

## Pseudoscalar form factors of the $\Delta(1232)$ in Baryon Chiral Perturbation Theory

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**Abstract:** The pseudoscalar current-delta transition form factors are calculated in the framework of relativistic baryon chiral perturbation theory including order  $\mathcal{O}(q^4)$ . The complex-mass scheme, which is suitable and well-defined for unstable particles, is taken into account as a renormalization scheme. We discuss the low energy behavior of the pseudoscalar  $\tilde{g}(Q^2)$  and  $\tilde{h}(Q^2)$  form factors and note that the momentum dependence of the form factors is dominated by the pion pole contributions.

**Key words:** Delta resonance, baryon chiral perturbation theory, complex-mass scheme

### 1. Introduction

The  $\Delta(1232)$  resonance is the most distinguished excitation state of the nucleon which has been worked extensively both in experimental and the theoretical aspect. Its quantum numbers are  $I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$  and the mass is very close to the nucleon bringing along a strong coupling to the  $\pi N$  channel. It has been shown that  $\Delta(1232)$  can be included as an explicit degree of freedom into the baryon chiral perturbation theory (BChPT) in both the resonance domain and  $\pi N$  threshold region (see, e.g., Refs. [1–4]). Chiral perturbation theory (ChPT) is an effective theory of quantum chromodynamics (QCD) at low energies which analyzes the interaction of pions with nucleons and external fields [5–7]. For the purpose to get finite and physical results, one applies dimensional regularization (DR) in combination with a modified minimal subtraction scheme ( $\tilde{M}\overline{S}$ ). As a result, divergent parts are removed and a consistent power counting (PC) is obtained. While in the mesonic sector PC works very well thanks to the small mass of the pions, for the baryonic sector things turn out to be more complicated since the nucleon mass does not vanish in the chiral limit [8]. The solution is to choose a suitable renormalization scheme. Extended on mass shell scheme (EOMS) is one of them that solves the problem including a finite subtraction of PC violating terms in addition to the DR+ $\tilde{M}\overline{S}$  in the relativistic BChPT framework [9, 10]. Thus, only those terms that violate the PC are subtracted and a consistent theory is achieved at the end. We calculate the  $\Delta(1232)$  transition form factors via pseudoscalar current in the framework of relativistic BChPT with the complex-mass scheme (CMS) which is the precise formalism inferred from the EOMS to unstable particles. It has been proved that the CMS is an effective method used in the Standard model at first and applied in diverse calculations within the effective theory of strong interactions [11–15].

This is the first time the pseudoscalar form factors are calculated in BChPT treating the  $\Delta(1232)$  as an unstable particle. Analyzing the  $\Delta(1232)$  baryon experimentally is a challenge because of the very short

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lifetime of it,  $10^{-23}$  s. For that reason, there is a lack of measurements of the pseudoscalar form factors of the  $\Delta(1232)$  in particular. Pseudoscalar current to  $\Delta(1232)$  transition form factors have been investigated recently in the framework of Lattice QCD [17] and light-cone QCD sum rules [16]. We confront our results with those obtained in these different approaches. Lattice QCD and QCD Sum rules are some of the methods to understand the low-energy behavior of QCD. While ChPT and Sum rules are the analytic approaches to work with QCD, Lattice QCD is a numerical approach to solve QCD Lagrangian. ChPT is based on the spontaneously and explicitly broken chiral symmetry which plays a fundamental role at those low energies. We work with a chiral Lagrangian with explicit  $\Delta(1232)$  baryon and physical pion mass at the energies below 1 GeV, whereas Ref. [17] is effective in a higher energy regime using relatively large pion mass ( $\geq 297$  MeV) compared to our method. As concerns the light-cone QCD sum rule approach used in Ref. [16], the relevant degrees of freedom are quarks and gluons as in the Lattice method instead of baryons and mesons in ChPT. Further, it works in a higher momentum transfer region around  $10 \text{ GeV}^2$ . Thus, the method used here gives complementary information for the form factors of the relevant transition which is presented.

