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Various paths to the spontaneous growth of p-form fields

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Abstract: We show that p-form fields can go through spontaneous growth due to various couplings in gravity theories, forming a new example of spontaneous tensorization. Generalizing the spontaneous scalarization theory of Damour and Esposito-Farèse where the original idea has been applied to different fields from vectors to spinors has received high levels of interest in recent years. We first review this existing literature on spontaneous growth in gravity, and then apply the known mechanisms to p-forms. We show that one can induce spontaneous growth in p-forms for each of the regularized instability mechanisms, which was not the case for other types of fields. We obtain theories with the common property that they lead to large deviations from general relativity in strong gravity as is usual in spontaneous tensorization. This is especially interesting for gravitational wave observations, a direct probe of this regime.

Key words: Alternative theories of gravitation, p-form fields, neutron stars, gravitational waves

1. Introduction

Gravitational wave astronomy has been the main driver of research in classical gravitational physics in recent years, and possible modifications of general relativity (GR) became a leading topic of interest [1–3]. One of the ideas that has come to the forefront is spontaneous growth, where fields that are nonminimally coupled to the metric grow spontaneously around compact objects, neutron stars (NSs) and black holes (BHs) [2, 3]. These fields form stable clouds, and lead to large deviations from GR. Theories with this feature have two main advantages. First, they are relatively easy targets for gravitational wave telescopes due to their prominent signatures in the strong-field regime, the most relevant cases for gravitational wave detections. Second, the deviation from GR becomes tiny for weak fields; hence, they conform to the already existing tests in this regime. In short, spontaneous growth theories are direct targets for gravitational wave astronomy: they will be either confirmed or severely restricted by observations in the coming decades unlike many other gravity theories which can stay indistinguishable from GR in the foreseeable future [3].

The first example of spontaneous growth was introduced in scalar-tensor theories by Damour and Esposito-Farèse (DEF) [4]. In this theory, the nonminimal coupling of a scalar field to the metric in the so-called Einstein frame leads to a tachyonic instability in the presence of NSs. An identically vanishing scalar field is a solution of the theory, corresponding to GR, but it is not a stable solution. Arbitrarily small scalar fields grow exponentially at first, but eventually the growth is suppressed by nonlinear interactions, and the system settles to a nontrivial stable configuration of a NS immersed in a scalar cloud. Such a "tame" instability is called a "regularized instability" [5]. The scalar fields typically attain large values by the time they

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stop growing, and cause large order-of-unity deviations from GR. This phenomenon is dubbed "spontaneous scalarization."

It was recently discovered that the DEF theory was one of many theories of gravity that feature the spontaneous growth and eventual stabilization of fields. First, the growth mechanism is not specific to scalars, e.g., one can have a vector-tensor [6] or spinor-tensor [7] theory of spontaneous growth. Second, instabilities other than tachyons, such as ghosts, can also trigger the initial exponential growth [5]. Third, the form of the nonminimal coupling can be different from that of scalar-tensor theories, and one can observe spontaneous growth in fields coupled to curvature terms as well as matter [8–11], or disformally coupled fields [12]. The generic phenomenon is called "spontaneous tensorization", and all the aforementioned theories have relatively large observational signatures in terms of gravitational waves, making them especially relevant for contemporary research.

In this study, we will investigate the spontaneous growth of p-form fields. At first, this may look like simply generalizing spontaneous scalarization to other fields, the first form of extending the DEF theory that we mentioned. However, we will see that one can have various theories of gravity with p-forms containing all the different mechanisms we mentioned above: spontaneous growth through ghosts, nonmatter couplings and disformal couplings. This is interesting, since it is not always possible to find more than one type of regularized instability for a given field [7]. Vector fields can be considered 1-forms, and they are encountered commonly as the carrier of electromagnetic fields, both in classical and quantum theories [13]. Higher form fields have not been observed in nature, but they are prominent in quantum gravity theories such as the Kalb-Ramond field in string theory [14] and modifications of electromagnetism [15].

