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Generating velocity-dependent potential in all partial waves

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Abstract: Velocity/energy-dependent potential to a parent nonlocal interaction is constructed for all partial waves by Taylor series expansion method and the related s and p-wave phase shifts for N-N and α-N systems are computed by application of modified phase equation. Our phase shifts are in good agreement with standard data.

Key words: Separable nonlocal potential, Taylor series expansion, velocity-dependent local potential, phase equation, N-N and α-N systems

1. Introduction

Several attempts were made to construct phase equivalent potentials [1–9] from knowledge of the scattering phase shifts. Also, the formalism of supersymmetric (SUSY) quantum mechanics has been employed by a number of workers [10–14] to produce phase equivalent potentials. At high energies the validity of the potential theory is itself questionable, because the multiparticle production and relativistic effects cannot be accounted for in a static potential approach. In general, the short-range part of the interaction is originated because of multiple pion exchange where the recoil of the nucleons cannot be ignored. Under the situation N-N interaction cannot be defined by one radial variable s, i.e. the relative separation of 2 interacting particles. It also depends on the immediate vicinity of the 2 interacting nucleons to account for the effects of the recoil. Thus, the N-N interaction should be represented by V(s,s$'$) which depends on 2 variables s and s$'$. Thus the local potential V(s) δ(s- s$'$) is a limiting form of a general nonlocal potential.

The use of separable nonlocal interactions for different angular momentum states is well established in nucleon-nucleon and nucleon-nucleus scattering. Several methods [6–9] for generating phase equivalent local potentials to nonlocal separable interactions exist in the literature. These methods make a comparative study between the phase shifts of parent nonlocal potentials and the energy-dependent equivalent local potentials. Talukdar et al. [9] constructed a phase equivalent potential to Yamaguchi [15] one by Taylor series expansion of the nonlocal wave function involved in the integro-differential equation. The present work is an extension of this work for all partial waves to study the nucleon-nucleus system.

In section 2, we develop our method for constructing phase equivalent velocity-dependent local potential to a nonlocal one. Results and discussions are presented in section 3. Finally, conclusion is given in section 4.

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2. Phase equivalent local potential

At a positive energy $E = k^2$, the Schrödinger equation for a nonlocal potential $V_\ell(s, s')$ in the framework of all partial wave analysis is given by

$$\left[\frac{d^2}{ds^2} + k^2 - \frac{\ell(\ell+1)}{s^2}\right] \phi_\ell(k, s) = \int_0^\infty V_\ell(s, s')\, \phi_\ell(k, s')\, ds'. \tag{1}$$

Expanding the radial wave function $\phi_\ell(k, s')$ about s using Taylor's series one can able to obtain

$$\phi_\ell(k, s') = \sum_{\lambda+0}^{\infty} \frac{(s'-s)^\lambda}{\lambda!} \frac{d^\lambda}{ds^\lambda} \phi_\ell(k, s)\,. \tag{2}$$

Substitution of Eq. (2) in Eq. (1) leads to

$$\left[\frac{d^2}{ds^2} + k^2 - \frac{\ell(\ell+1)}{s^2}\right] \phi_\ell(k, s) = \sum_{\lambda=0}^{\infty} \frac{d^\lambda}{ds^\lambda}\, \phi_\ell(k, , s) \int_0^\infty \frac{(s'-s)^\lambda}{\lambda!} V_\ell(s, s')\, ds'. \tag{3}$$

Considering the terms up to $\lambda = 2$ one obtains

$$\left[\frac{d^2}{ds^2} + k^2 - \frac{\ell(\ell+1)}{s^2}\right] \phi_\ell(k, s) = \phi_\ell(k, s)\, T_\ell^{(0)}(s) + \phi'_\ell(k, s)\, T_\ell^{(1)}(s) + \phi''_\ell(k, s)\, T_\ell^{(2)}(s) \tag{4}$$

with

$$T_\ell^{(0)}(s) = \int_0^\infty V_\ell(s, s')\, ds'\,, \tag{5}$$

$$T_\ell^{(1)}(s) = \int_0^\infty (s'-s)\, V_\ell(s, s')\, ds' \tag{6}$$

and

$$T_\ell^{(2)}(s) = \int_0^\infty \frac{(s'-s)^2}{2!} V_\ell(s, s')\, ds'. \tag{7}$$

