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Relativistic dynamics of a scalar boson confined by Woods–Saxon potential in a nucleus

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Abstract: We undertook a theoretical study of a scalar boson confined by Woods–Saxon potential in a nucleus via the Duffin–Kemmer–Petiau equation. We analytically obtained the eigenvectors and energy levels through the hypergeometric functions. Single-particle energy levels of a boson in the ^{208}Pb nucleus were calculated numerically.

Key words: Duffin–Kemmer–Petiau equation, Woods–Saxon potential, Pekeris approximation

1. Introduction

The Duffin–Kemmer–Petiau (DKP) equation is a relativistic wave equation that has the same form for spin-0 and spin-1 particles, which are analyzed by the Klein–Gordon (for spin-0 particles) and Proca (for spin-1 particles) equations. The DKP equation is a first-order equation, as is the Dirac equation. These two equations are also structurally similar to each other. The gamma matrices in the Dirac equation are replaced by the beta matrices in the DKP equation [1–3]. The DKP equation has explored meson-nucleus scattering with minimal scalar and vector interaction that has been used for photon nucleus scattering in the Dirac equation [4]. The DKP equation reduces to the Klein–Gordon equation in the absence of scalar interaction and gives the same results as the Klein–Gordon equation in minimal coupled vector interaction [5,6]. The DKP equation is used to obtain the relativistic dynamics of bosons subject to a variety of nonminimal vector coupling potentials. These potentials are not expressed in the Klein–Gordon equation [7,8].

The DKP equation has been misinterpreted by some authors due to the richness of its interactions. For spin-1 particles, the correct use of nonminimal vectorial interaction in the DKP equation was given by Castro [9]. The DKP equation has an original formulation in the 3+1 space-time dimension. The DKP equation for both spin-0 and spin-1 was reduced to 1+1 space-time dimension. However, the spin-1 version of the DKP equation is unitarily equivalent to its spin-0 version in 1+1 dimension, as was shown by Lunardi [10]. Recently, the DKP equation was utilized to obtain the scattering states of scalar bosons subject to the vector Yukawa potential [11]. The DKP equation with time-dependent interaction was used to study the scalar bosons in (1+1) and (2+1) dimensional space-time [12]. The scattering and bound states of the vector boson in the presence of Aharonov–Bohm flux were studied with the DKP equation by using self-adjoint extension. This showed that these states depend on the spin projection [13]. The massive DKP theory was utilized to reproduce relativistic Bose–Einstein condensation for both spin-0 and spin-1 field at finite temperature [14]. The DKP formalism also has applications in curved space-times [15–18]. The DKP was solved to understand elastic meson-nucleus scattering in the presence of minimal plus nonminimal vector Coulomb potential [19].

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The purpose of this paper is to investigate the behavior of scalar bosons confined by Woods–Saxon potential in the central field by the DKP equation. In recent years, this problem has been solved by using the Nikiforov–Uvarov (NU) method [20,21]. However, in those works, the authors made a mistake due to the method they used. This was because the NU method cannot perform an accurate examination of this problem in physical boundary conditions. In this paper, the correct solutions of the problem are given. In Section 2, the DKP equation is introduced for scalar bosons in the presence of scalar and vector interaction in a central field. In Section 2.1, the DKP equation is solved for a scalar boson confined in Woods–Saxon potential in a central field using Pekeris approximation. The conclusions are given in Section 3.

2. DKP equation in the central field

The DKP Hamiltonian for scalar and vector interactions is [2]:

$$\left(\vec{\beta}\bullet\vec{p}c+mc^2+U_s+\beta^0U_v^0\right)\psi\left(r\right)=\beta^0E\psi \quad , \tag{1}$$

where

$$\psi\left(r\right)=\begin{pmatrix}\psi_{upper}\\ i\psi_{lower}\end{pmatrix} \text{ with } \psi_{upper}=\begin{pmatrix}\varphi\\ \phi\end{pmatrix} \text{ and } \psi_{lower}=\begin{pmatrix}A_1\\ A_2\\ A_3\end{pmatrix} \tag{2}$$

