})^2}. \tag{15}$$

The behavior of $1/r$ and $1/r^2$ and their approximations are plotted in Figure 1 for $\sigma = 0.05$. It can be seen that for small σ, the Pekris approximation is valid.

By introducing a new variable, $s = e^{-\alpha r}$, and using the Pekris approximation, the equivalent Schrödinger in Eq. (2) can be transformed to the hypergeometric differential equation in Eq. (5) with the following expressions:

$$\bar{\tau}(s) = 1 - s, \quad \sigma(s) = s(1 - s), \quad \bar{\sigma}(s) = As^2 + Bs + C. \tag{16}$$

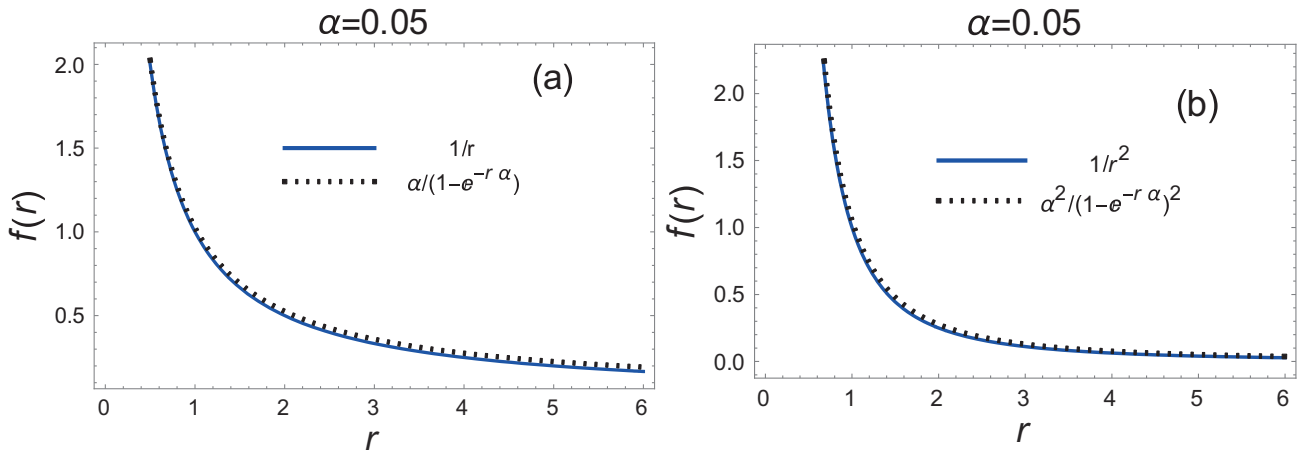

Figure 1. (a) 1/r and its approximation; (b) $1/r^2$ and its approximation.

These expressions can be used to determine two functions of $\pi(s)$, $\tau(s)$, and constant k. Applying the expressions $\pi'(s)$, $\tau'(s)$, k, and $\sigma''(s)$ in Eqs. (10) and (11), the following eigenvalue relation is obtained as a function of quantum number n:

$$2(B+2C) = \left(1+\sqrt{1-4A-4B-4C}\right)\left(1+2\sqrt{-C}\right)+$$

$$2n(3+\sqrt{1-4A-4B-4C}+2\sqrt{-C}+n). \tag{17}$$

In order to determine the wave functions in the NU method, we must first obtain the two functions $of\,\phi(s)$ and $\rho(s)$ from Eqs. (8) and (14):

$$\phi(s) = (1-s)^{\frac{1}{2}(1+\sqrt{1-4A-4B-4C})} s^{\sqrt{-C}}, \tag{18}$$

$$\rho(s) = (1-s)^{\sqrt{1-4A-4B-4C}} s^{2\sqrt{-C}}. \tag{19}$$

Substituting Eq. (19) into the Rodrigues relation given in Eq. (13), we obtain $y_n(s)$. By multiplying $\phi(s)$ in $y_n(s)$, the wave function can be written as:

$$\psi(s) = B_n (1-s)^{\frac{1}{2}\left(1+\sqrt{1-4A-4B-4C}\right)-\sqrt{1-4A-4B-4C}} s^{-\sqrt{-C}}$$

$$\frac{d^n}{ds^n}((1-s)^{\sqrt{1-4A-4B-4C}} s^{2\sqrt{-C}}((1-s)s)^n). \tag{20}$$