In the current work, the pseudoscalar form factors of the  $\Delta(1232)$  are presented in the relativistic BChPT at  $\mathcal{O}(q^4)$  using CMS. In Sec. II, the effective Lagrangian and the numerical values of the low energy constants (LECs) are introduced. A brief overview of the power counting and renormalization scheme considered in this work is given in Sec. III. Sec. IV gives the pseudoscalar current transition matrix element of the  $\Delta(1232)$  and calculation of the pseudoscalar form factors. In Sec. V the results as a function of the  $Q^2$  are outlined followed by a discussion of the findings. Sec. VI contains a summary.

## 2. Effective Lagrangian

The relevant most general effective Lagrangian for the calculation of the transition form factors up to and including order  $\mathcal{O}(q^4)$  reads

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N \Delta}^{(1)} + \mathcal{L}_{\pi N \Delta}^{(2)} + \mathcal{L}_{\pi \Delta}^{(1)} + \dots, \quad (1)$$

where the ellipsis denotes terms of higher-order than four. The locally invariant pure mesonic Lagrangians at chiral order  $\mathcal{O}(q^2)$  and  $\mathcal{O}(q^4)$  are given by [7, 8] as

$$\begin{aligned} \mathcal{L}_2 &= \frac{F^2}{4} (\text{Tr}[D_\mu U (D^\mu U)^\dagger] + \text{Tr}[\chi U^\dagger + U \chi^\dagger]), \\ \mathcal{L}_4 &= \frac{l_3 + l_4}{16} (\text{Tr}[\chi U^\dagger + U \chi^\dagger])^2 + \frac{l_4}{8} \text{Tr}[D_\mu U (D^\mu U)^\dagger] \text{Tr}[\chi U^\dagger + U \chi^\dagger] \\ &\quad + \frac{l_7}{16} (\text{Tr}[\chi U^\dagger - U \chi^\dagger])^2 + \frac{h_1 + h_3 - l_4}{4} \text{Tr}[\chi \chi^\dagger] \\ &\quad + \frac{h_1 - h_3 - l_4}{16} \{ (\text{Tr}[\chi U^\dagger + U \chi^\dagger])^2 + \text{Tr}[\chi U^\dagger - U \chi^\dagger]^2 - 2(\text{Tr}[\chi U^\dagger \chi U^\dagger + U \chi^\dagger U \chi^\dagger]) \} + \dots, \end{aligned} \quad (2)$$

respectively with  $\chi = 2B(s + ip)$ ,  $s$  and  $p$  denote the scalar and pseudoscalar external sources. Only relevant terms are presented at the mesonic  $\mathcal{O}(q^4)$  Lagrangian. In the calculation of the diagrams simply the LECs  $l_3$  and  $l_4$  contribute to the matrix element with the numerics  $l_3 = 1.55 \times 10^{-3}$ ,  $l_4 = 2.66 \times 10^{-3}$  [6]. The scalar sources are neglected in the definition of  $\chi$  as only pseudoscalar sources are of interest.  $F$  is the pion decay constant,  $F_\pi = F[1 + \mathcal{O}(\hat{m})] = 92.4 \text{ MeV}$  [18] and  $B$  is associated with the quark condensate  $\langle \bar{q}q \rangle$  in the chiral

limit with  $B = 2.28$  GeV depending on the average quark mass [19]. We employ the  $SU(2)$  isospin symmetry  $m_u = m_d = \hat{m}$ , and the lowest order prediction for the pion mass squared is  $M_\pi^2 = 2B\hat{m}$ . The triplet of pion fields is contained in the  $2 \times 2$  matrix  $U$ ,

$$U(x) = u^2(x) = \exp\left(\frac{i\Phi(x)}{F}\right), \quad \Phi = \vec{\tau} \cdot \vec{\phi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad (3)$$

where  $\tau^a$  are the Pauli matrices and the covariant derivative  $D_\mu U = \partial_\mu U$ . The external vector sources and the axial sources in the definition of the covariant derivative are ignored as they are not relevant here. Following leading order (LO) and next to leading order (NLO) Lagrangians are also needed [3, 8, 20]

