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Tracker-aided experiments for the determination of viscosity coefficients of liquids using the communicating vessels method with weighted regression analysis

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Abstract: Tracker-aided experiments using communicating vessels have been performed to determine the viscosity coefficients of liquids at room temperature with weighted linear regression analysis. The calculation involves weighted linear regression analysis of $-\ln[(2z_1/z_0) - 1]$ against time, where z_1 and z_0 are the instantaneous liquid level and the initial liquid level in one of the vessels with the higher initial liquid level, respectively, and a computer-aided χ^2 goodness of fit test. Based on the maximum Reynolds number obtained, it is concluded that the liquid flow was laminar, as required by the theory. From the χ^2 goodness of fit test it is concluded that the relation between $-\ln[(2z_1/z_0) - 1]$ and time is linear, in agreement with theory. As samples, water and rectified spirit were used. The viscosity coefficients of both liquids obtained from experiments using various capillary tube sizes agreed with accepted values.

Key words: Liquid viscosity, communicating vessels method, Tracker, weighted linear regression analysis, chi-square goodness of fit test

1. Introduction

Various methods have been developed to determine the coefficients of viscosity of liquids. One of the oldest methods is the vertical capillary tube method based on the application of Newton's law of fluid friction. As a modification of the vertical capillary tube method, Ortega et al. determined the coefficient of viscosity of water by observing its flow through a horizontal capillary tube between two communicating vessels [1].

Meanwhile, advances in information technology have brought various software tools that can aid in observing the position of moving objects by video analysis. A good example of such software is Tracker, which was designed in the framework of Java applications by the Open Source Physics Project to be used in physics laboratories [2].

This paper reports Tracker-aided experiments for the determination of viscosity coefficients of liquids at room temperature using the communicating vessels method first developed by Ortega et al. [1]. The final calculation of viscosity coefficients involves the results of weighted linear regression analysis of $-\ln[(2z_1/z_0) - 1]$ against time, where z_1 and z_0 are respectively the instantaneous liquid level and the initial liquid level in one of the vessels with the higher initial liquid level. To justify the assumption of the linear equation, a computer-aided χ^2 goodness of fit test [3] was also performed. The objectives of this study are to develop an inexpensive apparatus for the determination of viscosity coefficients of liquids using the communicating vessels method suitable for an undergraduate physics laboratory and to test the validity of the theoretical model as well as the

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assumption of a laminar liquid flow. The maximum Reynolds number (described in the next section) has been calculated to test the laminarity of the liquid flow.

The term "viscosity coefficient" as used in this paper refers to dynamic or absolute viscosity coefficient, η, to distinguish it from the quantity η/ρ (ρ is mass density) known as kinematic viscosity coefficient [4].

2. Theory

2.1. Viscous vs. turbulent force

Liquid flow due to its friction against the surface of a tube may be classified into two types: (a) laminar or streamline flow, where the friction force is said to be viscous, fulfilling Newton's law of liquid flow; and (b) turbulent flow involving a turbulent or inertial friction force. The Reynolds number is a dimensionless quantity, which, for a liquid flow in a capillary tube, can be defined as:

$$Re = \frac{\rho D v}{\eta}, \tag{1}$$

where ρ, D, and v are liquid mass density, the inner diameter of the capillary tube, and flow velocity, respectively. For flow in a pipe of inner diameter D, experimental observations show that for "fully developed" flow, laminar flow occurs when Re $<$ 2300 and turbulent flow occurs when Re $>$ 2900. At the lower end of this range, a continuous turbulent flow will form, but only at a very long distance from the inlet of the pipe. The flow in between will begin to transition from laminar to turbulent and then back to laminar at irregular intervals, called intermittent flow. This is due to the different speeds and conditions of the liquid in different areas of the pipe's cross-section, depending on other factors such as pipe roughness and flow uniformity. Laminar flow tends to dominate in the fast-moving center of the pipe while slower-moving turbulent flow dominates near the wall. As the Reynolds number increases, the continuous turbulent flow moves closer to the inlet and the intermittency in between increases, until the flow becomes fully turbulent at Re $>$ 2900 [5].