5. Diquark-triquark cluster scheme of the Θ^+ pentaquark

The Θ^+ pentaquark structure is a complicated five-body system of $uudd\bar{s}$. In the chiral soliton model and the hadronic molecule model, Θ^+ is considered as a system consisting of two clusters, one baryon with three quarks and one meson with a quark and one antiquark [29–32]. In a chiral soliton model, the two clusters, a ud diquark and a $ud\bar{s}$ triquark, are in a relative P-wave with $J^p = 1/2^+$ state [8]. In [8] the authors roughly estimated $m_{diq} = 720\,MeV$, $m_{triq} = 1260\,MeV$, and $\mu_{di-tri} = 458\,MeV$ where μ_{di-tri} denotes the reduced mass for the

relative motion of the diquark-triquark system. In this model, the binding energy of the two clusters is roughly $(m_{triq} + m_{diq} - m_{pentaq}) = 440\,MeV$.

One may assume that Θ^+ consists of five quarks in an S-wave and therefore has negative parity, $J^p = 1/2^-$ (a baryon (uud) and a meson $(d\bar{s})$) [33]. The mass of the $d\bar{s}$ meson with spin of zero is $m_d = 497.72\,MeV$, and the mass of the proton is $m_{uud} = 938.28\,MeV$. This configuration is not bound and the repulsive terms of the interaction force may then overcome the attractive terms and the S-wave pentaquark rearranged into the usual KN system. Other configuration of S-wave pentaquark may be the $d\bar{s}$ meson with spin of one and mass of 892 MeV. Hence, the reduced mass of Θ^+ is $\mu_{di-tri} = 457.277\,MeV$ and the binding energy will be $290.28\,MeV$.

In what follows, we assume that any cluster behaved like a particle and ignored the internal structure of each cluster. Moreover, the interactions operated only between the two clusters. The two clusters consisted of $d\bar{s}$ diquark with spin one and mass of 892 MeV and uud triquark with spin of 1/2 and mass of $938.28\,MeV$ in a relative S-wave.

6. Results

By introducing the exponential variable of $s = e^{-\alpha r}$ and using Pekris approximation, the equivalent Schrödinger equation of Eq. (2) was transformed to the hypergeometric differential equation of Eq. (5) with the three constants of A, B, and C and Yukawa potential:

$$A = \frac{2(E_n - V_0\alpha)\mu}{\hbar^2\alpha^2} B = \frac{2(-2E_n + V_0\alpha)\mu}{\hbar^2\alpha^2} C = -l(1+l) + \frac{2E_n\mu}{\hbar^2\alpha^2}. \tag{21}$$

By inserting the expressions of Eq. (21) into Eq. (17) and solving it for the eigenenergy of E_n we obtain:

$$\begin{aligned} E_n = & \frac{1}{16\hbar^2 (l-n)^2 (1+l+n)^2 \mu} (-\hbar^4(4l^3(1+4n(3+n)) + l^4(2+8n(3+n)) - 2ln(-1+3\sqrt{(1+2l)^2} \\ & + n(4+7\sqrt{(1+2l)^2} + n(10+2\sqrt{(1+2l)^2}+3n))) + n^2(5-3\sqrt{(1+2l)^2} + n(20-4\sqrt{(1+2l)^2} \\ & + n(31+5\sqrt{(1+2l)^2} + 2n(7+\sqrt{(1+2l)^2}+n)))) - 2l^2(-1+n(-13+3\sqrt{(1+2l)^2} + n(7\sqrt{(1+2l)^2} \\ & + n(10+2\sqrt{(1+2l)^2}+3n)))))\alpha^2 + 4h^2(8l^3 + 4l^4 - l^2(-5+\sqrt{(1+2l)^2} + 2n(-1+\sqrt{(1+2l)^2}+n)) \\ & - l(-1+\sqrt{(1+2l)^2} + 2n(-1+\sqrt{(1+2l)^2}+n)) + 2n(1-\sqrt{(1+2l)^2} + n(3-2\sqrt{(1+2l)^2} \\ & + n(4+n))))V_0\alpha\mu + 4(-1-2l(1+l)+\sqrt{(1+2l)^2} - 2n(1-\sqrt{(1+2l)^2}+n))V_0^2\mu^2). \end{aligned} \tag{22}$$