$$\begin{aligned} \mathcal{L}_{\pi N}^{(1)} &= \bar{\Psi} [i\not{D} - m_N + \frac{g_A}{2} \not{\psi} \gamma_5] \Psi, \\ \mathcal{L}_{\pi N \Delta}^{(1)} &= g \bar{\psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\nu}(z_1) \omega_\nu^j \Psi + \text{h.c.}, \\ \mathcal{L}_{\pi N \Delta}^{(2)} &= \bar{\psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\nu}(z_2) [ib_3 \omega_{\nu\beta}^j \gamma^\beta + i \frac{b_8}{m_N} \omega_{\nu\beta}^j iD^\beta] \Psi + \text{h.c.}, \\ \mathcal{L}_{\pi \Delta}^{(1)} &= -\bar{\psi}^\mu [(i\not{D} - m_\Delta) g^{\mu\nu} - i(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + i\gamma_\mu \not{D} \gamma_\nu + m_\Delta \gamma_\mu \gamma_\nu \\ &\quad + \frac{g_1}{2} g_{\mu\nu} \not{\psi} \gamma_5 + \frac{g_2}{2} (\gamma_\mu u_\nu + u_\mu \gamma_\nu) \gamma_5 + \frac{g_3}{2} \gamma_\mu \not{\psi} \gamma_5 \gamma_\nu] \psi^\nu, \end{aligned} \quad (4)$$

where the isospin structure in the LO Lagrangian of the  $\pi\Delta$  is suppressed. Note that the nucleon axial vector coupling in the chiral limit,  $g_A = 1.2723 \pm 0.0023$  [21], the nucleon isospin doublet,  $\Psi$  and  $\Theta^{\mu\nu}(z) = g^{\mu\nu} + z\gamma^\mu \gamma^\nu$  with off-shell parameter  $z$ . The off-shell parameters  $z_1$  and  $z_2$  in the interaction Lagrangian of the pion, nucleon and Delta are redundant and the physical observables are independent on it [22, 23]. Here, we use  $z_1 = z_2 = -1$  equivalently. The covariant derivative of  $\Psi$  and other chiral building blocks are defined as

$$\begin{aligned} D_\mu \Psi &= (\partial_\mu + \Gamma_\mu) \Psi, \\ \Gamma_\mu &= \frac{1}{2} [u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - il_\mu) u^\dagger], \\ u &= i [u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger], \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \end{aligned} \quad (5)$$

with  $\omega_\nu^i = \frac{1}{2} \text{Tr}(\tau^i u_\nu)$  and  $\omega_{\nu\beta}^i = \frac{1}{2} \text{Tr}(\tau^i [D_\nu, u_\beta])$ . We make use of the quark model estimation for the nucleon to Delta transition axial vector coupling constant  $g = \frac{3}{5} \sqrt{2} g_A = 1.08$  [3]. The LECs  $b_3$  and  $b_8$  have been determined from a fit in a restricted energy domain to the total cross section data for the reaction  $\pi N \rightarrow \pi\pi N$  in Ref. [24]. Given the relations for these couplings in there, the values  $b_3 = -0.47 \text{ GeV}^{-1}$  and  $b_8 = 0.55 \text{ GeV}^{-1}$  are used in the calculation. Note that the empirical values  $m_N = 938.3 \text{ MeV}$ ,  $M_\pi = 139.6 \text{ MeV}$ ,  $z_\Delta = (1210 - i50) \text{ MeV}$  for the parameters [25].  $\psi_\mu^i$  acts for the vector-spinor isovector-isospinor Delta as Rarita-Schwinger field [26] and  $\xi_{ij}^{\frac{3}{2}}$  is a matrix representation of the projection operator for the isospin-3/2 component of the Delta field. For a detailed description of the building blocks, see Refs. [27, 28]. In particular, here the relation  $g_2 = g_3 = -g_1$  is taken into account, which is obtained with the usage

of nontrivial constraints among the coupling constants in Ref. [27]. Furthermore, we employ the value of this LO  $\pi\Delta\Delta$ -coupling determined by Ref. [24] of  $g_1 = 1.68 \pm 1.38$  in the calculation. This value of  $g_1$  has been extracted from the same fit where the LECs  $b_3$  and  $b_8$  are determined accordingly.