We will first give a literature summary of spontaneous scalarization and various ways of generalizing it in Section 2. Readers familiar with the field may directly proceed to Section 3, where we present our novel results on p-form fields and how they can spontaneously grow via different mechanisms. In Section 4, we will summarize our results and give a perspective about their place in the general literature of alternative theories of gravitation. We will use the geometric units $G = c = 1$.

2. Spontaneous scalarization and its generalizations

The quintessential spontaneous growth theory is that of DEF [4], based on scalar-tensor theories given by the action

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R-\overbrace{2g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi}^{T_\phi}-\overbrace{2m_\phi^2\phi^2}^{V_\phi}\right]+S_{\rm m}\left[f_{\rm m},\tilde{g}_{\mu\nu}\right] \tag{1}$$

where

$$\tilde{g}_{\mu\nu}=A^2(\phi)g_{\mu\nu} \tag{2}$$

for some conformal scaling function A, and $f_{\rm m}$ collectively represents matter degrees of freedom. This is the so-called "Einstein frame" where the metic we use has the exact Hilbert form in the action, and the matter coupling is nonminimal. We will use this exclusively in this study as opposed to the "Jordan frame" where $\tilde{g}_{\mu\nu}$ is used as the metric variable; hence, the matter coupling to this metric is minimal, but the metric action has a scalar dependence. We will sometimes use quantities associated with $\tilde{g}_{\mu\nu}$, such as the stress energy tensor arising from deviation with respect to it, $\tilde{T}^{\mu\nu}$, which will carry a tilde to distinguish them from their counterparts associated with $g_{\mu\nu}$.

As explained in the introduction, $\phi = 0$ is a solution of this theory, and is exactly GR; however, such solutions are unstable in the presence of NSs. The stable solutions are scalarized stars where the scalar field amplitude is high near the star, leading to large deviations from GR, but the amplitude decays away from the star; hence, the theory satisfies the known bounds from weak-field tests of gravity. The former property makes this theory relevant for gravitational wave science, and the latter ensures that it is not pushed to an unnatural part of the parameter space like other scalar-tensor theories [16]. The mass term m_ϕ was not present in the original formulation, but has become necessary after binary star observations severely restricted the massless version [17].

How are these desirable properties achieved by a simple scalar-tensor theory, and what places it apart from other versions such as that of Brans-Dicke [18]? A more detailed explanation can be found in [17]; here we will summarize the basics. The DEF theory makes the choice $A(\phi) = e^{\beta\phi^2/2}$, i.e. a quadratic leading term in the conformal scaling rather than the linear one in the Brans-Dicke theory. The effect of this can be easily seen in the equation of motion (EOM) for the scalar field.

$$\begin{aligned} \Box_g \phi &= \left(-8\pi A^4 \frac{d\left(\ln A(\phi)\right)}{d(\phi^2)} \tilde{T} + m_\phi^2\right)\phi \\ &\approx \left(-4\pi\beta\tilde{T} + m_\phi^2\right)\phi \equiv m_{eff}^2\ \phi. \end{aligned} \tag{3}$$

We look at the linearized theory in the second line, which shows that our choice of A leads to a coupling that behaves like an effective mass-square term for small perturbations. In fact, this is true for any conformal coupling with the Taylor expansion $A(\phi) = 1 + \beta\phi^2/2 + \dots$. If matter is not heavily relativistic, then $\tilde{T} = -\tilde{\rho} + 3\tilde{p} \approx -\tilde{\rho} < 0$; hence, negative values of β with large absolute value mean $m_{eff}^2 < 0$, a degree of freedom with imaginary effective mass. Low wave-number modes of such a field has imaginary frequencies; hence, their time evolution is exponential. This is called a tachyon, and is the cause for the instability of the $\phi = 0$ solution. On the other hand, we want this growth to end, an ever-growing field is unphysical. This is achieved by the specific form of A: as ϕ grows, A is suppressed exponentially, which also suppresses the negative part of the effective mass square in Eq. 3. This means that the instability automatically shuts off as it reaches high enough values. Lastly, even though it may look like any matter distribution can go through spontaneous growth, a closer examination shows that wavelengths of the scalar field comparable to the size of the matter region inititate the instability, and for the natural choice of $|\beta| \sim 1$ only the most compact matter distributions, NSs, scalarize [17].