With the help of some mathematical rearrangement Eq. (4) yields

$$\left[\frac{d^2}{ds^2} + k^2 - \frac{\ell(\ell+1)}{s^2}\right] \phi_\ell(k, s) = V_\ell^a(k, s)\, \phi_\ell(k, s) + V_\ell^b(k, s)\, \phi'_\ell(k, s). \tag{8}$$

The quantities $V_\ell^a(k, s)$ and $V_\ell^b(k, s)$ used in the above expression are given by

$$V_\ell^a(k, s) = \frac{T_\ell^{(0)}(s) - k^2 T_\ell^{(2)}(s) + \frac{\ell(\ell+1)}{s^2} T_\ell^{(2)}(s)}{1 - T_\ell^{(2)}(s)} \tag{9}$$

and

$$V_\ell^b(k,s) \;=\; \frac{T_\ell^{(1)}(s)}{1-\,T_\ell^{(2)}(s)} \quad . \tag{10}$$

Rank-one separable potentials with its simplest mathematical form are used in different areas of physics. The rank one separable potential considered here reads as

$$V_\ell(s,s') \;=\; \lambda_\ell\, v_\ell(s)\, v_\ell(s') \tag{11}$$

where

$$v_\ell(s) \;=\; 2^{-\ell}\,(\ell!)^{-1} s^\ell\, e^{-\beta_\ell s}. \tag{12}$$

Substituting Eqs. (11) and (12) in Eqs. (5), (6), and (7) lead to

$$T_\ell^{(0)}(s) \;=\left[2^{-\ell}\,(\ell!)^{-1}\right]^2 s^\ell\, e^{-\beta_\ell s}\,\lambda_\ell \int_0^\infty s'^\ell\, e^{-\beta_\ell s'}\, ds', \tag{13}$$

$$T_\ell^{(1)}(s) \;=\; \left[2^{-\ell}\,(\ell!)^{-1}\right]^2\; s^\ell\, e^{-\beta_\ell s}\;\lambda_\ell \int_0^\infty (s'-s)\; s'^\ell\, e^{-\beta_\ell s'} ds', \tag{14}$$

and

$$T_\ell^{(2)}(s) \;=\; \left[2^{-\ell}\,(\ell!)^{-1}\right]^2\; s^\ell\, e^{-\beta_\ell s}\,\frac{\lambda_\ell}{2!} \int_0^\infty (s'-s)^2\; s'^\ell\, e^{-\beta_\ell s'} ds'. \tag{15}$$

For the partial waves $\ell = 0\,\&\,1$ Eqs. (13), (14) and (15) are given by

$$T_0^{(0)}(s) \;=\; \frac{\lambda_0}{\beta_0} e^{-\beta_0 s}, \tag{16}$$

$$T_1^{(0)}(s) \;=\; \frac{\lambda_1 s}{4\beta_1^2} e^{-\beta_1\, s}, \tag{17}$$

$$T_0^{(1)}(s) \;=\; \frac{\lambda_0(1-s\beta_0)}{\beta_0^2} e^{-\beta_0 s}, \tag{18}$$

$$T_1^{(1)}(s) \;=\; \frac{\lambda_1 s\,(2-s\beta_1)}{4\beta_1^3} e^{-\beta_1 s}, \tag{19}$$

$$T_0^{(2)}(s) \;=\; \frac{\lambda_0 e^{-\beta_0 s}}{2\beta_0^3}\left(2+s^2\beta_0^2-2s\beta_0\right) \tag{20}$$

and

$$T_1^{(2)}(s) \;=\; \frac{\lambda_1\, s\;\; e^{-\beta_1 s}}{8\beta_1^4}\left(6+s^2\beta_1^2-4s\beta_1\right). \tag{21}$$