### 3. Power counting and renormalization scheme

A power counting scheme is necessary to decide on the relative importance of Feynman diagrams. In ordinary mesonic ChPT, there is an exact correspondence between the loop expansion and the chiral expansion in powers of small quark masses and external momenta. To incorporate heavier particles, such as the delta baryon, into the theory as an explicit degree of freedom one needs to choose an expansion parameter consistently with the systematics of the theory. We make use of the so-called small scale expansion (SSE) or  $\epsilon$  expansion [3] with the expansion parameter

$$\epsilon \in \left\{ \frac{q}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi}, \frac{\delta}{\Lambda_\chi} \right\}, \quad (6)$$

where the difference in the nucleon-delta mass  $\delta \equiv m_\Delta - m_N$  is included in the same chiral order as the pion mass and the momentum transfer  $q, M_\pi \sim \delta \ll \Lambda_\chi$  with the typical scale of the theory  $\Lambda_\chi \approx 1\text{GeV}$ .

One has to renormalize the ultraviolet divergencies occurring from loop diagrams to obtain physically meaningful results. By removing divergencies, the renormalized parameters are obtained equal to their physical values in the chiral limit. CMS is the generalization of the on mass shell scheme to unstable particles wherein the finite width effects are employed. In this renormalization scheme, we split the bare parameters of the Lagrangian into complex renormalized parameters and counter terms. The renormalized complex mass  $z_R$  is chosen as a complex pole of the full propagator of the unstable particle in the chiral limit:  $m_0 = (m_R + i\frac{\Gamma_R}{2}) + \delta m \equiv z_R + \delta m$  with the renormalized width of the resonance  $\Gamma_R$ , the bare mass  $m_0$  and the counter term  $\delta m$ . The renormalized masses are included in the propagators and treated the counter terms perturbatively. In this way, physically measurable observables are identified by a set of real-valued functions with complex arguments.

### 4. The $\Delta(1232) \rightarrow \Delta(1232)$ pseudoscalar transition form factors

The pseudoscalar current matrix element between initial and final Delta states can be parametrized as

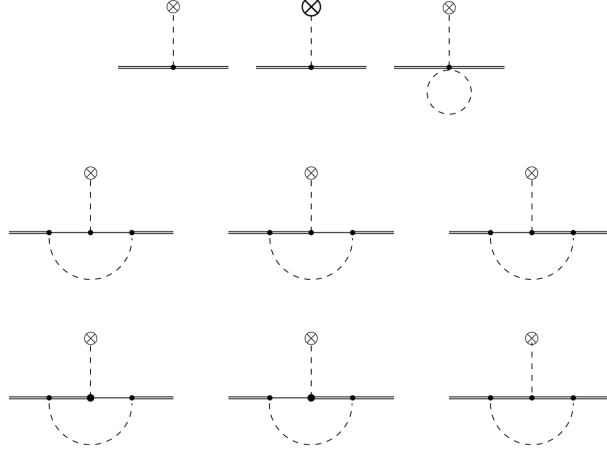
$$\langle \Delta(p_f, s_f) | P(x) | \Delta(p_i, s_i) \rangle = \bar{u}_\alpha(p_f, s_f) \left[ -\frac{1}{2} \left( g^{\alpha\beta} \gamma^5 \tilde{g}(Q^2) + \frac{q^\alpha q^\beta}{4m_\Delta^2} \gamma^5 \tilde{h}(Q^2) \right) \right] u_\beta(p_i, s_i), \quad (7)$$

where  $P(x) = \bar{\psi}(x) \gamma_5 \frac{\tau^3}{2} \psi(x)$  and  $Q^2 = -q^2$ ,  $q^\alpha = p_f^\alpha - p_i^\alpha$  are the physically relevant pseudoscalar density operator and momentum transfer, respectively. The initial and final Rarita-Schwinger vector-spinors  $u_\beta(p_i), \bar{u}_\alpha(p_f)$  satisfy the following constraint equations