The first idea to generalize the DEF theory simply recognizes that the tachyons and their suppression at high field values are not specific to scalars. For the simplest choice, replace the scalar with a vector [6]

$$\begin{aligned} &\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - F^{\mu\nu}F_{\mu\nu} - 2m_X^2 X^\mu X_\mu\right] \\ &\qquad + S_{\rm m}\left[f_{\rm m}, A_X^2(x)g_{\mu\nu}\right],\ x = g^{\mu\nu}X_\mu X_\nu\ , \end{aligned} \tag{4}$$

where $F_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$ is the field strength tensor. Recall that all the desirable properties of the DEF case are due to the specific form of the conformal scaling A. In analogy, if we choose a quadratic leading dependence on X_μ such as $A_X = e^{\beta_X X_\mu X^\mu/2}$, the vector EOM becomes

$$\nabla_\rho F^{\rho\mu} = \left(-4\pi A_X^4 \beta_X \tilde{T} + m_X^2\right) X^\mu\ . \tag{5}$$

The linearized EOM again has a negative mass-square for NSs, leading to solutions where the star is immersed in a vector field cloud. The astrophysical significance of such configurations are similar to those of the DEF theory, but they provide richer phenomenology. We call this phenomenon "spontaneous vectorization", and the growth of fields in gravity theories in this manner in general is named "spontaneous tensorization." [6].

The second path to generalize the DEF theory recognizes that the tachyon is not the only type of instability that leads to spontaneous growth. For the tachyon, the effective potential term of the scalar has the "wrong" sign, leading to a mass-square term with the wrong sign. If the effective kinetic term has the wrong sign, then the highest derivative term in the EOM changes sign, and we have a "ghost" field. Such terms are equivalent to having derivative couplings as in the action [5]

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}[R - 2\nabla_\mu\phi\nabla^\mu\phi - 2m_\phi\phi^2] \\ + S_{\rm m}[f_{\rm m}, A_\partial^2(K)g_{\mu\nu}] \ , \ K = g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \ , \tag{6}$$

which gives the EOM

$$\nabla_\mu\left[(-8\pi\tilde{T}A_\partial^3A'_\partial + 1)\nabla^\mu\phi\right] = m_\phi^2\phi \ . \tag{7}$$

The principal part (the term with the highest order derivative) of the linearized EOM reads

$$(-4\pi\tilde{T}\beta_\partial + 1)\Box\phi = \dots \ , \tag{8}$$

if we use $A_\partial = e^{\beta_\partial K/2}$ or some similar function of K. Hence, the wave operator rather than the mass term changes sign, but the final result is the same: $\phi = 0$ is unstable. We should note that NS solutions with such derivative couplings are possible, but they contain sharp structures which can be possibly ruled out by astrophysical observations [5].

The third way to generalize the DEF theory gives up the matter coupling idea which we have employed so far, and couples the field that is to grow spontaneously to some other term, such as the Gauss–Bonnet term [8–10]

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}[R - 2\nabla_\mu\phi\nabla^\mu\phi + \lambda^2 f(\phi)\mathcal{R}] \ . \tag{9}$$

Here, $\mathcal{R} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss–Bonnet invariant and $f(\phi)$ is some coupling function. Note that the term the scalar field couples to is pure curvature, that is, it is constructed out of the metric only. The EOM is

$$\Box\phi = -\frac{\lambda^2}{4}\left.\frac{df}{d\phi}\right|_{\phi=0}\mathcal{R} \ . \tag{10}$$

If the first derivative of the coupling function vanishes, such as $f(\phi) = 1 - e^{\beta\phi^2/2}$, then the linearized EOM becomes

$$\Box\phi \approx -\frac{\lambda^2}{4}\left.\frac{d^2f}{d\phi^2}\right|_{\phi=0}\mathcal{R}\phi \equiv m_{eff}^2\ \phi \ . \tag{11}$$

This behavior is identical to that of the scalar in the DEF theory, small perturbations are tachyonic for a correct choice of β, and most of the astrophysical results follow. This theory has the additional advantage that any object that generates enough curvature, not just matter, can spontaneously scalarize, and scalarized black hole solutions have been calculated [8, 9].

