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Quantum kinetic equation for fermionic fluids and chiral kinetic theory

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Abstract: We first review how one can establish the quantum kinetic equation for fluids of spin-1/2 particles. Then we present the construction of the semiclassical relativistic chiral kinetic equation of the fluid in the presence of the external electromagnetic fields. We derive the resulting nonrelativistic chiral kinetic theory. We calculated the particle number current density and showed that chiral effects are correctly generated. Moreover, it satisfies the anomalous continuity equation.

Keywords: Quantum kinetic equation, chiral kinetic theory, Coriolis force

1. Introduction

One of the most important milestones in physics is the discovery of the field equation of spin-1/2 particles by Dirac. This one particle equation is essential to understand transport properties of fermions which can be established by utilizing the Wigner function constructed for free fermionic particles. When the external electromagnetic fields are present, Dirac spinors can be employed to define a gauge invariant Wigner function. It satisfies a quantum kinetic equation (QKE) which depends on electromagnetic field strength manifestly [1, 2]. When the collective behavior of fermions are considered one may treat them as fluid. Then one should take into account vorticity of the fluid. This can also be visualized as the rotation of the reference frame. However, QKE has no explicit dependence on the noninertial effects.

In the last decade QKE has been used extensively in the inspection of the transport phenomena of massless fermions as it was reviewed recently in [3]. QKE has been employed especially to study heavy-ion collisions where there appears a chiral plasma. There are some anomalous effects: The chiral separation effect (CSE) and chiral magnetic effect (CME). They are similar to the chiral vortical effect (CVE) and local polarization effect (LPE). However, QKE is not aware of the vorticity of fluid in contrast to electromagnetic field strength. To deal with magnetic and vortical phenomena on the same footing we proposed to modify of QKE by making use of the enthalpy current [4, 5].

Although, dealing with the modified QKE has several advantages, it has not been derived from an action in contrast to the electromagnetic part. In [6] we presented a Lagrangian formulation which leads to the appropriate modification of QKE. Here we will first review this formalism and then the

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chiral kinetic theory (CKT) resulting from it [4] will be presented briefly.

2. Full action

The action,

$$S_{Dirac} = \frac{1}{2}\int d^4x\,\bar{\psi}\,(i\hbar\partial\!\!\!/ - m)\,\psi, \tag{2.1}$$

leads to the Dirac equation

$$(i\hbar\partial\!\!\!/ - m)\,\psi = 0, \tag{2.2}$$

which the free spin-1/2 particles obey. Here $\partial\!\!\!/ \equiv \gamma^\mu\partial_\mu$.

To take into account vorticity of the fermionic fluid which is subjected to electromagnetic fields, we introduce two gauge fields A_μ and a_μ. First one is the electromagnetic gauge field whose action is

$$S_A \;\; = \;\; -Q\int d^4x\;\bar{\psi}A\!\!\!/\,\psi - \frac{1}{4}\int d^4x\;F^{\mu\nu}F_{\mu\nu}. \tag{2.3}$$

The electromagnetic field strength is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.4}$$

Q is the electric charge. $F_{\mu\nu}$ is invariant under the gauge transformation

$$A_\mu(x) \to A_\mu(x) - \partial_\mu\Lambda(x). \tag{2.5}$$

The real four-vector field a_α, is the other gauge field. It is coupled to Dirac fermion,

$$S_\zeta = \zeta\int d^4x\bar{\psi}\,a\!\!\!/\,\psi, \tag{2.6}$$

where ζ is the related coupling constant. Now, introduce the complex scalar field $b(x)$ which is coupled to the latter gauge field as

$$S_b = \frac{1}{2}\int d^4x\left[(\partial^\alpha b - ia^\alpha b)^\star\,(\partial_\alpha b - ia_\alpha b) - V\,(b^\star b)\right]. \tag{2.7}$$

We propose

$$S = S_{Dirac} + S_A + S_\zeta + S_b, \tag{2.8}$$

as the total action which describes a fermionic fluid in the presence of vorticity and electromagnetic fields. Within this work the metric is $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. As we will present in the next section the fields b and a_α describe the fluid.