$$\begin{aligned} \bar{u}_\alpha(p_f, s_f) \not{p}_f &= z_\Delta \bar{u}_\alpha(p_f, s_f), & \not{p}_i u_\beta(p_i, s_i) &= z_\Delta u_\beta(p_i, s_i), \\ \bar{u}_\alpha(p_f, s_f) \gamma^\alpha &= 0, & \gamma^\beta u_\beta(p_i, s_i) &= 0, & \bar{u}_\alpha(p_f, s_f) p_f^\alpha &= 0, & p_i^\beta u_\beta(p_i, s_i) &= 0, \end{aligned} \quad (8)$$

with a complex mass  $z_\Delta$  and on-shell conditions  $p_i^2 = z_\Delta^2$ ,  $p_f^2 = z_\Delta^2$ . Because the Delta is a spin and isospin-3/2 particle it is described as a vector-spinor isovector-isospinor field  $\psi_{\mu\nu,ab}$ . We denote here only the Lorentz-vector and the Dirac-spinor indices and omit the isovector and isospinor indices. Tree-level and one-loop Feynman

diagrams contributing to the delta matrix element of the pseudoscalar current are sketched in Figure 1. There is no diagram of chiral order two and four at tree-level. Only tree-level diagrams of  $\mathcal{O}(q^1)$  and  $\mathcal{O}(q^3)$  contribute to the considering transition. Vertex Functions are the sums of all diagrams, which participate in the given process, multiplied by factors of the wave function renormalization constant,  $Z$ . Thus, a calculation at  $\mathcal{O}(q^4)$  requires to consider the wave function renormalization constant for the Delta, as the chiral order of  $Z$  starts from two. On the other hand, because the physical values of the coupling constants are used in the calculation, the evaluation of the wave function renormalization constant of the Delta is not necessary.



**Figure 1.** Contributions to the pseudoscalar current-delta transition form factors up to and including order  $\mathcal{O}(q^4)$ . The double, solid and dashed lines correspond to the delta, nucleon, and pion respectively. While the filled circles denote the first and second-order interaction vertices increasing in size progressively, the crossed circles represent the interaction with the external pseudoscalar source at orders two and four (the bold one at the top row in the center).

Calculation of the diagrams has been done using Mathematica packages FeynCalc, LoopTools, and Package-X. After we calculated the diagrams of Figure 1, used the constraint relations in Eq. (8) and simplified the algebra, it has been concluded the results have a tensor structure as in the following

$$\mathcal{S}^{\mu\nu}(p_i, p_f, \mu, \nu) = k_1 g^{\mu\nu} + k_2 p_i^\mu p_f^\nu. \quad (9)$$

Here,  $k_1$  and  $k_2$  are the coefficients of the Lorentz structures described by matching the result of the diagrams  $\mathcal{S}^{\mu\nu}(p_i, p_f, \mu, \nu)$  with the structure of Eq. (7). In this way, the form factors  $\tilde{g}$  and  $\tilde{h}$  have been extracted from the analytic expressions of the diagrams. Our results for the pseudoscalar form factors  $\tilde{g}$  and  $\tilde{h}$  are shown in Figure 2. A discussion of the results is given in the next section.