β^0 is the usual 5×5 matrix and U_s and U_v^0 represent the scalar and vector interactions, respectively. The equation is written as:

$$\left(mc^2+U_s\right)\phi=\left(E-U_v^0\right)\varphi+\hbar c\vec{\nabla}\bullet\vec{A},$$

$$\vec{\nabla}\phi=\left(mc^2+U_s\right)\vec{A}, \tag{3}$$

$$\left(mc^2+U_s\right)\varphi=\left(E-U_v^0\right)\phi,$$

where $\vec{A}=\left(A_1,A_2,A_3\right)$. In Eq. (2), $\psi\left(r\right)$ is simultaneously an eigenfunction of J^2 and J_3, given as:

$$J^2\begin{pmatrix}\psi_{upper}\\ \psi_{lower}\end{pmatrix}=\begin{pmatrix}L^2\psi_{upper}\\ \left(L+S\right)^2\psi_{lower}\end{pmatrix}=J\left(J+1\right)\begin{pmatrix}\psi_{upper}\\ \psi_{lower}\end{pmatrix} \quad , \tag{4}$$

$$J_3\begin{pmatrix}\psi_{upper}\\ \psi_{lower}\end{pmatrix}=\begin{pmatrix}L_3\psi_{upper}\\ \left(L_3+s_3\right)^2\psi_{lower}\end{pmatrix}=M\begin{pmatrix}\psi_{upper}\\ \psi_{lower}\end{pmatrix} \quad , \tag{5}$$

where the total angular momentum $J=L+S$, which commutes β^0, is a constant of motion. The general solution of Eq. (1) is:

$$\psi_{JM}\left(r\right)=\begin{pmatrix}f_{nJ}\left(r\right)Y_{JM}\left(\Omega\right)\\ g_{nJ}\left(r\right)Y_{JM}\left(\Omega\right)\\ i\sum_L h_{nJL}\left(r\right)Y_{JL1}^M\left(\Omega\right)\end{pmatrix} \quad , \tag{6}$$

where $Y_{JM}\left(\Omega\right)$ are the spherical harmonics of order J, $Y_{JL1}^M\left(\Omega\right)$ are the normalized vector spherical harmonics, and $f_{nJ}\left(r\right)$, $g_{nJ}\left(r\right)$, and $f_{nJL}\left(r\right)$ are radial wave functions. The equations above give the following coupled differential equations:

$$\left(E-U_v^0\right) f_{nJ}(r)=\left(mc^2+U_s\right) g_{nJ}(r), \tag{7a}$$

$$\hbar c\left(\frac{d}{dr}-\frac{J}{r}\right) f_{nJ}(r)=\frac{-1}{\alpha_J}\left(mc^2+U_s\right) h_{nJJ+1}(r), \tag{7b}$$

$$\hbar c\left(\frac{d}{dr}+\frac{J+1}{r}\right) f_{nJ}(r)=\frac{1}{\zeta_J}\left(mc^2+U_s\right) h_{nJJ-1}(r), \tag{7c}$$

$$-\alpha_J\left(\frac{d}{dr}+\frac{J+2}{r}\right) h_{nJJ+1}(r)+\zeta_J\left(\frac{d}{dr}-\frac{J-1}{r}\right) h_{nJJ-1}(r)$$

$$\frac{1}{\hbar c}\left(\left(mc^2+U_s\right) f_{nJ}(r)-\left(E-U_v^0\right) g_{nJ}(r)\right), \tag{7d}$$

where $\alpha_J=\sqrt{(J+1)/J+2}$, $\varsigma_J=\sqrt{J/J+2}$, $f_{nJ}(r)=F(r)/r$, $g_{nJ}(r)=G(r)/r$, and $h_{nJJ\pm1}(r)=H_{\pm1}(r)/r$. Taking $U_s=0$ and eliminating $G(r)$, $H_1(r)$, and $H_{-1}(r)$ in terms of $F(r)$, the DKP equation in the presence of a time-like component of four-dimensional vector potential is given by [2,20]:

$$\left(\frac{d^2}{dr^2}-\frac{J(J+1)}{r^2}+\frac{\left(E-U_v^0\right)}{(\hbar c)^2}-\frac{m^2c^4}{(\hbar c)^2}\right) F_{nJ}(r)=0. \tag{8}$$

2.1. The solutions of DKP equation with Woods-Saxon potential in the presence of a central field

In this section, we obtain the corrected wave function and energy eigenvalues of scalar bosons confined by Woods–Saxon potential in the presence of a central field by the DKP equation.