Using Eqs. (16) –(21) the Eqs. (9) and (10) can be written for the partial waves $\ell = 0\,\&\,1$ as

$$V_0^a(k,s) = \lambda_0 \frac{2\beta_0^2 - k^2\left(2+s^2\beta_0^2-2s\beta_0\right)}{2\beta_0^3-\lambda_0 e^{-\beta_0 s}\left(2+s^2\beta_0^2-2s\beta_0\right)} e^{-\beta_0 s}, \tag{22}$$

$$V_0^b(k,s) = \frac{2\lambda_0\,\beta_0(1-s\beta_0)\,e^{-\beta_0 s}}{2\beta_0^3-\lambda_0 e^{-\beta_0 s}\left(2+s^2\beta_0^2-2s\beta_0\right)}, \tag{23}$$

$$V_1^a(k,s) = \lambda_1 s \frac{2\beta_1^2 - \left(k^2-2/s^2\right)\left(6+s^2\beta_1^2-4s\beta_1\right)}{8\beta_1^4-\lambda_1 s e^{-\beta_1 s}\left(6+s^2\beta_1^2-4s\beta_1\right)} e^{-\beta_1 s} \tag{24}$$

and

$$V_1^b(k,s) = \frac{2\lambda_1\,\beta_1 s\;\;(2-s\beta_1)\;e^{-\beta_1 s}}{8\beta_1^4-\lambda_1 s e^{-\beta_1 s}\left(6+s^2\beta_1^2-4s\beta_1\right)}. \tag{25}$$

The regular solution $u_\ell(k,s)$of Eq. (8) can be written as

$$\phi_\ell(k,s) = \alpha_\ell(k,s)\;\sin\left(ks+\frac{\ell\pi}{2}+\delta_\ell(k,s)\right). \tag{26}$$

The quantities $\delta_\ell(k,s)$ and $\alpha_\ell(k,s)$ represent the phase and amplitude functions, respectively.

The regular Green function appropriate to Eq. (8) is [16]

$$G_\ell^{(R)}(s,s') = \begin{cases} \frac{-\left(\hat{j}_\ell(ks)\hat{\eta}_\ell(ks')-\hat{\eta}_\ell(ks)\hat{j}_\ell(ks')\right)}{k} & for \quad s'<s \\ 0 & for \quad s'>s \end{cases}. \tag{27}$$

In view of Eq. (27) the general solution of Eq. (8) reads as

$$\phi_\ell(k,s) = \alpha_\ell(k,s)\;\left[\hat{j}_\ell(ks)\,\cos\delta_\ell(k,s)\;\;-\hat{\eta}_\ell(ks)\,\sin\delta_\ell(k,s)\right] \tag{28}$$

with

$$\begin{aligned}\alpha_\ell(k,s)\;\cos\delta_\ell(k,s) = 1 - k^{-1}\left[\int_0^s \hat{\eta}_\ell(ks')\,V_\ell^a(k,s')\,\phi_\ell(k,s')ds'\right.\\ \left.+\int_0^s \hat{\eta}_\ell(ks')\,V_\ell^b(k,s')\,\phi_\ell'(k,s')ds'\right]\end{aligned} \tag{29}$$

and

$$\alpha_\ell(k,s)\;\sin\delta_\ell(k,s) = -k^{-1}\left[\int_0^s \hat{j}_\ell(ks')\,V_\ell^a(k,s')\,\phi_\ell(k,s')ds' \;\;+\int_0^s \hat{j}_\ell(ks')\,V_\ell^b(k,s')\,\phi_\ell'(k,s')ds'\right]. \tag{30}$$