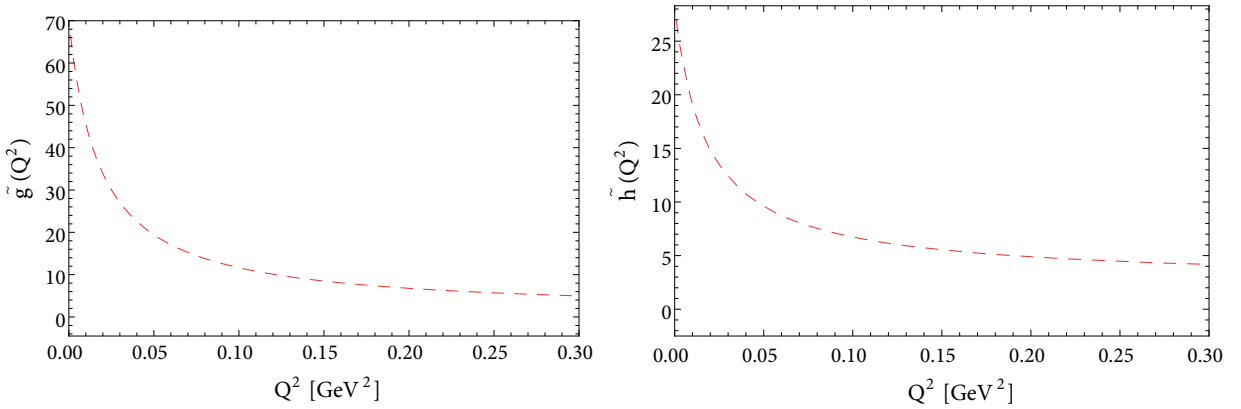
## 5. Results

The calculation of the tree-level diagrams which are shown in Figure 1 gives the contribution only to the form factor  $\tilde{g}$

$$\begin{aligned} \tilde{g}(Q^2)_1 &= \frac{4B g_1 z_\Delta}{9(M_\pi^2 + Q^2)}, \\ \tilde{g}(Q^2)_2 &= \frac{8B g_1 z_\Delta M_\pi^2}{9F^2(M_\pi^2 + Q^2)}(l_3 + l_4), \end{aligned} \quad (10)$$

where the subscripts 1 and 2 represent the contributions of the first and second tree-level diagrams. We show the one-loop contributions to the  $\tilde{g}$  and  $\tilde{h}$  which also expresses  $Q^2$  momentum dependence in Figure 2. Clearly seen that the form factors are dominated by the pion pole contribution at low energies as in the case of the induced pseudoscalar form factor of the nucleon [29]. This is not a surprise once looking at the contributed Feynman diagrams of the transition. That is, the diagrams where there are an external pion and correspondingly the pole of that propagators are the main contribution. Moreover, the spontaneous breaking of the chiral symmetry leads to a large pion cloud effect, which is responsible for the observed pion pole dominance structure in the low- $Q^2$  domain. It has been also seen that the form factor  $\tilde{g}$  is more dominant than  $\tilde{h}$  and the dominant contributions come from the tree-level diagrams, while loop corrections are small.

The relativistic BChPT is a low-energy regime method compared with, e.g., Lattice QCD. Hence, a one-to-one comparison is not always possible. Nevertheless, one can say that the lattice and Sum rule results for the same quantities are in good agreement with the present study, as they both display a pion pole dominance.



**Figure 2.**  $Q^2$  dependence of  $\tilde{g}$  and  $\tilde{h}$  form factors.

It can be seen from Eq. (7) only the form factor  $\tilde{g}(Q^2)$  can be extracted at  $Q^2 = 0$ . At this zero transfer momentum limit, the pseudoscalar form factor reduces to  $\tilde{g}(0) = 68.52$ . Once this value is compared with the one obtained in the sum rule approach, wherein a dipole parametrization has been used for the fit of  $\tilde{g}$ , it is seen that the difference is pretty large. In our calculation, the numerics substantially governs by the values of the constant B, which is proportional to the quark condensate, and the coupling  $g_1$ . The constant B is defined by the average quark masses via the Gell-Mann-Oakes-Renner formula [19],  $M_\pi^2 = 2B\hat{m}$  which varies widely depending on the masses. The pion-Delta coupling  $g_1$  has also different values on the literature, e.g., Ref. [30] presents its value as  $-1.21$ , extracting from a phase shift fit analysis based on the  $\pi N$  scattering amplitude. Further, it is also emphasized that accurate verification of the value of  $g_1$  is not provided because of manifesting itself only in the loop contribution [20]. For other works discussing the numerical value of  $g_1$ , see Refs. [20, 24] and references in there. We hold on to the fit in Ref. [24] where  $g_1$ ,  $b_3$ , and  $b_8$  are used simultaneously as free fit parameters. Therefore, the numerics could be formalized varying on the values of these LECs. On the other hand, it is difficult to make more comments because there are no precise measurements yet on the pseudoscalar form factors of the  $\Delta(1232)$  baryon. Works on the structure of the  $\Delta(1232)$  continue both on the experimental and theoretical sides to make these issues clear.