Eq. (22) for $l = 0$ will take a simple form:

$$E_n = -\frac{(\hbar^2(1+4n+n^2)\alpha - 2V_0\mu)^2}{8\hbar^2(1+n)^2\mu}, \tag{23}$$

and the radial wave function for $n = 0$ and $l = 0$ is:

$$\psi(r) = \frac{B_n(e^{-r\alpha})^{1+\sqrt{2}\sqrt{-\frac{E_0\mu}{\alpha^2\hbar^2}}}(-1+e^{r\alpha})}{r}. \tag{24}$$

The normalization factor B_n in Eq. (24) can be obtained by the normalization condition of the radial wave function:

$$\int_0^\infty \psi^*(r)\psi(r)\, r^2 dr = 1. \tag{25}$$

The calculated binding energy of the Θ^+ pentaquark and coefficients of Yukawa potential in the diquark-triquark model are listed in Table 1. The graph of potentials for different values of α and V_0 are also shown in Figure 2. The horizontal red line shows the binding energy of the Θ^+ pentaquark. The radial wave functions of the Θ^+ pentaquark for different values of α and V_0 for $n = 0$ and $l = 0$ are illustrated in Figure 3. As shown in Figure 3, different sets of α and V_0 give the same radial wave function for the ground state of the Θ^+ pentaquark.

Table 1. The calculated binding energy and coefficients of Yukawa potential in the diquark-triquark model of the Θ^+ pentaquark.

$\alpha\,(fm^{-1})$	$V_0(MeV\,fm)$	$E_0(MeV)$	$E_0 = -B\,(MeV)$
0.05	457.4	−290.22	-290.28 MeV
0.10	470.2	−290.25	
0.15	483.0	−290.29	
0.20	495.8	−290.33	
0.25	508.5	−290.23	

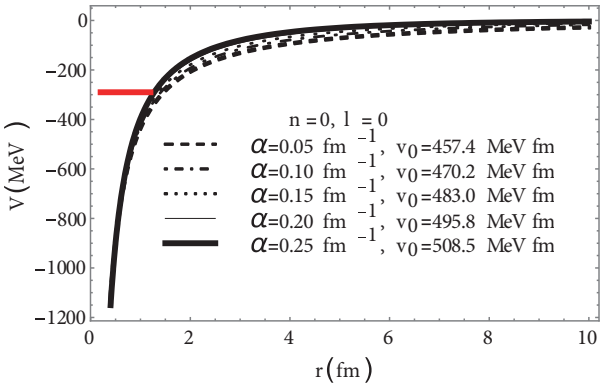

Figure 2. The Yukawa potential of the Θ^+ pentaquark for different values of α and V_0. The horizontal red line shows the binding energy of the Θ^+ pentaquark.

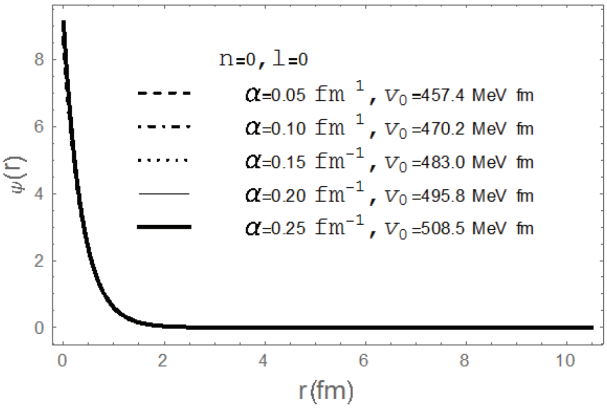

Figure 3. The radial wave function of the Θ^+ pentaquark for different values of α and V_0 for $n = 0$ and $l = 0$ for Yukawa potential in the diquark-triquark cluster model.