2.2. A system of two communicating vessels

The system of two communicating vessels used in this experiment for the determination of liquid viscosity coefficient (Figure 1) consists of two vertical communicating vessels connected by a horizontal capillary tube. Let us assume that the liquid level in vessel 1 is higher than that in vessel 2, and liquid flows from vessel 1 to vessel 2 through the capillary tube. As shown in Figure 1, z_0, z_1, and z_2 are the initial liquid height in vessel 1, the instantaneous liquid height in vessel 1, and instantaneous liquid height in vessel 2, respectively (relative to the height of the capillary tube), while p_1 and p_2 are hydrostatic pressures in vessel 1 and vessel 2, respectively, at the height of the capillary tube.

Assuming that the liquid flow through the capillary tube is laminar (Newtonian) and the liquid is incompressible, the flow rate, Q, follows Poiseuille's law:

$$Q = \frac{\pi \Delta p}{8 L \eta} R^4, \tag{2}$$

where R is the inner radius of the capillary tube, L is the length of the capillary tube, and Δp is the pressure difference between the ends of the capillary tube [4], or

$$Q = \frac{\pi \Delta p}{128 L \eta} D^4, \tag{3}$$

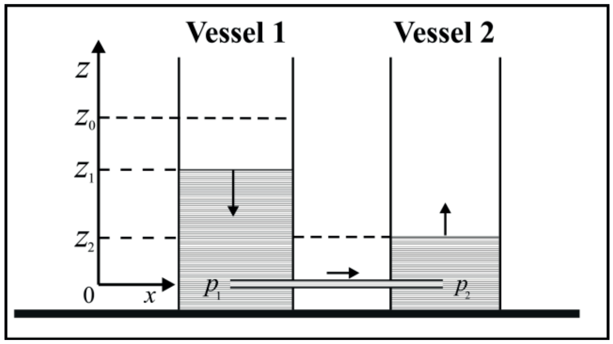

Figure 1. A system of two communicating vessels.

where D is the inner diameter of the capillary tube. The pressures p_1 and p_2 in the vessels are $p_1 = p_0 + \rho g z_1$ and $p_2 = p_0 + \rho g z_2$, where p_0 is the atmospheric pressure, and g the acceleration of gravity. Therefore,

$$\Delta p = p_1 - p_2 = \rho g (z_1 - z_2). \tag{4}$$

Assuming the liquid temperature remains constant during the flow, the total liquid volume will remain constant. Therefore, taking the initial liquid heights in vessel 1 as $z_1 = z_0$, and $z_2 = 0$ in vessel 2, and at any time $z_1 + z_2 = z_0$, then

$$\Delta p = \rho g (z_1 - z_2), \tag{5}$$

and we may write

$$Q = \frac{\pi D^4 \rho g}{128 L \eta}(2z_1 - z_0). \tag{6}$$

Taking the cross-section of both vessels to be $A = \pi d^2/4$, where d is the diameter of the vessels, the flow rate can also be written as

$$Q = -A\frac{dz_1}{dt}, \tag{7}$$

and from Eqs. (6) and (7) it follows that

$$\frac{dz_1}{dt} = \frac{D^4 \rho g}{32 L \eta d^2}(2z_1 - z_0). \tag{8}$$

Writing $\frac{D^4 \rho g}{32 L \eta d^2} = K$, the final equation is

$$-\ln[(2z_1/z_0) - 1] = 2Kt, \tag{9}$$

which can be written as a linear equation of the form

$$Y = a_0 + a_1 X, \tag{10}$$

where $Y = -\ln[(2z_1/z_0) - 1]$, and $X = t$.

Therefore, it is possible to perform a linear regression analysis to determine the coefficients a_0 and a_1, as well as their errors, and to calculate the viscosity coefficient of the liquid and its error from these quantities.

Given s_i, the measurement error of each measured value Y_i and then a_0, a_1, and their errors can be calculated using a weighted linear regression analysis through the following equations [3]:

$$a_1 = \frac{\sum \frac{1}{s_i^2} \sum \frac{X_i Y_i}{s_i^2} - \sum \frac{X_i}{s_i^2} \sum \frac{Y_i}{s_i^2}}{\Delta}, \tag{11a}$$

$$a_0 = \frac{\sum \frac{X_i^2}{s_i^2} \sum \frac{Y_i}{s_i^2} - \sum \frac{X_i}{s_i^2} \sum \frac{X_i Y_i}{s_i^2}}{\Delta}, \tag{11b}$$

where

$$\Delta = \sum \frac{1}{s_i^2} \sum \frac{X_i^2}{s_i^2} - \left(\sum \frac{X_i}{s_i^2} \right)^2, \tag{12}$$