Differentiating Eqs. (29) and (30) with respected to s one obtains

$$\begin{aligned}\alpha_\ell'(k,s)\;\cos\delta_\ell(k,s)\;-\;\alpha_\ell(k,s)\,\delta_\ell'(k,s)\;\sin\delta_\ell(k,s)\\ = -k^{-1}\hat{\eta}_\ell(ks)\;\left[V_\ell^a(k,s)\,\phi_\ell(k,s)\;+\;V_\ell^b(k,s)\,\phi_\ell'(k,s)\right]\end{aligned} \tag{31}$$

and

$$\begin{aligned}\alpha_\ell'(k,s)\;\sin\delta_\ell(k,s)\;+\alpha_\ell(k,s)\,\delta_\ell'(k,s)\;\cos\delta_\ell(k,s)\\ = -k^{-1}\hat{j}_\ell(ks)\;\left[V_\ell^a(k,s)\,\phi_\ell(k,s)\;+\;V_\ell^b(k,s)\,\phi_\ell'(k,s)\right].\end{aligned} \tag{32}$$

Multiplying Eq. (31) by $\cos\delta_\ell(k,s)$ and Eq. (32) by $\sin\delta_\ell(k,s)$ and adding them we get

$$\alpha'_\ell(k,s) \;=\; -k^{-1}\left[\,V^a_\ell(k,s)\,\phi_\ell(k,s) \;+\; V^b_\ell(k,s)\,\phi'_\ell(k,s)\right]\;\left[\hat{\eta}_\ell(ks)\cos\delta_\ell(k,s) \;+\hat{j}_\ell(ks)\sin\delta_\ell(k,s)\right]. \tag{33}$$

Similarly, Eq. (32) multiplied by $\cos\delta_\ell(k,s)$ when subtracted from Eq. (31) multiplied by $\sin\delta_\ell(k,s)$ results in

$$\begin{aligned}\alpha_\ell(k,s)\;\delta'_\ell(k,s) = \; -k^{-1}\left[V^a_\ell(k,s)\,\phi_\ell(k,s) + \;V^b_\ell(k,s)\,\phi'_\ell(k,s)\right]\\ \times\;\left[\hat{\eta}_\ell(ks)\sin\delta_\ell(k,s) \;-\hat{j}_\ell(ks)\cos\delta_\ell(k,s)\right].\end{aligned} \tag{34}$$

Differentiating Eq. (28) with respect to s and use of Eqs. (33) and (34) in the result along with some mathematical simplification leads to

$$\phi'_\ell(k,s) \;=\; \alpha_\ell(k,s)\;\left[\hat{j}'_\ell(ks)\;\cos\delta_\ell(k,s) \;-\;\hat{\eta}'_\ell(ks)\;\sin\delta_\ell(k,s)\right]. \tag{35}$$

Substitution of Eqs. (28) and (35) in Eq. (34) yields

$$\begin{aligned}\delta'_\ell(k,s) = \; k^{-1}\left[V^b_\ell(k,s)\;\left\{\hat{j}'_\ell(ks)\;\cos\delta_\ell(k,s) \;-\hat{\eta}'_\ell(ks)\;\sin\delta_\ell(k,s)\right\} - \;V^a_\ell(k,s)\right]\\ \times\;\left[\hat{\eta}_\ell(ks)\sin\delta_\ell(k,s) \;-\hat{j}_\ell(ks)\cos\delta_\ell(k,s)\right].\end{aligned} \tag{36}$$

To calculate the values of quantum mechanical scattering phase shifts for different nuclear systems, one can easily rely on an efficient as well as a simplest method popularly known as the Phase function method (PFM) [17–20]. In this method the wave function of the radial Schrödinger equation is separated into 2 parts viz. the amplitude part and phase part. For a local potential the phase function obeys a nonlinear differential equation. However, the situation differs in case of nonlocal potential. The nonlocal potential couples the wave function at one point with its value at all neighbouring points. Thereby the accumulation of the phase will depend on the values of the wave function for all values of s. For common nonlocal potential the phase equation possesses a complicated structure. However, it becomes somehow easier when one deals with the separable nonlocal potential.