For the case of modified Yukawa potential, which involves the repulsive term for short distances, the three constants of A, B, and C are defined as:

$$A = \frac{2(E_n - V_0\alpha)\mu}{\hbar^2\alpha^2} B = -\frac{2(2E_n + \alpha(-V_0 + V_1\alpha))\mu}{\hbar^2\alpha^2} C = -l(1+l) + \frac{2E_n\mu}{\hbar^2\alpha^2}. \tag{26}$$

By inserting the expressions of Eq. (26) into Eq. (17) and solving it for the eigenenergy of E_n for $l = 0$, we

obtain:

$$E_n = -\frac{1}{16\mu(\hbar^2 n(1+n)-2V_1\mu)^2}(16V_1(V_0-V_1\alpha)^2\mu^3+4\hbar^2\mu^2(8n^2V_0V_1\alpha+V_0^2(1-\sqrt{1+\frac{8V_1\mu}{\hbar^2}}$$
$$+2n(1+n-\sqrt{1+\frac{8V_1\mu}{\hbar^2}}))+V_1^2\alpha^2(1+\sqrt{1+\frac{8V_1\mu}{\hbar^2}}+2n(3-n+\sqrt{1+\frac{8V_1\mu}{\hbar^2}})))$$
$$-4\hbar^4 n\alpha\mu(2V_0(1-\sqrt{1+\frac{8V_1\mu}{\hbar^2}}+n(3+n(4+n)-2\sqrt{1+\frac{8V_1\mu}{\hbar^2}}))+V_1\alpha(1+\sqrt{1+\frac{8V_1\mu}{\hbar^2}}$$
$$+n(6+n^2+3\sqrt{1+\frac{8V_1\mu}{\hbar^2}}+2n(5+\sqrt{1+\frac{8V_1\mu}{\hbar^2}}))))+h^6n^2\alpha^2(5-3\sqrt{1+\frac{8V_1\mu}{\hbar^2}}$$
$$+n(-4(-5+\sqrt{1+\frac{8V_1\mu}{\hbar^2}})+n(31+5\sqrt{1+\frac{8V_1\mu}{\hbar^2}}+2n(7+n+\sqrt{1+\frac{8V_1\mu}{\hbar^2}})))), \quad (27)$$

and the radial wave function for $n = 0$ and $l = 0$ is:

$$\psi(r) = \frac{B_n(e^{-r\alpha})^{\sqrt{2}\sqrt{-\frac{E_0\mu}{\alpha^2\hbar^2}}}(1-e^{-r\alpha})^{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{8V_1\mu}{\hbar^2}}}}{r}. \quad (28)$$

The calculated binding energy of the Θ^+ pentaquark as well as coefficients of modified Yukawa potential in the diquark-triquark model are presented in Table 2. The graph of potentials for different values of α, V_0, and V_1 are shown in Figure 4. The horizontal red line shows the binding energy of the Θ^+ pentaquark. The radial wave functions of the Θ^+ pentaquark for different values of α, V_0, and V_1 for $n = 0$ and $l = 0$ are provided in Figure 5.

Table 2. The calculated binding energy and coefficients of modified Yukawa potential in diquark-triquark cluster model of the Θ^+ pentaquark.

$\alpha\,(fm^{-1})$	$V_0(MeVfm)$	$V_1(MeVfm^2)$	$E_0(MeV)$	$E_0 = -B\,(MeV)$
0.10	300	17.7	−290.37	
0.10	350	34.4	−290.60	
0.10	400	54.8	−290.70	
0.10	450	78.8	−290.53	
0.10	500	106.5	−290.50	–290.28 MeV
0.15	350	32.6	−290.68	
0.20	400	49.5	−290.73	
0.25	450	67.8	−290.72	
0.30	500	87.0	−290.65	

7. Conclusion

In this study, a cluster model was considered for the Θ^+ pentaquark by dividing the system into two clusters: a uud triquark and a $d\bar{s}$ diquark, which are in a relative S-wave. Considering the Yukawa potential and modified Yukawa potential as the interaction potential between clusters and solving the radial Schrödinger equation by the NU method, the values of potential coefficients were determined, which predicted the binding energy of