and their errors are

$$s_{a_1.} = \sqrt{\frac{\sum \frac{1}{s_i^2}}{\Delta}}, \tag{13a}$$

$$s_{a_0} = \sqrt{\frac{\sum \frac{X_i^2}{s_i^2}}{\Delta}}. \tag{13b}$$

From Eqs. (9) and (10) it is seen that

$$a_1 = 2 \left(\frac{D^4 \rho g}{32 L \eta d^2} \right), \tag{14}$$

and therefore the viscosity coefficient of the liquid and its error can be calculated from

$$\eta = \frac{D^4 \rho g}{16 L a_1 d^2}, \tag{15}$$

$$s_\eta = \left\{ \left(\frac{\partial \eta}{\partial D} s_D \right)^2 + \left(\frac{\partial \eta}{\partial \rho} s_\rho \right)^2 + \left(\frac{\partial \eta}{\partial g} s_g \right)^2 + \left(\frac{\partial \eta}{\partial L} s_L \right)^2 + \left(\frac{\partial \eta}{\partial a_1} s_{a_1} \right)^2 + \left(\frac{\partial \eta}{\partial d} s_d \right)^2 \right\}^{\frac{1}{2}}, \tag{16}$$

where $s_D, s_\rho, s_g, s_L, s_{a_1}$, and s_d are respectively the errors for the capillary tube inner diameter D, the liquid mass density ρ, acceleration of gravity g, pipe length L, the linear coefficient a_1, and the vessel diameter d, or

$$s_\eta = \eta \sqrt{ \left(\frac{4 s_D}{D} \right)^2 + \left(\frac{s_\rho}{\rho} \right)^2 + \left(\frac{s_g}{g} \right)^2 + \left(\frac{s_L}{L} \right)^2 + \left(\frac{s_{a_1}}{a_1} \right)^2 + \left(\frac{2 s_d}{d} \right)^2 }. \tag{17}$$

2.3. Calculation of the maximum Reynolds number

The maximum Reynolds number may be calculated using Eqs. (1) and (8), keeping in mind that the instantaneous liquid level velocity in vessel 1 is $v = -dz_1/dt$ and that the maximum value of z_1 is z_0, and the final expression is

$$Re_{\max} = \frac{\rho^2 D^3 g z_0}{32 L \eta^2}. \tag{18}$$

The error of the maximum Reynolds number may be calculated from

$$s_{Re_{\max}} = Re_{\max}\sqrt{\left(\frac{2s_\rho}{\rho}\right)^2 + \left(\frac{3s_D}{D}\right)^2 + \left(\frac{s_g}{g}\right)^2 + \left(\frac{s_{z_0}}{z_0}\right)^2 + \left(\frac{s_L}{L}\right)^2 + \left(\frac{2s_\eta}{\eta}\right)^2}, \tag{19}$$

where s_{z_0} is the error for z_0.

3. Experimental method

3.1. The system constants

The measurement system in this experiment consists of two communicating vessels connected through a capillary tube (Figure 1), where the capillary tube inner diameter and length can be varied. A plastic ruler was used to measure the vessel length and the capillary tube length, while a caliper was used to measure the vessel diameter and the capillary tube outer diameter. A beaker glass was used to pour the liquid into the vessel. An Ohaus scale was used to weigh the beaker glass and the liquid for the determination of liquid mass density.

The determination of the capillary tube inner diameter D needs special attention, since the inner surface may have irregularities along and around the surface. Moreover, from Eq. (15) it is seen that the viscosity coefficient of the liquid η contains the fourth power of D, so the contribution of the relative error in D to the total error will be large and therefore it is important to determine D very carefully. For this purpose, the author has developed a special technique to obtain the value of D by precisely (i.e. repeatedly) measuring the capillary tube outer diameter D_t using a caliper, the capillary tube volume V_p using a graduated cylinder, and the capillary tube length L using a plastic ruler. The volume of the capillary tube hole is the difference between the capillary tube outer volume and its glass shell, so the capillary tube inner diameter may be calculated from

$$D = \sqrt{\left(D_t^2 - \frac{4V_p}{\pi L}\right)}, \tag{20}$$

and the error from

$$s_D = \sqrt{\left(\frac{\partial D}{\partial D_t} s_{D_t}\right)^2 + \left(\frac{\partial D}{\partial V_p} s_{V_p}\right)^2 + \left(\frac{\partial D}{\partial L} s_L\right)^2}$$

or

$$s_D = \sqrt{\left(\frac{D_t}{D} s_{D_t}\right)^2 + \left(\frac{-2}{\pi D L} s_{V_p}\right)^2 + \left(\frac{-2V_p}{\pi L}\right)^2}. \tag{21}$$