One can combine the aforementioned paths, and obtain theories of spontaneous vectorization based on ghosts, so we have both tachyon- and ghost-based spontaneous growth for scalars and vectors [5]. Similarly, we can devise theories of generalized Einstein–Gauss–Bonnet theories with vectors instead of scalars [11]. In another alternative, one can come up with spontaneous growth theories that contain both scalars and vectors, in analogy to the Higgs mechanism, and preserve the gauge symmetry of the vectors [19]. However, not all mechanisms of spontaneous growth work for all fields. For example, the spontaneous growth of spinor fields can only be achieved via a mechanism which can be best described as ghost-based, but has differences from all the mechanisms we mentioned [7].

Another way to combine the ghosts and tachyons is using a disformal coupling rather than a conformal one. A scalar-dependent disformal coupling is of the form [20]

$$\tilde{g}_{\mu\nu} = A^2(\phi)\left[g_{\mu\nu} + \Lambda B^2(\phi)\partial_\mu\phi\partial_\nu\phi\right] \ . \tag{12}$$

Inserting this to Eq. (1) and varying the action, we get [12, 21]

$$\begin{aligned}\Box\phi = m_\phi^2\phi + \frac{4\pi}{1+\Lambda B^2\partial_\mu\phi\partial^\mu\phi}\times \\ \left\{\Lambda B^2\left[(\delta-\alpha)\,T^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + T^{\rho\sigma}\partial_\rho\partial_\sigma\phi\right] - \alpha T\right\} \ ,\end{aligned} \tag{13}$$

with $\alpha(\phi) \equiv A^{-1}(dA/d\phi)$ and $\delta(\phi) \equiv B^{-1}(dB/d\phi)$. It is slightly more cumbersome to analyze the linearization of this equation, but one can show that it contains elements from both tachyon- and ghost-based spontaneous growth [12]. Disformal transformations can also be generalized to other fields, for example one of the few possible vector-dependent transformations is given by

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B(x)X_\mu X_\nu \ , \tag{14}$$

which also leads to spontaneous vectorization.[1]

3. Spontaneous growth of p-form fields

A p-form (or a differential form) field $X_{\mu_1\ldots\mu_p}$ is a completely antisymmetric $(0,p)$ tensor field. Their basic properties and operators on them which we use below can be found in Ref. [22]. In $D \geq p$ dimensions, a p-form has the canonical action[2]

$$S_F = \int d^4x\sqrt{-g}\ \mathcal{L}_F = -\int d^4x\sqrt{-g}\ \overbrace{F^{\mu_1\ldots\mu_{p+1}}F_{\mu_1\ldots\mu_{p+1}}}^{\eta_F} \tag{15}$$

where

$$\mathbf{F} \equiv d\mathbf{X} \Rightarrow F_{\mu_1\ldots\mu_{p+1}} \equiv (p+1)\nabla_{[\mu_1}X_{\mu_2\ldots\mu_{p+1}]} \ , \tag{16}$$

[1]A detailed study of this co-authored by us is currently under review.

[2]Traditionally, there is also a factor of $\frac{1}{2(p+1)!}$ in the action, but such factors can be absorbed in a field redefinition, and we will omit them to make comparison to spontaneous vectorization easier.

$_{[\,]}$ denoting antisymmetrization. The theory has the gauge freedom $\mathbf{X} \to \mathbf{X} + d\lambda$, where λ is any $(p-1)$-form field thanks to the property $d(d\lambda) = 0$.

Similarities to massless vector fields are unmistakable from the field strength tensor to the gauge freedom. Indeed, the canonical massless vector field action is nothing but the case for $p = 1$ up to some overall constant factors. Note that one can add a mass term

$$S_X = \int d^4x\sqrt{-g}\ \mathcal{L}_X = -(p+1)m_X^2 \int d^4x\sqrt{-g}\,\overbrace{X^{\mu_1\dots\mu_p}X_{\mu_1\dots\mu_p}}^{\eta_X}\ , \tag{17}$$

but this breaks the gauge symmetry. The EOM corresponding to the action $S_F + S_X$ is

$$\nabla_\rho F^{\rho\mu_1\dots\mu_p} = m_X^2 X^{\mu_1\dots\mu_p}\ , \tag{18}$$

in complete analogy to that of the vector field.