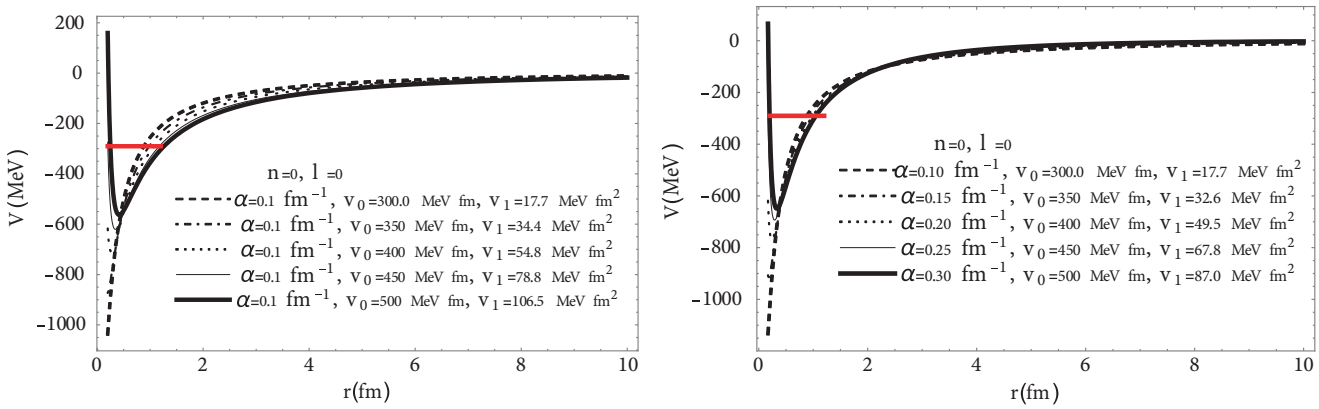

Figure 4. The modified Yukawa potential of the Θ^+ pentaquark for different values of α, V_0, and V_1. The horizontal red line shows the binding energy of the Θ^+ pentaquark.

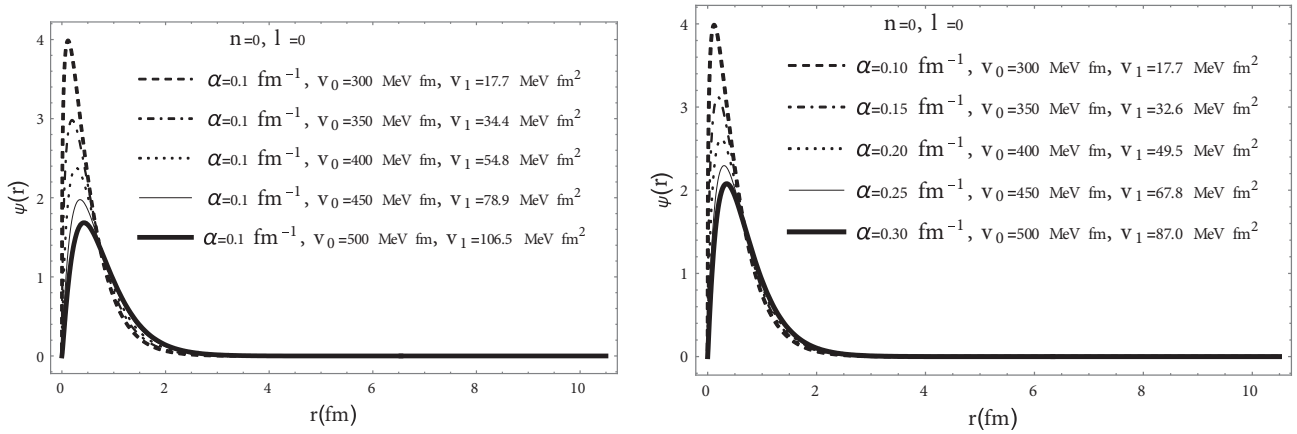

Figure 5. The radial wave function of the Θ^+ pentaquark for different values of α, V_0, and V_1 for $n = 0$ and $l = 0$ for modified Yukawa potential in the diquark-triquark cluster model.

the Θ^+ pentaquark. As shown in Figures 3 and 5, the radial wave functions of the Θ^+ pentaquark vanished at about 2 *fm* for both cases of Yukawa and modified Yukawa potentials, but in the case of modified Yukawa potential, it vanished near the origin while for the Yukawa potential it increased at the origin.

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