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On an econophysics model

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Abstract: Krugman has proposed a continuous in time and space model for the emergence of polycentric urban areas in the regional space. By using a discrete version of this model on spatial networks, we predicted the distribution of jobs among the different localities inside a couple of economic regions in Ohio and Texas. The time evolution of the distribution of jobs is governed by a market potential. Employment gradually moves towards locations considered relatively attractive if their market potential is above the spatial average. Similarly, jobs move away from locations with below-average market potential. First, I will show that the market potential satisfies the Fisher equation of natural selection. Second, I will determine the stationary distribution.

Keywords: Econophysics, Fisher equation of genetics

1. Introduction

Nihat Berker's inspirational range of research activities, including his recent application [1] of the statistical mechanics of spin glasses to understanding culture and music, opens new research avenues for physicists. The econophysics [2, 3] (or physics-inspired) model described shortly is in this vein.

Krugman's model [4] generates the time evolution of the distribution of jobs among the different localities inside an economic region. To enable us to connect to employment data we have discretized Krugman's model. The region's localities are nodes in a network. In the work done so far [5, 6], the interactions between any two nodes depend on the distance between the two localities. The market potential function governs the model dynamics. At time t and location x the market potential is determined by all fractions of jobs $n_{t,x}$ and by a network matrix $q_{x,y}$ connecting any two locations, x and y, in the region. Employment gradually moves towards locations with market potential above the spatial average. Conversely jobs move away from locations with below-average market potential.

In Section 2, we describe the continuous and discreet versions of the model. We prove that the time evolution of the market potential for the continuous model follows the Fisher fundamental equation of genetics [7] in Section 3. The time derivative of the market potential is proportional to its spatial variance. As a result, the market potential is a monotonically increasing function of time. The stationary distribution is determined in Section 4. We provide a numerical estimate of differences between the continuous model and its discreet approximation in Section 5. We compute the stationary distribution market potential for each of the two economic regions considered in references [5] and [6]. Since, in both cases, the current market potential is larger than the stationary distribution market potential, in light of the fact that the market potential increases monotonically in time, the stationary

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distribution will not occur in the future. Other directions of research on this model are discussed in Section 6.

2. Model

Krugman [4] has presented the model using continuous space and time variables:

$$\frac{\partial n(t,x)}{\partial t} = \left(P(t,x) - \overline{P}(t)\right) n(t,x), \tag{2.1}$$

where $n(t,x)$ is the fraction of all employment at time t and at location x. The market potential $P(t,x)$ and its spatial average $\overline{P}(t)$ are:

$$P(t,x) = \int q(x,y)\, n(t,y)\, dy \tag{2.2}$$

$$\overline{P}(t) = \int P(t,y)\, n(y)\, dy. \tag{2.3}$$

The matrix $q(x,y)$ contains the interaction between localities x and at y. The normalization condition is consistent with Eqs. (2.1) and (2.3):

$$\int n(t,x)\, dx = 1. \tag{2.4}$$

We [5] have discretized Krugman's model to enable its use with regional employment data. The discreet model dynamic evolution equation, see Eq. (2.1), is:

$$n_{t+1,x} - n_{t,x} = \left(P_{t,x} - \overline{P}_t\right) n_{t,x}\,, \tag{2.5}$$

where the time interval is 1. The market potential and its time average are:

$$P_{t,x} = \sum_y q_{x,y}\, n_{t,y} \tag{2.6}$$

$$\overline{P}_t = \sum_x \sum_y q_{x,y}\, n_{t,x}\, n_{t,y}\,, \tag{2.7}$$

where $n_{t,y}$ is the fraction of the total employment in a region at location y, at time t. The model preserves the spatial sum of the $n_{t,x}$ at any time t:

$$\sum_x n_{t,x} = 1. \tag{2.8}$$

Employment gradually moves to locations considered relatively attractive if their market potential is above average: $P_{t,x} > \overline{P}_t$. Similarly, employment tends to move away from locations with below-average market potential: $P_{t,x} < \overline{P}_t$. This average market potential is the term of comparison for establishing whether a municipality will attract or repel employment.

The matrix $q_{x,y}$ contains the interactions between localities x and y. The matrix $q_{x,y}$ may depend on the Euclidian distance $|x-y|$ as suggested by Krugman [4]. We have used this parametrization in our implementation [5, 6] of the model for the North East Ohio and Dallas-Fort Worth economies. As a result, in those implementations the network matrix is symmetrical: $q_{x,y} = q_{y,x}$. The symmetry of the network is assumed throughout this work.