Woods–Saxon potential is a short-range potential and is given by:

$$U_v^0(r)=\frac{U_0}{1+e^{(r-R)/a}}, \tag{9}$$

where U_0 is the depth of potential, R is the radius of potential, and a is the width of surface diffuseness [22]. The Woods–Saxon potential and its modifications are used for various applications in different branches of physics [23–25]. In particular, they have been used to understand the nuclear shell model and to describe the interaction of a neutron with the heavy nucleus [26]. Therefore, the analytical solutions of wave equations of nonrelativistic and relativistic particles confined by Woods–Saxon potential have been investigated using various methods [27–31].

Considering the Woods–Saxon potential given by Eq. (9) as the time-like component of the four-dimensional vector potential given in Eq. (8), the radial DKP equation can be written as:

$$\left(\frac{d^2}{dr^2}-\frac{J(J+1)}{r^2}+\frac{1}{(\hbar c)^2}\frac{V_0^2}{\left(1+e^{(r-R)/a}\right)^2}+\frac{(2EV_0)}{(\hbar c)^2\left(1+e^{(r-R)/a}\right)}-\frac{E^2-m^2c^4}{(\hbar c)^2}\right) F_{nJ}(r)=0. \tag{10}$$

There is no exact solution of this equation due to the centrifugal term $\frac{J(J+1)}{r^2}$. To overcome this challenge, Pekeris approximation will be used for the centrifugal term as follows [32]:

$$\frac{J(J+1)}{r^2}\cong\frac{J(J+1)}{R}\left(D_0+\frac{D_1}{\left(1+e^{(r-R)/a}\right)}+\frac{D_2}{\left(1+e^{(r-R)/a}\right)^2}\right), \tag{11}$$

where

$$D_0 = 1 - \frac{4}{\alpha} + \frac{12}{\alpha^2}, \quad D_1 = \frac{8}{\alpha} + \frac{48}{\alpha^2}, \quad D_2 = \frac{48}{\alpha^2} \quad , \tag{12}$$

with $\alpha = R/a$. Substituting Eq. (11) into Eq. (10) and using a new variable, $y(r) = 1/\left[1 + e^{(r-R)/a}\right]$, the DKP equation reduces to:

$$\left(\frac{d^2}{dy^2} + \frac{1-2y}{y(1-y)} \frac{d}{dy} + \frac{-\xi_1^2 y^2 + \xi_2^2 y - \xi_3^2}{[y(1-y)]^2} \right) F_{nJ}(y) = 0 \quad , \tag{13}$$

with

$$\xi_1 = \sqrt{a^2 \left(\frac{J(J+1) D_2}{R^2} - \frac{U_0^2}{(\hbar c)^2} \right)} \quad , \tag{14}$$

$$\xi_2 = -\sqrt{a^2 \left(\frac{J(J+1) D_1}{R^2} + \frac{2U_0 E_{nJ}}{(\hbar c)^2} \right)} \quad , \tag{15}$$

$$\xi_3 = \sqrt{a^2 \left(\frac{J(J+1) D_0}{R^2} - \frac{m^2c^4 - E_{nJ}^2}{(\hbar c)^2} \right)} \quad . \tag{16}$$

To obtain the solution, as done in other studies [20,21], the NU method can be used [33] to solve the second-order differential equation in the following form:

$$\left(\frac{d^2}{ds^2} + \frac{\tilde{\tau}(s)}{\sigma(s)} \frac{d}{ds} + \frac{\tilde{\sigma}(s)}{\sigma^2(s)} \right) \psi(s) = 0 \quad . \tag{17}$$

In this method, the energy eigenvalue equation is given by:

$$k + \pi'(s) = -n\tau'(s) - \frac{n(n-1)}{2} \sigma''(s) \quad , n = 0, 1, 2 \ldots, \tag{18}$$

where the prime denotes the derivative with respect to s, n is a constant, and the k values are obtained by considering that the discriminant of the square root has to be zero in Eq. (18). In Eq. (17), $\pi(s)$ and $\tau(s)$ are respectively defined as:

$$\pi(s) = \frac{\sigma'(s) - \tilde{\tau}(s)}{2} \pm \sqrt{\left(\frac{\sigma'(s) - \tilde{\tau}(s)}{2} \right)^2 - \tilde{\sigma}(s) + k\sigma(s)} \quad , \tag{19}$$