## 6. Conclusion

We investigated the low- $Q^2$  behavior of the pseudoscalar form factors of the  $\Delta(1232)$  resonance in relativistic BChPT up to the fourth chiral order. We performed the renormalization in the CMS for the unstable  $\Delta(1232)$  baryon. We provided the  $Q^2$  dependence of the  $\tilde{g}(Q^2)$  and  $\tilde{h}(Q^2)$  exhibiting a pion pole dominance structure. We also presented the value of the pseudoscalar form factor  $\tilde{g}$  in the limit of  $Q^2 \rightarrow 0$ . Furthermore, we compared the results with other techniques such as Lattice QCD. In conclusion, the fact that the results are consistent with one another confirms the accuracy of the chiral effective theory based on SU(2) isospin symmetry limit.

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## References

- [1] Pascalutsa V, Vanderhaeghen M. The Nucleon and delta-resonance masses in relativistic chiral effective-field theory. *Physics Letters B* 2006; 636: 31-39. doi: 10.1016/j.physletb.2006.03.023
- [2] Jenkins EE, Manohar AV. Chiral corrections to the baryon axial currents. *Physics Letters B* 1991; 259: 353-358. doi: 10.1016/0370-2693(91)90840-M
- [3] Hemmert TR, Holstein BR, Kambor J. Chiral Lagrangians and delta (1232) interactions: Formalism. *Journal of Physics G* 1998; 24: 1831-1859. doi: 10.1088/0954-3899/24/10/003
- [4] Epelbaum E, Gegelia J, Meißner UG, Yao DL. Baryon chiral perturbation theory extended beyond the low-energy region. *European Physics Journal of C* 2015; 75 (10): 499. doi: 10.1140/epjc/s10052-015-3728-7
- [5] Weinberg S. Phenomenological Lagrangians. *Physica A* 1979; 96 (1-2): 327-340. doi: 10.1016/0378-4371(79)90223-1
- [6] Gasser J, Leutwyler H. Chiral Perturbation Theory to One Loop. *Annals of Physics* 1984; 158: 142. doi: 10.1016/0003-4916(84)90242-2
- [7] Gasser J, Leutwyler H. Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark. *Nuclear Physics B* 1985; 250: 465-516. doi: 10.1016/0550-3213(85)90492-4
- [8] Gasser J, Sainio ME, Svarc A. Nucleons with Chiral Loops. *Nuclear Physics B* 1988; 307: 779-853. doi: 10.1016/0550-3213(88)90108-3
- [9] Fuchs T, Gegelia J, Japaridze G, Scherer S. Renormalization of relativistic baryon chiral perturbation theory and power counting. *Physical Review D* 2003; 68; 056005. doi: 10.1103/PhysRevD.68.056005
- [10] Fuchs T, Schindler MR, Gegelia J, Scherer S. Power counting in baryon chiral perturbation theory including vector mesons. *Physics Letters B* 2003; 575: 11-17. doi:10.1016/j.physletb.2003.09.060
- [11] Stuart RG. Pitfalls of radiative corrections near a resonance. In  $Z^0$  Physics, edited by Tran Thanh Van J. (Editions Frontières, Gif-sur-Yvette, 1990) p. 41.
- [12] Denner A, Dittmaier S, Roth M, Wackerth D. Predictions for all processes  $e^+e^- \rightarrow 4$  fermions +  $\gamma$ . *Nuclear Physics B* 1999; 560: 33-65. doi: 10.1016/S0550-3213(99)00437-X
- [13] Denner A, Dittmaier S. The Complex-mass scheme for perturbative calculations with unstable particles. *Nuclear Physics B Proceeding Supplements* 2006; 160: 22-26. doi: 10.1016/j.nuclphysbps.2006.09.025
- [14] Actis S, Passarino G. Two-Loop Renormalization in the Standard Model Part III: Renormalization Equations and their Solutions. *Nuclear Physics B* 2007; 777: 100-156. doi: 10.1016/j.nuclphysb.2007.04.027