3. Results and discussions

Using Eq. (36) along with the parameters [7,21,22] given in Table 1 the scattering phase shifts for different systems are computed for different values of laboratory energy and are depicted in Figures 1–5.

In general, the $^3\mathrm{s}_1$-state scattering phase shifts for n-p system, portrayed in figure 1, reproduce the data which are comparable with the standard results [23,24] with both sets of parameters under consideration up to 100 MeV. However, the scattering phase shifts for pure nonlocal interaction with Arnold-MacKeller (AM) [7] parameters slightly differ from the standard data both in intermediate and high energy range. While those with Laha-Bhoi (LB) [21] parameters are in exact agreement with refs [23] up to 150 Mev and beyond that they discern gradually from standard data [23,24]. On the other hand, the $^3\mathrm{s}_1$ scattering phase shifts for equivalent local potential with AM and LB [7, 21] parameters change their sign at laboratory energies 230 MeV and 347 MeV, respectively whereas the same change their signs at laboratory energies 375 MeV [23] and 350 MeV [24], respectively. The scattering phase shifts for this state computed with LB [21] parameters are in exact agreement with standard result up to laboratory energy of 400 MeV.

Table . List of potential parameters.

System	State	$\lambda\,(fm^{-2\ell-3})$	$\beta\ (fm^{-1})$
Nucleon-nucleon	$^3s_1$(n-p)	–7.533 (AM)	1.4054(AM)
	$^1s_0$(n-p/p-p)	–3.901(LB)	1.1 (LB)
	$^3p_0$(n-p/p-p)	–2.405	1.1 (VH)
	$^3p_2$(n-p/p-p)	–20.5	1.45
		–500.0	2.625
$\alpha - n$	$1/2^+$	–9.995	1.2
	$1/2^-$	–25.28	1.2
	$3/2^-$	–36.50	1.2
$\alpha - p$	$1/2^+$	–21.56	1.3
	$1/2^-$	–37.28	1.3
	$3/2^-$	–76.20	1.4

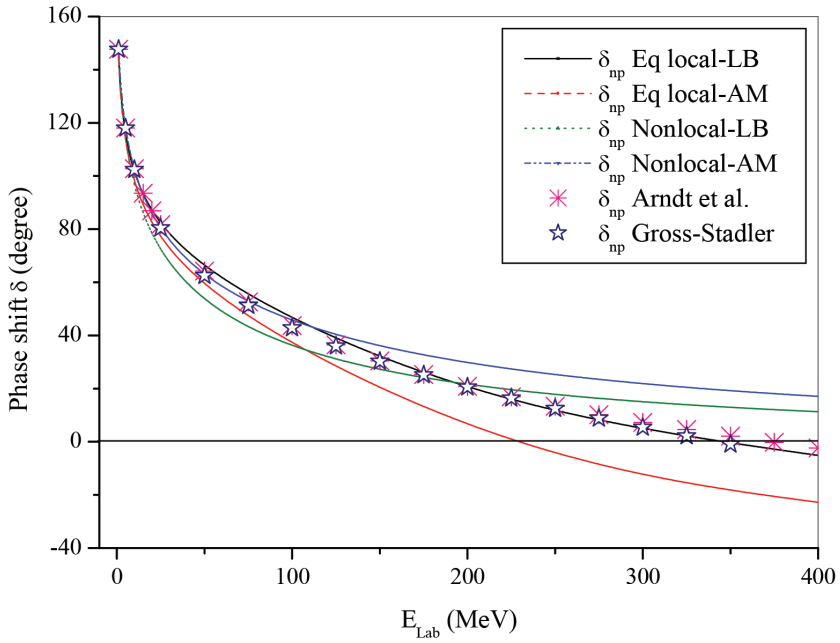

Figure 1. Phase shifts ($^3s_1$-State) for n-p system as a function of E_{Lab}.