$$\tau(s) = \tilde{\tau}(s) + 2\pi(s) \text{ with } \tau'(s) < 0. \tag{20}$$

The wave function can be separated by $\psi(s) = \phi(s) f(s)$. The asymptotic behavior of $\psi(s)$ is $\phi(s)$, which is obtained from:

$$\frac{\phi'(s)}{\phi(s)} = \frac{\pi(s)}{\sigma(s)} \quad . \tag{21}$$

Comparing Eq. (13) and Eq. (17), the following relationships are obtained:

$$\begin{aligned}\tilde{\sigma}(y) &= -\xi_1^2 y^2 + \xi_2^2 y - \xi_3^2, \\ \tilde{\tau}(y) &= 1 - 2y \quad , \\ \sigma(y) &= y(1-y).\end{aligned} \tag{22}$$

By determining the parameter of Eq.(18), an energy equation is found as follows:

$$n^2 + n - \xi_2^2 + (2\xi_3 + 2n - 1)\left(\sqrt{\xi_3^2 + \xi_1^2 - \xi_2^2} + \xi_3\right) = 0. \tag{23}$$

However, this energy eigenvalue equation is inaccurate due to the application of the wrongly selected method. The NU method does not scrutinize how the wave function behaves at the boundary condition near the origin $(r = 0 \longrightarrow y = 1)$. Therefore, the wave function in the boundary conditions must be carefully scrutinized.

To find the wave function $F_{nJ}(y) = \phi(y) f(y)$ we consider the asymptotic part of the wave function $\phi(y)$ from Eq. (21) as $\phi(y) = y^{\xi_3}(1-y)^{\eta}$. Thus, the wave function is:

$$F_{n,J}(y) = y^{\xi_3}(1-y)^{\eta} f(y) \quad , \tag{24}$$

with $\eta = \sqrt{\xi_3^2 + \xi_1^2 - \xi_2^2}$. This wave function satisfies the boundary condition in $F_{n,J}(r \to \infty, y \to 0) = 0$ and $F_{n,J}(r \to 0, y \to 1) = 0$. By replacing Eq. (24) in Eq. (13), we obtain:

$$\begin{aligned}&y(1-y) f_{nJ}^{''}(y) + (1 + 2\xi_3 + 2(1 + \xi_3 + \eta)) f_{nJ}^{'}(y) \\ &- ((\xi_3 + \eta)(1 + 2\xi_3) - \xi_2^2) f_{nJ}(y) = 0 \quad .\end{aligned} \tag{25}$$

This equation is known as the hypergeometric equation [34]:

$$y(1-y)\omega^{''}(y) - [c - (a + b + 1) y]\omega^{'}(y) - ab\omega(y) = 0 \quad , \tag{26}$$

and its solution is $\omega(y) = {}_2F_1(a, b, c, y)$. Comparing Eq. (25) with Eq. (26), the parameters a, b, and c are found as:

$$\begin{aligned}a &= \frac{1}{2}\left(1 + 2\xi_3 + 2\eta \mp \sqrt{1 - 4\xi_1^2}\right), \\ b &= \frac{1}{2}\left(1 + 2\xi_3 + 2\eta \pm \sqrt{1 - 4\xi_1^2}\right), \\ c &= 1 + 2\xi_3.\end{aligned} \tag{27}$$

Therefore, the solution of Eq. (26) is obtained as:

$$F_{n,J}(y) = y^{\xi_3}(1-y)^{\eta}\, {}_2F_1(a, b, c; y). \tag{28}$$

Now the correct energy eigenvalues can be calculated. For this, the wave function near the origin $(r \longrightarrow y \longrightarrow 1)$ is investigated. The following identity for the hypergeometric function can be used:

$${}_2F_1(a, b, c, y) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\, {}_2F_1(a, b, a + b - c + 1, 1 - y)$$

$$+\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}(1-y)^{c-a-b}\,{}_2F_1(c-a,c-b,c-a+b+1,1-y) \quad . \tag{29}$$