The liquid mass density ρ was determined using the basic method, i.e. by measuring its mass and volume, to obtain

$$\rho = \frac{m}{V}, \tag{22}$$

and the error

$$s_\rho = \sqrt{\left(\frac{s_m}{m}\right)^2 + \left(\frac{s_V}{V}\right)^2}. \tag{23}$$

The determination of the vessel inner diameter d was carried out by measuring its outer diameter, d_{outer}, using a ruler and its shell thickness, d_{shell}, using a caliper, to obtain

$$d = d_{outer} - d_{shell}, \tag{24}$$

and the error

$$s_d = \sqrt{\left(s^2_{d_{outer}} + s^2_{d_{shell}}\right)}. \tag{25}$$

The liquid temperature was measured using an ordinary mercury bulb thermometer on a 0–100 °C scale and remained a constant 25 °C for all experiments performed. The liquid was poured into vessel 1 and then the liquid flow from vessel 1 to vessel 2 was recorded using a digital camera. Subsequently, the video file produced was analyzed using Tracker by fixing z_0 at the moment the liquid level in vessel 2 was at the same level as the capillary tube. The video data recording and the determination of z_0 were repeated to obtain three datasets for the same video scene, and the same procedure was repeated for different capillary tube inner diameters and lengths.

The liquids used in this experiment were water, with known viscosity coefficient of $(1.08 \pm 0.07) \times 10^{-2}$ P [6], and spirit, which is a mixture of 94% methanol and water, with known viscosity coefficient of 0.543×10^{-2} P for pure methanol [7], all data being taken at 25 °C. These liquids were chosen since they are inexpensive and easily available, while the flow is easy to observe and does not leave dark traces on the tube after an experiment is performed.

3.2. Video analysis with Tracker

A digital camera was used to record the liquid flow in one of the vessels (let us call it vessel 1), and then the recorded video file was analyzed with Tracker. The open end of vessel 1 was closed with a hand palm after liquid was poured to prevent the liquid from flowing to vessel 2, and video recording started shortly before the hand palm was released from the open end of vessel 1 until liquid flow almost stopped or when liquid levels in both vessels were almost equal. The same procedure was repeated for different capillary tube diameters and lengths.

The recorded video was analyzed with Tracker using the following steps. First Tracker was turned on and activated through the option ***window***, ***right view***, and then the video was called through ***file*** and ***import***, and the video appeared. The video analysis was started by tracking the liquid level motion using the option ***track***, ***new***, and ***point mass*** to obtain the numerical value of z_0 at various values of time (t) selected for tracking. The numerical output of Tracker was a dataset containing z_1 at various times, t, which was subsequently copied and pasted into a Microsoft Excel 2010 table, where the quantity $-\ln[(2z_1/z_0) - 1]$ was calculated for every value of time t.

3.3. Calculation of viscosity coefficient

The weighted linear regression of the quantity $-\ln[(2z_1/z_0) - 1]$ against time was aided by a computational program in Windows Compaq Visual Fortran 6.5 compiling subroutines from [3], which produces the linear coefficients a_0 and a_1 (Eqs. (11a) and (11b)) as well as their errors (Eqs. (13a) and (13b)). Thereafter, from Eq. (15) the liquid viscosity coefficient (η) was calculated using the values of a_1, d, D, L, ρ, and g, while its error was calculated from Eq. (17). A goodness of fit test was also performed on the data to check whether the assumption of a linear relation was justified or not. For this purpose, the program also gave the reduced chi-square, defined as $\chi^2_\nu = \chi^2/\nu$, where $\nu = N - 2$ is the degrees of freedom, and $P(\chi^2 \geq \chi^2_{calc})$, the probability of obtaining a chi-square value from a random set of data larger than or equal to the calculated chi-square. The set of data pairs (X_i, Y_i), where $i = 1, 2, 3, ..., N$, is said to have a linear relation or, in other words, the dataset fit well to a linear function if $P(\chi^2 \geq \chi^2_{calc})$ has a value in the range of 10%–90% [8].