- [15] Actis S, Passarino G, Sturm C, Uccirati S. Two-Loop Threshold Singularities, Unstable Particles and Complex Masses. *Physics Letters B* 2008; 669: 62-68. doi: 10.1016/j.physletb.2008.09.028
- [16] Kucukarslan A, Ozdem U, Ozpineci A. Isovector axial vector and pseudoscalar transition form factors of  $\Delta$  in QCD. *Physical Review D* 2014; 90: 054002. doi: 10.1103/PhysRevD.90.054002
- [17] Alexandrou C, Gregory EB, Korzec T, Koutsou G, Negele JW et al. Determination of the  $\Delta(1232)$  axial and pseudoscalar form factors from lattice QCD. *Physical Review D* 2013; 87 (11): 114513. doi: 10.1103/PhysRevD.87.114513
- [18] Nakamura K, Hagiwara K, Hikasa K, Murayama H, Tanabashi M et al. [Particle Data Group]. Review of particle physics. *Journal of Physics G* 2010; 37: 075021. doi:10.1088/0954-3899/37/7A/075021
- [19] Gell-Mann M, Oakes RJ, Renner B. Behavior of current divergences under  $SU(3) \times SU(3)$ . *Physical Review* 1968; 175: 2195. doi:10.1103/PhysRev.175.2195
- [20] Fettes N, Meissner UG. Pion-nucleon scattering in an effective chiral field theory with explicit spin 3/2 fields. *Nuclear Physics A* 2001; 679: 629-670. doi: 10.1016/S0375-9474(00)00368-7
- [21] Patrignani C, Agashe K, Aielli G, Amsler C, Antonelli M et al. Particle Data Group, Review of Particle Physics. *Chinese Physics C* 2016; 40 (10): 100001. doi: 10.1088/1674-1137/40/10/100001
- [22] Tang HB, Ellis PJ. Redundance of Delta isobar parameters in effective field theories. *Physics Letters B* 1996; 387: 9-13. doi: 10.1016/0370-2693(96)00862-3
- [23] Krebs H, Epelbaum E, Meissner UG. Redundancy of the off-shell parameters in chiral effective field theory with explicit spin-3/2 degrees of freedom. *Physics Letters B* 2010; 683: 222-228. doi: 10.1016/j.physletb.2009.12.023
- [24] Siemens D, Bernard V, Epelbaum E, Krebs H, Meissner UG. The reaction  $\pi N \rightarrow \pi\pi N$  in chiral effective field theory with explicit  $\Delta(1232)$  degrees of freedom. *Physical Review C* 2014; 89 (6): 065211. doi: 10.1103/PhysRevC.89.065211
- [25] Tanabashi M, Hagiwara K, Hikasa K, Nakamura K, Sumino Y et al. [Particle Data Group]. Review of Particle Physics. *Physical Review D* 2018; 98 (3): 030001. doi:10.1103/PhysRevD.98.030001
- [26] Rarita W, Schwinger J. On a theory of particles with half integral spin. *Physical Review* 1941; 60: 61. doi: 10.1103/PhysRev.60.61
- [27] Wies N, Gegelia J, Scherer S. Consistency of the  $\pi$  Delta interaction in chiral perturbation theory. *Physical Review D* 2006; 73: 094012. doi: 10.1103/PhysRevD.73.094012
- [28] Scherer S, Schindler MR. Quantum chromodynamics and chiral symmetry. *Lecture Notes in Physics* 2012; 830: 1-48. doi: 10.1007/978-3-642-19254-8-1
- [29] Ünal Y, Küçükarslan A, Scherer S. Contribution of the  $\alpha_1$  meson to the axial nucleon-to-delta transition form factors. *Physical Review D* 2019; 99 (1): 014012. doi: 10.1103/PhysRevD.99.014012
- [30] Yao DL, Siemens D, Bernard V, Epelbaum E, Gasparyan AM et al. Pion-nucleon scattering in covariant baryon chiral perturbation theory with explicit Delta resonances. *Journal of High Energy Physics* 2016; 1605: 038. doi: 10.1007/JHEP05(2016)038