One should note that in D dimensions, all p-form fields vanish for $p > D$ due to antisymmetry. Moreover, there is a natural one-to-one mapping, the Hodge dual, between p-forms and $(D-p)$-forms; hence, one need not consider form fields with $p > D/2$. p-forms commonly appear in theories with extra dimensions, but we will use $D = 4$, which can be generalized in a trivial manner.

In the following subsections, we will apply all the mechanisms of spontaneous growth we have discussed so far to p-form fields.

3.1. Tachyon-based spontaneous p-form field growth

The simplest theory that spontaneously grows p-forms is given by the action

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}\,[R + \mathcal{L}_F + \mathcal{L}_X] + S_{\rm m}[f_{\rm m}, \tilde{g}_{\mu\nu}]\ . \tag{19}$$

where

$$\tilde{g}_{\mu\nu} = A_X^2(\eta_X)g_{\mu\nu} \tag{20}$$

Recall that $\eta_X = X^{\mu_1\dots\mu_p}X_{\mu_1\dots\mu_p}$; hence, this theory is a complete analog of tachyon-based spontaneous vectorization in Eq. 4. For our generic choice $A_X = e^{\beta_X\eta_X/2}$, the EOM is

$$\nabla_\rho F^{\rho\mu_1\dots\mu_p} = \overbrace{\left(-8(p+1)^{-1}\pi A_X^4\beta_X\tilde{T} + m_X^2\right)}^{m_{eff}^2} X^{\mu_1\dots\mu_p}\ . \tag{21}$$

Appropriate values of β_X lead to a negative m_{eff}^2; hence, a tachyon, and all the the succeeding discussion on vectors is also valid for this theory of spontaneous p-form growth.

We should repeat that the mass term m_X is not needed for spontaneous growth, it actually suppresses it. However, it is likely needed to conform to observational bounds, as in the scalarization and vectorization cases.

3.2. Ghost-based spontaneous p-form field growth

Like scalars, it is also possible to spontaneously grow p-form fields through a ghost-like instability. This is achieved by the action

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}\,[R + \mathcal{L}_F + \mathcal{L}_X] + S_{\rm m}[f_{\rm m}, \tilde{g}_{\mu\nu}]\ , \tag{22}$$

where

$$\tilde{g}_{\mu\nu} = A_F^2(\eta_F) g_{\mu\nu} \tag{23}$$

Namely, the action is exactly in the same form as Eq. 19, but the conformal scaling is now a function of the kinetic term; hence, contains derivatives. The effect of this can be easily seen when we vary the action for the choice $A_F = e^{\beta_F \eta_F/2}$ and obtain the EOM

$$\nabla_\rho \left[(-8\pi A_F^4 \beta_F \tilde{T} + 1) F^{\rho\mu_1\cdots\mu_p} \right] = m_X^2 X^{\mu_1\cdots\mu_p} \ . \tag{24}$$

This is just like ghost-based spontaneous scalarization in Eq. 7. The principal part of Eq. 24 is

$$(-8\pi A_F^4 \beta_F \tilde{T} + 1) \nabla_\rho F^{\rho\mu_1\cdots\mu_p} = \ldots \ , \tag{25}$$

we see that it changes sign when β_F is large enough. This is in complete analogy to the scalar case; hence, we expect the p-form field to grow spontaneously due to this ghost-like degree of freedom, and all the consequences of such an instability follow as before.