3. Market potential time evolution

We start by expressing the average market potential $\overline{P}(t)$ by substituting Eq. (2.2) into Eq. (2.3):

$$\overline{P}(t) = \int\int q(x,y)\, n(t,x)\, n(t,y)\, dxdy. \tag{3.1}$$

We differentiate both sides of Eq. (3.1) with respect to t:

$$\frac{d}{dt}\overline{P}(t) = \int\int q(x,y)\left[\frac{\partial n(t,x)}{\partial t}\, n(t,y) + \frac{\partial n(t,y)}{\partial t}\, n(t,x)\right] dxdy. \tag{3.2}$$

We substitute the time derivatives on the right hand side of Eq. (3.2) by using the time evolution Eq. (2.1):

$$\frac{d}{dt}\overline{P}(t) = \int\int q(x,y)\left[P(t,x) - \overline{P}(t) + P(t,y) - \overline{P}(t)\right] n(t,x)\, n(t,y)\, dxdy. \tag{3.3}$$

Finally, by using Eqs. (2.2) and (2.3) on the right hand side of Eq. (3.3), we get:

$$\frac{d}{dt}\overline{P} = 2 * var(P), \tag{3.4}$$

where $var(P) = \overline{P^2} - \overline{P}^2$ is the variance of the market potential. Since $var(P) > 0$, the mean market potential is a monotonically increasing function of time. Interestingly enough, the econophysics model prediction of Eq. (3.4) is qualitatively similar the Fisher's fundamental theorem of natural selection according to which: "The rate of increase in fitness of any organism at any time is equal to its genetic variance in fitness at that time". Also Kardar and collaborators [8] have used a generalization of Fisher's equation in a study of HIV infection propagation. The analog of the genetics fitness is the economics market potential $\overline{P}/2$. The same equation holds approximatively in the discreet model, Eqs. (2.5) to (2.8), if quadratic corrections $(P_{t+1} - P_t)^2$ are neglected. In Section 5, we will estimate the discrepancy between the continuous and discreet model by using Eq. (3.4).

4. Stationary distribution

The stationary distribution for the discreet model is obtained by using Eq. (2.5):

$$n_{t+1,x} - n_{t,x} = \left(P_{t,x} - \overline{P}_t\right) n_{t,x} = 0. \tag{4.1}$$

Hence for any x:

$$P_{t,x} - \overline{P}_t = 0. \tag{4.2}$$

In view of Eq. (2.6), and noting that the average market potential $\overline{P}_t$ is independent of position, Eq. (4.2) can be written as a matrix equation:

$$qn = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \overline{P}. \tag{4.3}$$

The stationary distribution is:

$$n = q^{-1} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \overline{P}, \tag{4.4}$$

where q^{-1} is the inverse matrix. Using the normalization Eq. (2.8), we calculate the average market potential:

$$\overline{P} = \frac{1}{\sum_z \sum_y (q^{-1})_{z,y}}. \tag{4.5}$$

The stationary distribution follows:

$$n_x = \frac{\sum_y (q^{-1})_{x,y}}{\sum_z \sum_y (q^{-1})_{z,y}}. \tag{4.6}$$

5. Numerics

We have used [5, 6] numerical implementation of the discreet model to predict the time evolution of employment in the localities of a couple of economic regions: North East Ohio and Dallas-Fort Worth. The matrix of interactions [4] between any two locations x and y is:

$$q_{x,y} = A\, exp\left(- \frac{|x-y|}{D_a} \right) - B\, exp\left(- \frac{|x-y|}{D_b} \right), \tag{5.1}$$

where $|x-y|$ is the Euclidean distance between the centroids of localities x and y. The first term represents the attraction pull and the second represents the repelling push on jobs. The four parameters A, B, D_a and D_b were estimated from the data, consisting of the location and employment numbers for the North East Ohio region (261 localities) and for the Dallas-Fort Worth region (359 localities). We refer the reader to references [5, 6] for model predictions for the two economic regions.

Here we check the accuracy of the discreet model compared to the original Krugman model by computing $\frac{\frac{d\overline{P}}{dt}}{2*var(P)}$ vs time. As one can see in Figure 1 the discrepancy from unity (predicted in Eq. (3.4) for the continuous model) is about 1% for the whole time period considered. In Figure 2, we show the time dependence of the average market potential as function of time for the two economic regions. Consistent with the Fisher equation, Eq. (3.4), the average market potential is a monotonically increasing function of time. We also find the average potential to be larger than the average potential associated with the stationary distribution, computed using Eq. (4.5). Hence the model predicts for both regions that the stationary distribution will not be reached in the future.