Looking closely into Figure 2 it is observed that the $^1s_0$ scattering phase shifts for both n-p and p-p systems with equivalent local interaction, computed using the parameters given in Table 1, are in close agreement with the standard data [23,24]. The phase shifts for $^1s_0$ state for n-p and p-p systems as computed by us attain maxima of 61.34^0 and 50.09^0 at laboratory energies of 5 MeV and 10 MeV, respectively. Our peak values for the phase shifts fall below the standard results [23] by approximately 3^0. The phase shifts values for n-p and p-p systems for this state change their signs at E_{Lab} = 239 MeV and E_{Lab} = 229 MeV, respectively which match exactly with the standard data [23,24] whereas for pure nonlocal interaction, both n-p and p-p phases are comparable to refs. [23,24] up to 125 MeV and then diverge from standard results. This is obvious because a one term nonlocal potential cannot account for the change in sign of the phase shifts.

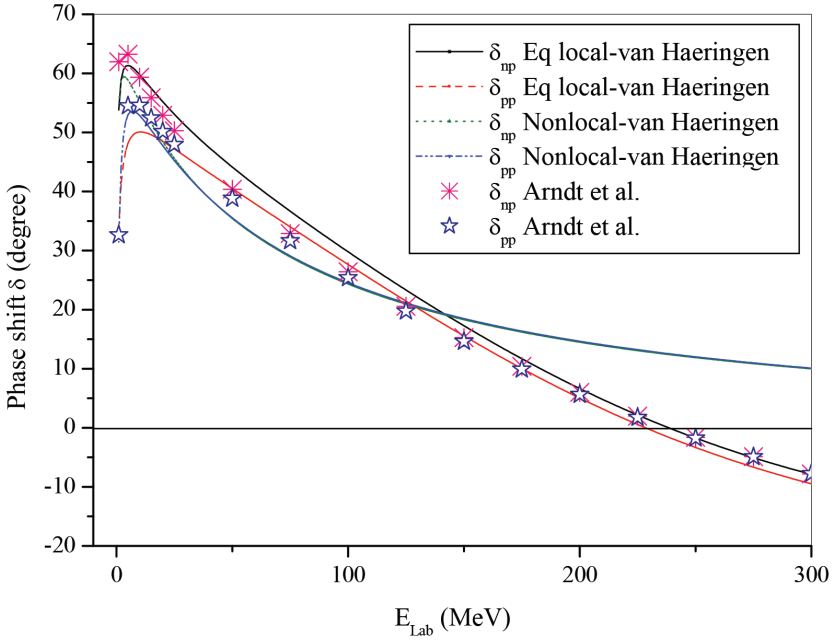

Figure 2. Phase shifts (^{1}s$_0$-State) for n-p and p-p systems as a function of E$_{Lab}$.