3. Fermionic fluid

The action

$$S_{Fluid} = S_b + S_\zeta, \tag{3.1}$$

was proposed in [7] for being able to formulate magnetohydrodynamics as a covariant field theory. Although we are mainly motivated by their considerations, our formulation has subtle differences.

Let us introduce the real fields $s(x)$ and $\theta(x)$ to express the complex field $b(x)$ as

$$b = se^{i\theta}, \tag{3.2}$$

so that (2.7) leads to

$$S_b = \frac{1}{2}\int d^4x \left[\partial^\alpha s \partial_\alpha s + s^2 \left(\partial^\alpha \theta - a^\alpha\right)\left(\partial_\alpha \theta - a_\alpha\right) - V(s^2)\right]. \tag{3.3}$$

It can easily be observed that under the transformations

$$a_\alpha(x) \to a_\alpha(x) - \partial_\alpha \lambda(x), \quad \theta(x) \to \theta(x) - \lambda(x), \tag{3.4}$$

the action (3.3) is left invariant. The equations of motion obtained from S_{Fluid} are

$$s^2\left(\partial^\alpha\theta - a^\alpha\right) - \zeta\bar{\psi}\gamma^\alpha\psi = 0, \tag{3.5}$$

$$\partial_\alpha[s^2\left(\partial^\alpha\theta - a^\alpha\right)] = 0. \tag{3.6}$$

In terms of the operators representing the fermionic particles $\hat{\psi}$, $\hat{\bar{\psi}}$, we define the number current density by

$$j^\alpha = \langle :\hat{\bar{\psi}}\gamma^\alpha\hat{\psi}: \rangle = \int d^4q\, q^\alpha f(x,q). \tag{3.7}$$

Colons denote normal ordering and $f(x.q)$ is the distribution function. As we will show this system can be considered as a fluid: Introduce the fluid four-velocity satisfying

$$u^\alpha u_\alpha = 1. \tag{3.8}$$

Now, any four-vector can be written as $k^\alpha = (u\cdot k)u^\alpha + k_\perp^\alpha$, where $k_\perp \cdot u = 0$. By making use of this decomposition one can show that (3.7) yields

$$j^\alpha = nu^\alpha, \tag{3.9}$$

where n is the particle number density. In the mean field approach one can view $\hat{\psi}$ as an assembly of wave-packets. Then, a fermionic fluid element can be considered as a wave-packet, so that

$$\bar{\psi}\gamma^\alpha\psi \equiv nu^\alpha. \tag{3.10}$$

Plugging this into (3.5) yields

$$nu^\alpha = \zeta^{-1}s^2\left(\partial^\alpha\theta - a^\alpha\right). \tag{3.11}$$

(3.6) leads to conservation of the current density,

$$\partial_\alpha \left(n u^\alpha \right) = 0. \tag{3.12}$$

Under the assumption of a slowly varying b, the equation of motion of s field gives

$$\left(\partial^\alpha \theta - a^\alpha \right) \left(\partial_\alpha \theta - a_\alpha \right) = \nu^2. \tag{3.13}$$

Here $\nu^2 \equiv dV(s^2)/ds^2$. Employing (3.11) in (3.13) leads to

$$n^2 \zeta^2 = s^4 \nu^2. \tag{3.14}$$

Then n can be expressed as a function of s as

$$n = \frac{s^2 \nu}{\zeta}. \tag{3.15}$$

Inserting (3.15) into (3.11) leads to

$$\partial^\alpha \theta - a^\alpha = \nu u^\alpha. \tag{3.16}$$

Utilizing (3.16), after taking the derivative of (3.13), we get

$$\nu u^\alpha (\partial_\alpha \partial_\beta \theta - \partial_\alpha a_\beta - w_{\beta\alpha}) = \nu \partial_\beta \nu, \tag{3.17}$$

where

$$w_{\beta\alpha} = \partial_\beta a_\alpha - \partial_\alpha a_\beta. \tag{3.18}$$