3.4. The acceleration of gravity

As can be seen from Eqs. (15) and (17), the calculation of the viscosity coefficient of the liquid needs knowledge of g, the local acceleration of gravity. However, there has been no accepted officially reported value of g in Yogyakarta. Therefore, it has been determined in this study using the most refined 2000 amendment of the World Geodetic System 1984 (WGS 84) ellipsoidal gravity formula [9]:

$$g = 9.7803253359 \frac{1 + 0,00193185265241 \sin^2 \varphi}{\sqrt{1 - 0.00669437999013 \sin^2 \varphi}} \quad (\mathrm{m/s^2}), \tag{26}$$

where ϕ is latitude. The location of the Yogyakarta city center in the Special Region of Yogyakarta was found ("measured") online from https://www.worldatlas.com to be $-7.782780°$ south latitude and $110.36°$ east longitude at an elevation of 125 m above sea level. A more precise location where the experiment was conducted, i.e. the Laboratory of Physics, Campus III, Ahmad Dahlan University, Yogyakarta, was found elsewhere at https://latitude.to to be $-7.81023°$ south latitude and $110.38850°$ east longitude. Neglecting the effect of elevation and putting the value $\phi = 7.81023°$ into Eq. (26), we obtain $g = 9.781278866$ m/s^2.

4. Results and discussion

4.1. System constants

For the determination of the capillary tube inner diameter D and its error using Eqs. (20) and (21), two capillary tubes were available with different inner diameters. For the capillary tube of smaller inner diameter, the values of $D_t = (0.605 \pm 0.001)$ cm, $V_p = (4.030 \pm 0.029)$ cm^3, and $L = (14.971 \pm 0.008)$ cm were obtained, giving $D = (0.153 \pm 0.009)$ cm, while for the capillary tube of larger inner diameter, the values of $D_t = (0.696 \pm 0.001)$ cm, $V_p = (5.017 \pm 0.024)$ cm^3, and $L = (15.253 \pm 0.007)$ cm were obtained, giving $D = (0.256 \pm 0.005)$ cm.

The water density and its error were calculated from Eqs. (22) and (23) using $m = (45.334 \pm 0.010)$ g and $V = (46.070 \pm 0.067)$ cm^3, giving $\rho = (0.984 \pm 0.002)$ g/cm^3, close to the accepted values of 0.99730 g/cm^3 at 24 °C and 0.99678 g/cm^3 at 26 °C for air-free water [7]. Similarly for the spirit density and its error, $m = (35.480 \pm 0.105)$ g and $V = (44.948 \pm 0.073)$ cm^3, giving $\rho = (0.789 \pm 0.003)$ g/cm^3, which is close to the accepted value of 0.8090 g/cm^3 for a 94% methanol solution in water at 20 °C [10], and in agreement with the value of 0.792 g/cm^3 for pure methanol at 20 °C [11].

Vessel outer diameter measurement using a caliper gave (5.859 ± 0.011) cm. There are two capillary tube inner diameters; the smaller diameter is (0.153 ± 0.009) cm and the larger is (0.256 ± 0.005) cm. Capillary tube lengths are (14.964 ± 0.007) cm, (22.556 ± 0.01) cm, and (30.499 ± 0.025) cm for the smaller diameter tubes and (10.452 ± 0.011) cm, (15.253 ± 0.007) cm, and (24.648 ± 0.007) cm for the larger diameter tubes.

4.2. Water

Table 1 shows our experimental data for water, where D and L are respectively the inner diameter and length of the capillary tube. The fifth column shows the values of the viscosity coefficient and their errors. The last column shows the maximum Reynolds numbers and their errors.

The fourth column in Table 1 shows values of $P\left(\chi^2 \geq \chi^2_{hit}\right)$ between 10% and 90% in all cases, which means that the linear theoretical relation between $-\ln(2z_1/z_0 - 1)$ and time is fulfilled. The linear relation can also be seen graphically in Figures 2 and 3. Figure 2 shows deviation of one end data point far from each

Table 1. Experimental data for water.