3.3. Spontaneous growth of p-form fields beyond matter coupling

So far, we only considered transformations of the metric in the matter coupling. However, we have seen that this is not the only way to obtain an instability, and presented how to couple a scalar to a curvature term in Eq. 9. This type of theory can also be generalized to p-form fields as

$$\frac{1}{16\pi} \int d^4x \sqrt{-g} [R + \mathcal{L}_F + \lambda^2 f(\eta_X) \mathcal{R}] \ . \tag{26}$$

If the coupling function has leading quadratic dependence on $\mathbf{X}$ such as $f(\eta_X) = 1 - e^{\beta_X \eta_X/2}$, the linearized EOM is

$$\nabla_\rho F^{\rho\mu_1\cdots\mu_p} \approx \overbrace{\frac{\lambda^2 \beta_X}{2(p+1)} \mathcal{R}}^{m_{eff}^2} X^{\mu_1\cdots\mu_p} \ . \tag{27}$$

This equation behaves like that of a tachyon if $\beta_X < 0$, and results in a phenomenology similar to the scalar case. We only considered $D = 4$ here, where the Gauss–Bonnet term is the only option if we want to avoid anomalies, but other Lovelock curvature terms are possible in higher dimensions [2]. We also did not consider an intrinsic p-form field mass m_X, but it is possible to add S_X to the action and still have a tachyonic instability. The only major difference would be in the far field behavior of the field as in spontaneous scalarization, which can make agreement with binary star system observations easier.

One might be tempted to think that we can have a ghost-based version of this theory through a derivative coupling, i.e.

$$\frac{1}{16\pi} \int d^4x \sqrt{-g} [R + S_F + \lambda^2 f(\eta_F) \mathcal{R}] \ . \tag{28}$$

Even though the reversal of the sign of the principal part in the EOM occurs in this theory as well, third derivatives of the metric also appear due to the derivatives of $\mathcal{R}$ [11]. More than two time derivatives are generically indicative of unphysical ghost degrees of freedom, the kind that does not eventually stop growing. Because of this, theories involving higher curvature terms are thought not to be generalizable to derivative couplings [11].

We should also add that one can replace the Gauss–Bonnet invariant with another action term, such as the Chern–Simons term, or an expression that depends on some other field, which can also incite the required instability [23]. However, we need to be careful not to have any undesired ghosts, which is the case for any curvature term other than that of Gauss–Bonnet if we restrict ourselves to 4 dimensions. Nevertheless, such theories are still of interest, considered as effective theories of a yet unknown more fundamental theory [2, 3].

3.4. Disformal transformations based on p-form fields

We discussed in the previous section that conformal scaling is not the only way to modify the metric that couples to the matter field in Eq. 12. One can generalize such transformations to depend on p-forms. Namely, consider the action in Eq. 19 or Eq. 22, but with a metric transformation of the form

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_X(\eta_X)\ \overbrace{X_{\rho_1\ldots\rho_{p-1}(\mu|}X^{\rho_1\ldots\rho_{p-1}}{}_{|\nu)}}^{X^2_{(\mu\nu)}} \tag{29}$$

in analogy to Eq. 14. $_{(\mu|\ldots|\nu)}$ represents symmetrization over μ,ν, but none of the indices in between, as required by the symmetry of the metric. The EOM is

$$\begin{aligned}\nabla_\rho F^{\rho\mu_1\cdots\mu_p} = {} & m_X^2 X^{\mu_1\cdots\mu_p} \\ & -8(p+1)^{-1}\pi\sqrt{\chi}B'_X\tilde{T}^{\rho\sigma}X^2_{(\rho\sigma)}X^{\mu_1\cdots\mu_p} \\ & -8(p+1)^{-1}\pi\sqrt{\chi}B_X\tilde{T}^{\sigma\mu_p}g_{\sigma\rho}X^{\mu_1\cdots\mu_{p-1}\rho}\end{aligned} \tag{30}$$

with $\sqrt{\chi} \equiv \sqrt{-\tilde{g}}/\sqrt{-g}$ and $B'_X \equiv \partial B_X/\partial\eta_X$.