By taking the derivative of (3.16) and employing it in (3.17) we attain

$$\nu u^\alpha (u_\beta \partial_\alpha \nu + \nu \partial_\alpha u_\beta - w_{\beta\alpha}) = \nu \partial_\beta \nu. \tag{3.19}$$

For $\nu \neq 0$, it gives

$$\nu u^\alpha \partial_\alpha u_\beta = \partial_\beta \nu - u^\alpha u_\beta \partial_\alpha \nu + u^\alpha w_{\beta\alpha}. \tag{3.20}$$

Acceleration which is the proper time derivative of the fluid velocity, can be calculated by using (3.20) as

$$\frac{du_\beta}{d\tau} = u^\alpha \partial_\alpha u_\beta = \frac{\partial_\beta \nu}{\nu} - u^\alpha u_\beta \frac{\partial_\alpha \nu}{\nu} + \frac{u^\alpha w_{\beta\alpha}}{\nu}. \tag{3.21}$$

Let us deal with the Euler equations

$$\frac{du_\beta}{d\tau} = \frac{\partial_\beta P}{\rho + P} - u^\alpha u_\beta \frac{\partial_\alpha P}{\rho + P} - \frac{F_\beta}{\rho + P}. \tag{3.22}$$

ρ is the energy density and P is the pressure. The external force F_α can be gravitational, electromagnetic or the Coriolis force [8].

Except the last terms, (3.21) and (3.22) are identical as far as

$$\frac{dP}{\rho + P} = \frac{d\nu}{\nu}, \tag{3.23}$$

is satisfied. Let there be no heat exchange and the fluid be composed of only one kind of particle. Then, assuming that (3.23) is satisfied one obtains

$$\nu = \xi \frac{\rho + P}{n} = \xi h. \tag{3.24}$$

ξ is a positive constant and $h = (\rho + P)/n$ is the specific enthalpy. For an ideal fluid it can be shown that

$$\nu = \xi' \frac{\rho}{n}, \tag{3.25}$$

where ξ' is an arbitrary constant. Let p^μ be the momentum of the wave packet center. Then we can write $\rho/n = u \cdot p$. Consequently, we get

$$\nu = \xi' u \cdot p. \tag{3.26}$$

On the other hand one can show that $u^\alpha w_{\alpha\beta} \equiv \xi' \mathcal{C}_\beta$ is the relativistic Coriolis force:

$$\mathcal{C}_\beta = \epsilon_{\beta\alpha\mu\nu} u^\mu \omega^\nu p^\alpha.$$

Here we introduced the vorticity four-vector

$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \Omega_{\alpha\beta} u_\nu, \tag{3.27}$$

where

$$\Omega_{\alpha\beta} = \frac{1}{2}(\partial_\alpha u_\beta - \partial_\beta u_\alpha), \tag{3.28}$$

is the kinematic vorticity tensor.

By making use of the properties of the fluid velocity one can write

$$w^{\mu\nu} = \xi' w_C^{\mu\nu} + \kappa (u \cdot p) \Omega^{\mu\nu}, \tag{3.29}$$

where κ is an arbitrary constant and

$$w_C^{\mu\nu} \;\; = \;\; (\partial^\mu u^\alpha) p_\alpha u^\nu - (\partial^\nu u^\alpha) p_\alpha u^\mu. \tag{3.30}$$

Observe that $w^{\mu\nu}$ is the circulation (vorticity) tensor for $\kappa = 2$ and $\xi' = 1$ [8]. We conclude that the fluid composed of the Dirac particles is represented by the scalar field b and the vector field a_α whose strength tensor is given by (3.29).