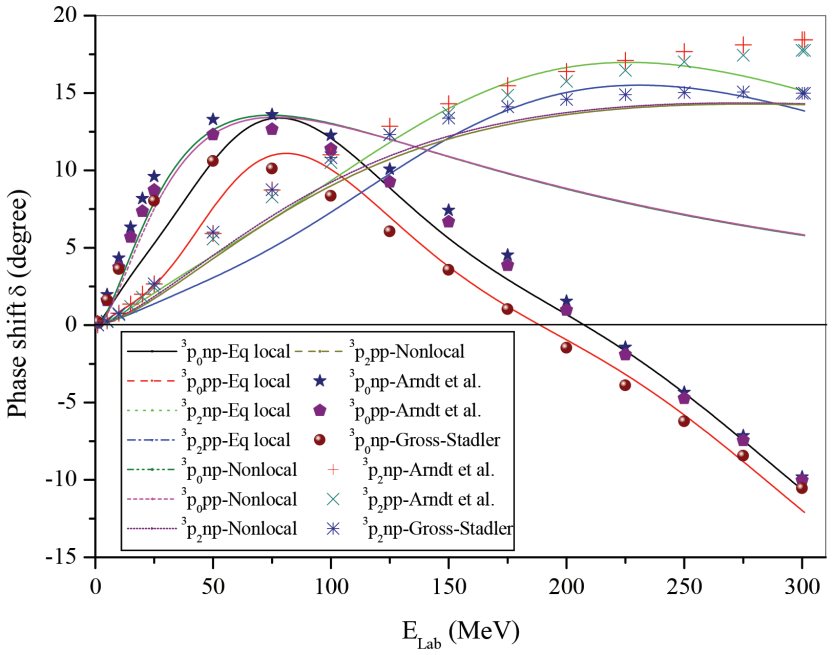

Figure 3. Phase shifts (^{3}p$_0$ & ^{3}p$_2$ States) for n-p and p-p systems as a function of E$_{Lab}$.

The scattering phase shifts for ^{3}p$_0$ and ^{3}p$_2$ states for both n-p and p-p systems as a function of laboratory energy are shown in the Figure 3. For ^{3}p$_0$ state of n-p and p-p systems the phase shifts increase with energy, attain a maximum then gradually decrease and change their signs. Our results for n-p system with equivalent

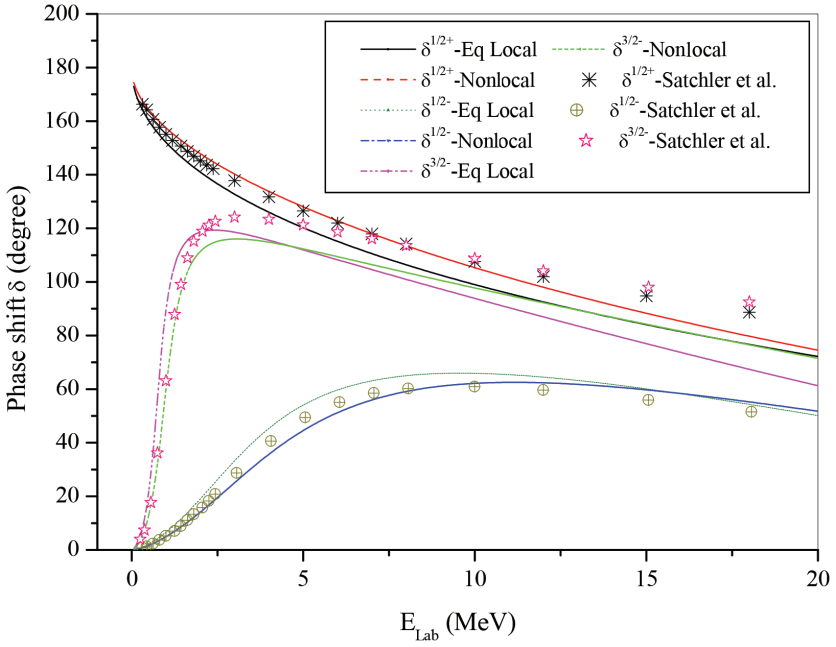

Figure 4. Phase shifts ($1/2^+$, $1/2^-$ & $3/2^-$ States) for $\alpha - n$ system as a function of E_{Lab}.