It is known that ${}_2F_1(a,b,c,0)=1$. Therefore, using Eq. (29) and boundary condition $F_{nJ}(r\longrightarrow 0, y\longrightarrow 1)=0$ and an approximation $1+e^{R/a}\approx e^{R/a}$ leads to:

$$\frac{\Gamma(a+b-c)}{\Gamma(c-a-b)}\frac{\Gamma(c-b)}{\Gamma(b)}\frac{\Gamma(c-b)}{\Gamma(a)}e^{2\delta R/a}=-1 \quad , \tag{30}$$

where $\delta=i\lambda$ and $\gamma=\sqrt{\xi_2^2-\xi_1^2-\xi_3^2}$. Thus, from the quantum condition and using $e^{-2iarg\Gamma(y)}=\frac{\Gamma(y)}{\Gamma(y)}$, the corrected energy eigenvalue equation is obtained as follows:

$$arg\Gamma(2i\lambda)-arg\Gamma\left(\frac{1}{2}\left(1+2\xi_3+2i\lambda-\sqrt{1-4\xi_1^2}\right)\right)-arg\Gamma\left(\frac{1}{2}\left(1+2\xi_3+2i\lambda-\sqrt{1-4\xi_1^2}\right)\right)+\frac{R\lambda}{a}$$

$$=(n+1/2)\pi, \quad n=0,\pm1,\pm2,\ldots \tag{31}$$

To numerically calculate the corrected energy eigenvalues from Eq. (31) the parameters are set as $m_{K^-c^2=493.677}$ MeV, $U_0=-67.54$ MeV, $R=7.6136$ fm, $a=0.65$ fm^{-1}, and $\hbar c=197.329$ MeV [20,29,35]. These parameters belong to the ^{208}Pb nucleus. The numerical results are listed in the Table, which shows that the single-particle energy levels of the kaon confined by the Woods–Saxon potential in the ^{208}Pb nucleus decrease as quantum numbers n and J increase.

Table 1. Energy levels E_{nJ} (MeV) of K^- (kaon) particles confined by the Woods–Saxon potential for (n , J) states.

n	J	E_{nJ}
0	0	–433.03300525732527
	1	–434.06718583350555
	2	–436.12898522457374
1	0	–450.77925900152399
	1	–451.80308001111999
	2	–453.84424689451373
2	0	–474.34735195773544
	1	–475.35419286222988
	2	–477.36188112729855
3	0	–494.07778168241688

The radial wave function given by Eq. (28) in terms of parameters in Eq. (27) is obtained as:

$$F_{n,J}=N\left(\frac{1}{1+e^{(r-R)/a}}\right)^{\xi_3}\left(1-\frac{1}{1+e^{(r-R)/a}}\right)^{\eta}{}_2F_1\left(\frac{1}{2}\left(1+2\xi_3+2i\lambda-\sqrt{1-4\xi_1^2}\right);\right.$$

$$\left.\frac{1}{2}\left(1+2\xi_3+2i\lambda-\sqrt{1-4\xi_1^2}\right);1+2\xi_3;\frac{1}{1+e^{(r-R)/a}}\right) \quad , \tag{32}$$

where N is obtained from the normalization condition. The Figure shows the unnormalized wave function as a function of r for several n quantum numbers.

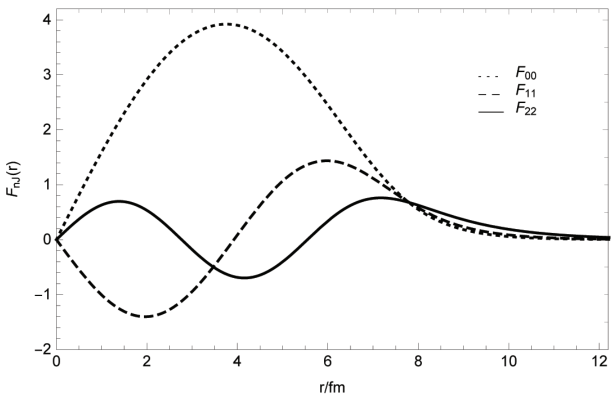

Figure 1. The unnormalized wave function for different n.

3. Conclusion

The DKP equation was solved to describe the dynamics of a scalar boson confined in the Woods–Saxon potential of the nucleus by using Pekeris approximation. It has been realized that the NU method cannot take into account the behavior of a boson's wave function confined by Woods–Saxon potential in boundary conditions. Therefore, the wave function obtained in terms of a hypergeometric function has been carefully examined. The corrected energy eigenvalue equation and corresponding eigenfunction have been obtained. Some numerical results were given in the Table. The results obtained in this paper might be useful in particle and nuclear physics.

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