4. Quantum kinetic equation

Let us deal with the Dirac equation

$$[\gamma^\mu (i\hbar \partial_\mu - \zeta a_\mu - Q A_\mu) - m]\psi = 0. \tag{4.1}$$

It is invariant under the gauge transformations (2.5), (3.4), accompanied by

$$\psi(x) \to e^{i(\zeta \lambda(x) + Q\Lambda(x))/\hbar} \psi(x). \tag{4.2}$$

By following the formulation given in [2], we define the gauge invariant Wigner operator by

$$\hat{W}(x,p) = \int d^4y\, e^{-ip\cdot y/\hbar}\bar{\psi}(x_1)U(A,a;x_1,x_2)\otimes\psi(x_2), \tag{4.3}$$

where we introduced

$$U(A,a;x_1,x_2) \equiv \exp\left[-iQ\gamma^\mu\int_0^1 dsA_\mu(x_2+sy)\right]\exp\left[-i\zeta\gamma^\mu\int_0^1 dsa_\mu(x_2+sy)\right]. \tag{4.4}$$

Here $\otimes$ represents tensor product and $x_1^\mu \equiv x^\mu + y^\mu/2,\ x_2^\mu \equiv x^\mu - y^\mu/2$. The Wigner function is defined by taking the average of (4.2):

$$W(x,p) = \langle :\hat{W}(x,p):\rangle. \tag{4.5}$$

By making use of the Dirac equation (4.1), we establish the QKE

$$\left[\gamma\cdot\left(\pi+\frac{i\hbar}{2}D\right)-m\right]W(x,p)=0, \tag{4.6}$$

with

$$\begin{aligned} D^\mu &\equiv \partial^\mu - j_0(\Delta)\left[QF^{\mu\nu}+\zeta w^{\mu\nu}\right]\partial_{p\nu}, \\ \pi^\mu &\equiv p^\mu - \frac{\hbar}{2}j_1(\Delta)\left[QF^{\mu\nu}+\zeta w^{\mu\nu}\right]\partial_{p\nu}. \end{aligned}$$

$\Delta \equiv \frac{\hbar}{2}\partial_p\cdot\partial_x$ and $j_0(x),\ j_1(x)$ denote the spherical Bessel functions. Here, ∂_μ and ∂_p^ν contained in Δ act only on, respectively, $[QF^{\mu\nu}+\zeta w^{\mu\nu}]$ and $W(x,p)$.

Observe that (2.3) yields the Maxwell equations given in terms of the electric and magnetic fields E_μ, B_μ which are related to the field strength and the fluid 4-velocity u_μ by

$$F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\rho}u_\alpha B_\rho. \tag{4.7}$$

On the other hand, by imposing the equations of motion following from the variation of the action (2.8) with respect to a_α and b, $w_{\mu\nu}$ is expressed as in (3.29).

5. Chiral kinetic equations and the semiclassical approximation

One decomposes the Wigner function in terms of the independent 4×4 matrices constructed by the gamma matrices:

$$W = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right). \tag{5.1}$$

Inserting this into (4.6) yields the equations which the fields satisfy. However, the scalar field $\mathcal{F}$, the pseudoscalar field $\mathcal{P}$, and the antisymmetric tensor $\mathcal{S}_{\mu\nu}$ are irrelevant in constructing chiral transport equations, i.e. for $m=0$. We are interested in the quantum kinetic equations of the axial field $\mathcal{A}_\mu$ and the vector field $\mathcal{V}_\mu$. We unify them in the chiral vector fields

$$\mathcal{J}_\chi^\mu = \frac{1}{2}(\mathcal{V}^\mu + \chi\mathcal{A}^\mu).$$

$\chi = 1$ ($\chi = -1$) corresponds to the right-handed (left-handed) fermion. Their quantum kinetic equations are

$$p_\mu \mathcal{J}^\mu_\chi = 0, \tag{5.2}$$
$$\tilde{\nabla}^\mu \mathcal{J}_{\chi\mu} = 0, \tag{5.3}$$
$$\hbar\epsilon_{\mu\nu\alpha\rho}\tilde{\nabla}^\alpha \mathcal{J}^\rho_\chi = -2\chi(p_\mu \mathcal{J}_{\chi\nu} - p_\nu \mathcal{J}_{\chi\mu}). \tag{5.4}$$