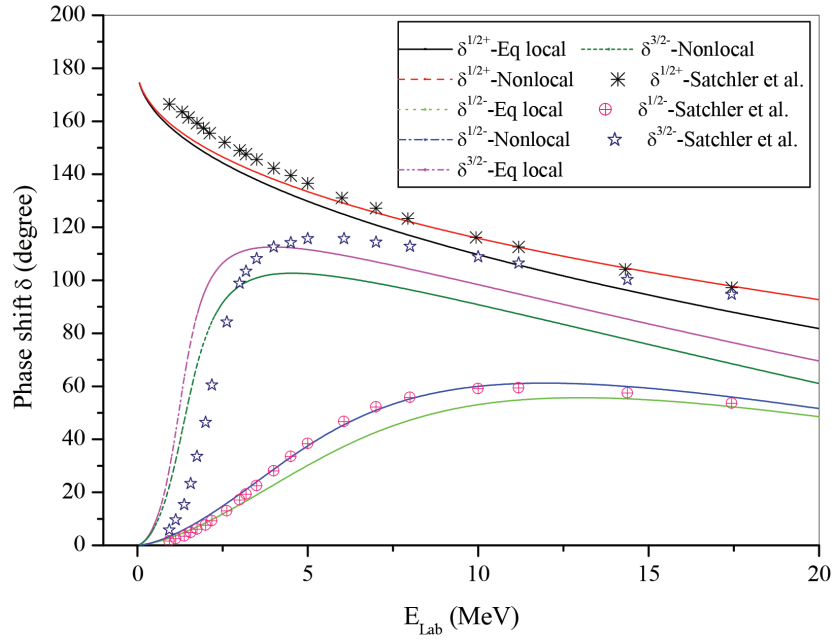

Figure 5. Phase shifts ($1/2^+$, $1/2^-$ & $3/2^-$ States) for $\alpha - p$ system as a function of E_{Lab}.

local potential are in good agreement with Arndt et al. [23] but slightly higher than Gross and Stadler [24]. For p-p system the results for phase shifts are lower than ref. [24] but retrace the correct trend. Our n-p phase shifts for $^3p_0$ states change their sign at proper place while p-p phases differ by few MeV. Both n-p and p-p phases for nonlocal potential are well matched with standard results up to 100 MeV and beyond that give poor

fitting with refs. [23,24] as the potential has one term of attractive nature. For $^3\mathrm{p}_2$ state our data are in better agreement than $^3\mathrm{p}_0$ state with Arndt et al. [23] and Gross-Stadler [24]. However, our equivalent potential data are superior to its nonlocal counterpart.

To explore this method to many nucleon systems we extend our work for $\alpha - nucleon$ systems. For $\alpha - n$ and $\alpha - p$systems we have calculated the phase shifts for the partial wave states $\ell = 0$ & 1 and match our results with those of Satchler et al. [25]. These results shown in figure 4 and figure 5 match quite well with the ref [25]. For nonlocal separable potential the scattering phase shifts for $½^-$ state are in close agreement with Ahmed et al. [26]. It is noticed that results for equivalent local potentials are in better agreement with Satchler et al. [25] than pure nonlocal interaction.

4. Conclusion

We have extended the method of Talukdar et al. [9] to all partial waves and obtained good agreement in phase shift values for nucleon-nucleon and alpha-nucleon systems. Also, we have developed the phase equation of MacKellar and May [17] for all partial waves. For $\ell > 1$, however, we have verified that this method does not work satisfactorily. This may arise due to the strong centrifugal repulsion in effective interaction. As our parent nonlocal potential is a one term attractive type potential it has no ability to reproduce phase shifts for those states which give negative phase values. To that end we have considered those states which give positive phase shifts. A one term attractive nonlocal potential obviously has no ability to produce sign change in phase shift values for s-wave and thus the $^1\mathrm{s}_0$ and $^3\mathrm{s}_1$ n-p/p-p phase shifts have positive values. However, the equivalent local potential for the same nonlocal one contains 2 terms, 1 energy-dependent and the other 1 is independent. Thus, this equivalent interaction has the quality to develop correct nature and values of s-wave n-p/p-p phases. Our method can easily be extended for nucleus-nucleus systems and for the nonlocal potential of higher rank. It is concluded by noting that our energy-dependent local potentials reproduce better results than its parent nonlocal interactions.

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