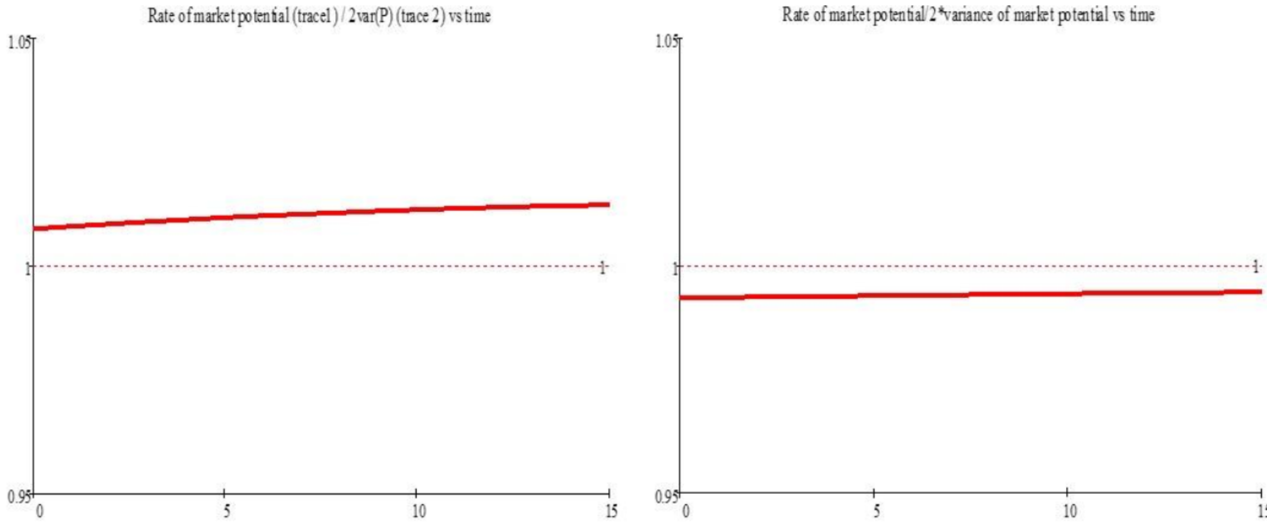

Figure 1. North East Ohio economy (left panel) and Dallas-Fort Worth economy (right panel); $\frac{\frac{d\overline{P}}{dt}}{2*var(P)}$ vs time.

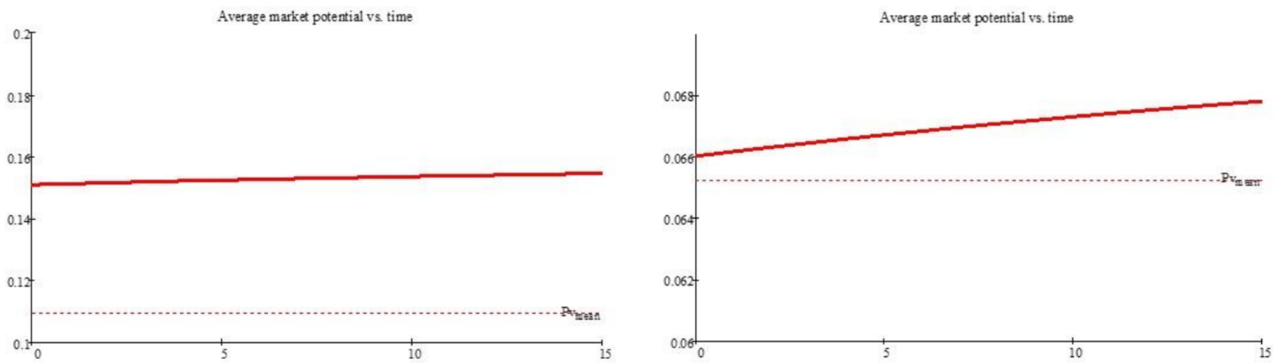

Figure 2. North East Ohio economy (left panel) and Dallas-Fort Worth economy (right panel); average market potential vs. time; dotted line market potential for stationary distribution.

6. Future research

We plan to analyze the stability of the stationary distribution, determining whether in the long run the jobs distribution settles to the stationary distribution or not. We will expand the model to other type of interactions. For example, we will replace the Euclidian distance by an effective distance that accounts for the available modes of transportation. Furthermore, we will consider interactions that depend on the population size and political power. As a result the network matrix is not symmetrical. We will study whether the Fisher equation still holds for the nonsymmetrical model.

Acknowledgement

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