We deal with the semiclassical approximation where the fields are expanded in powers of the Planck constant $\hbar$ and the zeroth and first order terms are kept. In the $m = 0$ limit we set $\kappa = 0$, and choose $\xi' = 1$. In the semiclassical scheme $\pi_\mu = p_\mu$ and D_μ is substituted with

$$\tilde{\nabla}^\nu \equiv \partial^\nu - \left[QF^{\nu\beta} + w_C^{\nu\beta}\right]\partial_{p\beta}, \tag{5.5}$$

where we suppressed ζ. Now we express the fields as $\mathcal{J}^\mu_\chi = \mathcal{J}^{(0)\mu}_\chi + \hbar\mathcal{J}^{(1)\mu}_\chi$ and consider only the equations at most first-order in $\hbar$. The leading order solution of (5.2) and (5.4) is

$$\mathcal{J}^{(0)\mu}_\chi = p^\mu\delta(p^2)f^0_\chi. \tag{5.6}$$

The distribution function is given by $f^0_\chi = \sum_{s=\pm1}\theta(sn\cdot p)f^0_{s,\chi}(x,p)$, where $s = 1$ and $s = -1$ indicate the particle and antiparticle, respectively.

In the comoving frame $\mathcal{J}^{(1)}_{\chi\mu}$ which satisfies (5.2) and (5.4), possesses the general form

$$\begin{aligned}\mathcal{J}^{(1)\mu}_\chi &= p^\mu f^1_\chi\delta(p^2) + \frac{1}{2}\chi Q\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}p_\nu f^0_\chi\delta'(p^2)\\ &+ \chi\epsilon^{\mu\nu\alpha\rho}p_\nu(\partial_\alpha u_\beta)p^\beta u_\rho f^0_\chi\delta'(p^2) + \mathcal{K}^\mu.\end{aligned} \tag{5.7}$$

$\delta'(p^2) = -\delta(p^2)/p^2$ and the distribution function is written as $f_\chi \equiv f^0_\chi + \hbar f^1_\chi$. The solutions obtained in [9–11] suggest that

$$\mathcal{K}^\mu = S^{\mu\nu}(\tilde{\nabla}_\nu f^0_\chi)\delta(p^2).$$

$S^{\mu\nu}$ is the spin:

$$S^{\mu\nu} = \frac{\chi}{2n\cdot p}\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma.$$

The last equation (5.3) which should also be satisfied yields

$$\begin{aligned}\tilde{\nabla}_\mu\mathcal{J}^\mu_\chi &= \delta\left(p^2 + \hbar\chi Q\frac{u_\mu\tilde{F}^{\mu\nu}p_\nu}{u\cdot p}\right)\{p\cdot\tilde{\nabla}\\ &\quad+\frac{\hbar\chi Q}{u\cdot p}S^{\mu\nu}E_\mu\tilde{\nabla}_\nu - \frac{\hbar\chi}{u\cdot p}p_\mu\tilde{\Omega}^{\mu\nu}\tilde{\nabla}_\nu\\ &\quad+\frac{\hbar\chi}{u\cdot p}(\tilde{\Omega}^{\mu\nu}p_\mu u_\nu)\Omega^{\sigma\rho}p_\rho\partial^{(p)}_\sigma\}f_\chi = 0.\end{aligned}$$

Here $\tilde{\Omega}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\rho}\Omega^{\alpha\rho}$. We have the freedom of replacing f^1_χ with

$$f^1_\chi \Rightarrow \chi\frac{S^{\mu\nu}\Omega_{\mu\nu}}{u\cdot p}f^0_\chi+f^1_\chi.$$