To see the unstable mode in Eq. 30, let us linearize and recast it as

$$\begin{aligned}\nabla_\rho F^{\rho\mu_1\cdots\mu_p} &\approx (m_X^2\delta_\rho^{\mu_p} - 8(p+1)^{-1}\pi B_X(0)\tilde{T}^{\sigma\mu_p}g_{\sigma\rho})X^{\mu_1\cdots\mu_{p-1}\rho} \\ &= \mathcal{M}_\rho^{\mu_p}X^{\mu_1\cdots\mu_{p-1}\rho}\ .\end{aligned} \tag{31}$$

$\mathcal{M}_\rho^{\mu_p}$ acts as a mass-square tensor, and if it has any negative eigenvalues, the degree of freedom corresponding to the related eigenvalue behaves as a tachyon. A negative eigenvalue can always be achieved by appropriate values of $B_X(0)$ and $\tilde{T}^{\sigma\mu_p}$. This can be more directly seen for a spacetime with a diagonal metric and stress-energy tensor, as is usually encountered in spherically symmetric spaces such as nonrotating NSs. Then, the linearized EOM becomes

$$\nabla_\rho F^{\rho\mu_1\cdots\mu_p} \approx (m_X^2 - 8(p+1)^{-1}\pi B_X(0)\tilde{T}^{\mu_p\mu_p}g_{\mu_p\mu_p})X^{\mu_1\cdots\mu_{p-1}\mu_p}\ , \tag{32}$$

where there is no summation over the index μ_p. The effective mass-square term is clearly negative for sufficiently large negative values of $B_X(0)$.

Having an instability starts the spontaneous growth process, but in order to have a stable solution, the growth must shut off eventually. This can be achieved by having $B_X(\eta_X \to \infty) = 0$, so that the modification to the mass term in the fully nonlinear theory in Eq. 30 gets suppressed at high p-form field values. This means that our usual choice $B_X(\eta_X) = \Lambda_X e^{\beta_X \eta_X/2}$ satisfies the above requirements. We added the constant factor Λ_X in this case since having a negative eigenvalue for $\mathcal{M}_\rho^{\mu_p}$ requires $B_X(0)$ to be different from unity, and possibly large.

Lastly, we add that Eq. 29 is not the only possible p-form dependent disformal coupling. For example, one can also use the field strength tensor as in

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \Lambda_F B_F(\eta_X) \overbrace{F_{\rho_1\dots\rho_p(\mu|} F^{\rho_1\dots\rho_p}_{|\nu)}}^{F^2_{(\mu\nu)}} \tag{33}$$

where Λ_F is introduced to render B_F dimensionless. We will not examine this theory in detail, but it leads to ghost-based spontaneous growth due to the derivative terms in the coupling.

4. Conclusion

We have adapted all the known mechanism that lead to spontaneous scalarization in gravity to the case of p-form fields. This is not a trivial task, since there are fields where only one mechanism of spontaneous growth is possible, like spinors [7], or there is no known mechanism at all, such as rank-2 tensors [5]. One helpful fact we utilized is that vector fields can be expressed as 1-form fields, and vectors are very amenable to spontaneous growth through various mechanisms which can be generalized to p-forms.

We have explicitly investigated the spontaneous growth of p-forms through matter couplings that arise from both conformal and disformal transformations, each of which can lead to ghost-like or tachyon-like instabilities. We have also seen that one can couple the form field to a curvature term, and still obtain spontaneous growth, with the added benefit that black holes can also grow p-form clouds since matter is not required.

Different spontaneous growth mechanisms are not equally plausible. Tachyon-based mechanisms generically respect the known bounds from observations. However, ghost-based mechanisms generically lead to sharp structures in compact objects such as cusps in the density profile [5]. Even though we are not aware of any observations that directly rule out such features, we expect them to have easily recognizable signatures; hence, they should be quickly confirmed or ruled out in the near future.

As with much of the recent work on spontaneous tensorization, the case of p-form fields are not only valuable in their own right, but rather they are also the newest example that spontaneous growth is ubiquitous in gravity theories. We believe that the best approach to understand spontaneous tensorization is analyzing the common properties of various theories in order to better understand the spontaneous growth phenomena itself. All spontaneous tensorization theories share the appealing property that they have large deviations from GR in strong gravitational fields; hence, the era of gravitational wave science is an especially exciting time to relate these theoretical ideas to observations.

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