No.	D (cm)	L (cm)	$P\left(\chi^2 \geq \chi^2_{hit}\right)$ %	η (10^{-2}P)	$Re_{\max}$
1	0.153	14.964	62.93	1.11 ± 0.27	147 ± 30
2		22.556	19.65	0.91 ± 0.22	91 ± 18
3		30.499	52.40	0.79 ± 0.20	59 ± 12
4	0.256	10.452	17.46	1.21 ± 0.11	1220 ± 135
5		15.253	56.58	1.17 ± 0.09	753 ± 89
6		24.648	51.74	1.23 ± 0.10	407 ± 47

corresponding fitted line for capillary tube lengths of 10.452 cm and 15.253 cm at fixed capillary tube inner diameter of 0.153 cm, which is supposed to be due to rather difficult tracking since the liquid motion is slow. The deviation does not significantly affect the calculated viscosity coefficient since the data points still lie within the limits of experimental errors and their relative errors are large. As in the case shown in Figure 2, in Figure 3 the deviation of one end data point far from each corresponding fitted line can be seen for capillary tube lengths of 10.452 cm and 15.253 cm at fixed capillary tube inner diameter of 0.256 cm, which is supposed to be due to rather difficult tracking since the liquid motion is slow.

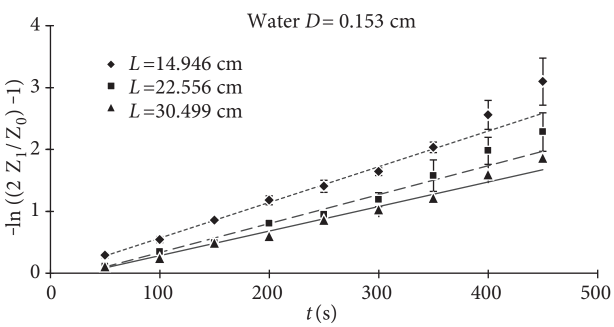

Figure 2. Graph of $-\ln(2z_1/z_0 - 1)$ against time for experiments with water for different capillary tube lengths L at fixed capillary tube inner diameter of 0.153 cm.

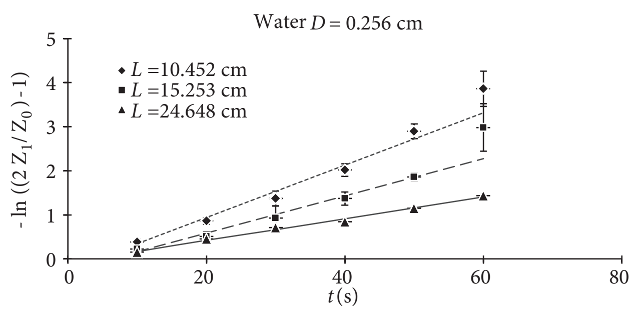

Figure 3. Graph of $-\ln(2z_1/z_0 - 1)$ against time for experiments with water for different capillary tube lengths L at fixed capillary tube inner diameter of 0.256 cm.

The fifth column in Table 1 shows the calculated values of the viscosity coefficient and their errors. It is seen that the values for different capillary tube sizes are consistent with each other, as expected, within the limits of experimental errors, and their weighted average gives $(1.15 \pm 0.05) \times 10^{-2}$ P, in agreement with a recent value of $(1.08 \pm 0.07) \times 10^{-2}$ P at 20 °C obtained in an experiment by observing the oscillation of a sphere with the aid of a webcam [6]. Previously using the same communicating vessels method as reported here, Ortega et al. [1] obtained 0.92×10^{-2} P at (22.5 ± 0.1) °C and 0.81×10^{-2} P at (28.3 ± 0.1) °C, slightly smaller than the value reported here.

The last column in Table 1 shows the maximum Reynolds numbers and their errors, and it is seen that all the calculated maximum Reynolds numbers are smaller than 2300, which means that the liquid flow is laminar as expected.

4.3. Spirit

Table 2 shows experimental data with spirit. The fourth column in Table 2 shows values of $P\left(\chi^2 \geq \chi^2_{hit}\right)$ between 10% and 90% in all cases, which means that the linear theoretical relation between $-\ln\left(2z_1/z_0 - 1\right)$ and time is fulfilled. The linear relation can also be seen graphically in Figures 4 and 5. Figure 4 shows deviations of one end data point far from the fitted lines for capillary tube lengths of 22.556 cm and 30.499 cm at fixed capillary tube inner diameter of 0.153 cm, which is supposed to be due to rather difficult tracking since the liquid motion becomes too slow. However, the deviation does not affect significantly the calculated viscosity coefficient since the data points have relatively large errors. Unlike the case shown in Figure 4, in Figure 5 there is no experimental data point for all capillary tube lengths that lies far from the fitted line, which is an indication that the precision of tracking in this case does not decrease as time increases, since the liquid motion is relatively faster than in the cases shown in Figures 2, 3, and 4. This can be justified by noting that, as one goes from Figure 2 through Figure 5, the approximate average liquid flow velocity for each case can be calculated roughly by dividing the capillary tube length by the total flow time, and the values are 0.1 cm/s, 0.4 cm/s, 0.1 cm/s, and 1 cm/s, respectively, in that order.