Therefore, the covariant semiclassical chiral equation is accomplished:

$$\begin{aligned}\delta\left(p^2+\hbar\chi Q\frac{u_\mu\tilde{F}^{\mu\nu}p_\nu}{u\cdot p}\right)\{p\cdot\tilde{\nabla}\left(1+\hbar\chi\frac{S^{\mu\nu}\Omega_{\mu\nu}}{u\cdot p}\right)\\ +\frac{\hbar\chi Q}{u\cdot p}S^{\mu\nu}E_\mu\tilde{\nabla}_\nu-\frac{\hbar\chi}{u\cdot p}p_\mu\tilde{\Omega}^{\mu\nu}\tilde{\nabla}_\nu\\ +\frac{\hbar\chi}{u\cdot p}(\tilde{\Omega}^{\mu\nu}p_\mu u_\nu)\Omega^{\sigma\rho}p_\rho\partial^{(p)}_\sigma\}f_\chi=0.\end{aligned}\tag{5.8}$$

Let us now derive nonrelativistic transport equation (TE) which results from the relativistic formulation. To achieve this goal we will integrate the latter over the zeroth-component of momentum variable, p_0 [12, 13]. One should take into account the fact that to extract the 3D theory from the 4D, the necessary condition is

$$\int d^4p\{\text{4D TE}\}=\int d^3p\{\text{3D TE}\}.$$

We make use of the relation

$$\int d^4p\Big\{E\cdot\omega\left(\frac{f^0_\chi}{p_0}-\frac{1}{2}\frac{\partial f^0_\chi}{\partial p_0}\right)+\frac{\omega\cdot pE\cdot p}{p_0^2}\left(\frac{2f^0_\chi}{p_0}-\frac{1}{2}\frac{\partial f^0_\chi}{\partial p_0}\right)\Big\}\delta(p^2)=0,\tag{5.9}$$

which is satisfied by the Fermi-Dirac distribution:

$$f^0_\chi=\frac{2}{(2\pi\hbar)^3}\sum_{s=\pm1}\frac{\theta(sn\cdot p)}{e^{s(u\cdot p-\mu_\chi)/T}+1}.\tag{5.10}$$

We integrate the relativistic theory in the frame $u^\mu=(1,\mathbf{0})$, $\omega^\mu=(0,\boldsymbol{\omega})$ and establish the nonrelativistic chiral transport equation (CTE) as

$$\big(\sqrt{\rho}^\chi_s\frac{\partial}{\partial t}+(\sqrt{\rho}\dot{\boldsymbol{x}})^\chi_s\cdot\frac{\partial}{\partial\boldsymbol{x}}+(\sqrt{\rho}\dot{\boldsymbol{p}})^\chi_s\cdot\frac{\partial}{\partial\boldsymbol{p}}\big)f^{eq}_{\chi,s}(t,\boldsymbol{x},\boldsymbol{p})=0,$$

with

$$\sqrt{\rho}^\chi_s = 1+\hbar sQ\chi\boldsymbol{\beta}_s\cdot\boldsymbol{B},\tag{5.11}$$

$$\begin{aligned}(\sqrt{\rho}\dot{\boldsymbol{x}})^\chi_s &= \boldsymbol{v}^\chi_s+\hbar\chi(\hat{\boldsymbol{p}}\cdot\boldsymbol{\beta}_s)(sQ\boldsymbol{B}+2\mathcal{E}^\chi_s\boldsymbol{\omega})\\ &\quad+\hbar sQ\chi\boldsymbol{E}\times\boldsymbol{\beta}_s-2\hbar\chi(\boldsymbol{\omega}\cdot\boldsymbol{\beta}_s)\boldsymbol{p},\end{aligned}\tag{5.12}$$