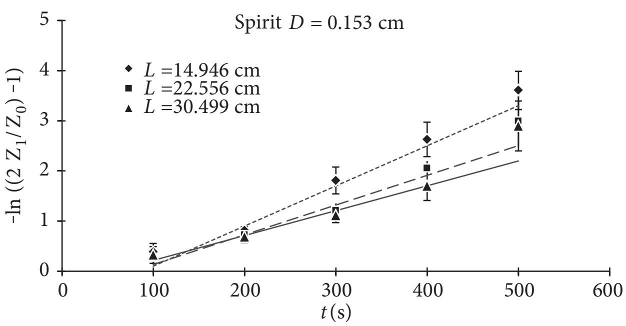

Figure 4. Graph of $-\ln\left(2z_1/z_0 - 1\right)$ against time for experiments with spirit for different capillary tube lengths L at fixed capillary tube inner diameter of 0.153 cm.

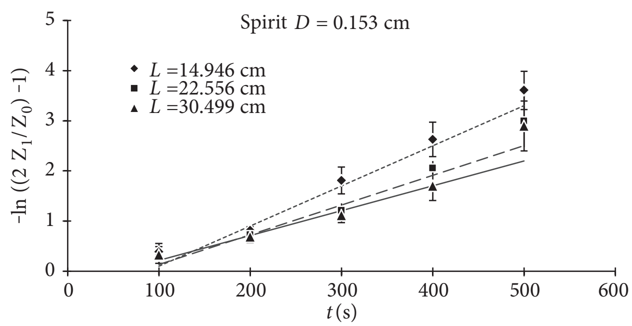

Figure 5. Graph of $-\ln\left(2z_1/z_0 - 1\right)$ against time for experiments with spirit for different capillary tube lengths L at fixed capillary tube inner diameter of 0.256 cm.

Table 2. Experimental data for spirit.

No.	D (cm)	L(cm)	$P\left(\chi^2 \geq \chi^2_{hit}\right)$ %	$\eta(10^{-2}P)$	$Re_{\max}$
1	0.153	14.964	16.82	0.64 ± 0.17	422 ± 87
2		22.556	22.27	0.58 ± 0.14	264 ± 54
3		30.499	40.60	0.51 ± 0.15	183 ± 38
4	0.256	10.452	24.40	0.55 ± 0.07	1734 ± 210
5		15.253	32.46	0.54 ± 0.05	1003 ± 133
6		24.648	44.22	0.56 ± 0.05	608 ± 72

The last column in Table 2 shows the maximum Reynolds numbers and their errors, and it is seen that all the calculated maximum Reynolds numbers are smaller than 2300, which means that the liquid flow is laminar as expected.

The fifth column in Table 2 shows the calculated values of viscosity coefficient and their errors. It is seen that the values for different capillary tube sizes are consistent with each other, as expected, within limits of experimental errors, and their weighted average is $(0.55 \pm 0.03) \times 10^{-2}$ P, in agreement with the accepted value for pure methanol of 0.543×10^{-2} P at 25 °C [7]; the small discrepancy of about 1.29% in the coefficient of viscosity can be accounted for by the 6% presence of water and ethanol (both with coefficients of viscosity close to 1.0×10^{-2} P). For comparison, our result agrees well with an extrapolated value of 0.5294×10^{-2} P for pure methanol at 0.10 MPa (about equal to atmospheric pressure) and 300 K by Xiang et al. [12].

4.4. Conclusions

To summarize, a set of Tracker-aided experiments using communicating vessels have been performed to determine the viscosity coefficients of water and spirit at room temperature with weighted linear regression analysis. All the experiments conducted using various capillary tube sizes showed a linear relation of $-\ln(2z_1/z_0 - 1)$ against time, as expected theoretically. The maximum Reynolds numbers indicate that the liquid flow is laminar in all cases. The coefficients of viscosity obtained are in good agreement with accepted values from previous studies.

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