$$\begin{aligned}(\sqrt{\rho}\dot{\boldsymbol{p}})^\chi_s &= sQ\boldsymbol{E}+sQ\boldsymbol{v}^\chi_s\times\boldsymbol{B}+\boldsymbol{v}^\chi_s\times\mathcal{E}^\chi_s\boldsymbol{\omega}\\ &\quad+\hbar\chi Q^2\boldsymbol{\beta}_s(\boldsymbol{E}\cdot\boldsymbol{B})-2\hbar\chi(\boldsymbol{\omega}\cdot\boldsymbol{\beta}_s)\boldsymbol{p}\times\mathcal{E}^\chi_s\boldsymbol{\omega}\\ &\quad-2\hbar sQ\chi(\boldsymbol{\omega}\cdot\boldsymbol{\beta}_s)\boldsymbol{p}\times\boldsymbol{B}.,\end{aligned}\tag{5.13}$$

where $\boldsymbol{\beta}_s = s\boldsymbol{p}/2|\boldsymbol{p}|^3$. Mass shell condition leads to the following dispersion relation,

$$\mathcal{E}_s^\chi = |\boldsymbol{p}|(1 - \hbar s Q \chi \boldsymbol{\beta}_s \cdot \boldsymbol{B}). \tag{5.14}$$

Hence the canonical velocity is

$$\boldsymbol{v}_s^\chi = \frac{\partial \mathcal{E}_s^\chi}{\partial \boldsymbol{p}} = \hat{\boldsymbol{p}}(1 + 2\hbar s Q \chi \boldsymbol{\beta}_s \cdot \boldsymbol{B}) - \hbar s Q \chi \beta_s \boldsymbol{B}.$$

It is worth noting that the Coriolis force appears as the third term of (5.13).

The particle number and current densities are defined as

$$n_s^\chi = \int [dp] (\sqrt{\rho})_s^\chi f_\chi^{eq,s}, \tag{5.15}$$

$$\boldsymbol{j}_s^\chi = \int [dp] (\sqrt{\rho}\dot{\boldsymbol{x}})_s^\chi f_\chi^{eq,s} + \boldsymbol{\nabla} \times \int [dp] \mathcal{E}_s^\chi \boldsymbol{b}_s^\chi f_\chi^{eq,s}, \tag{5.16}$$

where the measure is $[dp] = d^3p/(2\pi\hbar)^3$. By making use of (5.11) and (5.12), we calculate the continuity equation as

$$\frac{\partial n_s^\chi}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j}_s^\chi = \frac{\chi Q^2}{(2\pi\hbar)^2} \boldsymbol{E} \cdot \boldsymbol{B} \; f_\chi^{eq,s}|_{\boldsymbol{p}=0}.$$

The 4D equilibrium distribution function of a rotating fluid was given in [14]. In the frame which we work it becomes

$$f_\chi^{eq,s} = \frac{1}{e^{(\mathcal{E}_s^\chi - s\mu_\chi - \hbar s \chi \hat{\boldsymbol{p}} \cdot \boldsymbol{\omega}/2)/T} + 1}. \tag{5.17}$$

Here μ_χ denotes the chemical potentials of right- and left-handed particles which can be written in terms of the total and chiral chemical potentials as $\mu_{R,L} = \mu \pm \mu_5$. We are interested in calculating the vector current $\boldsymbol{j}_V = j_R + j_L$, and the axial current $\boldsymbol{j}_A = j_R - j_L$. By plugging (5.17) into (5.16) we observe that their calculations lead to the CME and CSE

$$\boldsymbol{j}_V^{CME} = \xi_B \boldsymbol{B}, \; \boldsymbol{j}_A^{CSE} = \xi_{B5} \boldsymbol{B},$$

where

$$\xi_B = \frac{Q\mu_5}{2\pi^2\hbar^2}, \quad \xi_{B5} = \frac{Q\mu}{2\pi^2\hbar^2}.$$

Moreover, the CVE and LPE are obtained as

$$\boldsymbol{j}_V^{CVE} = \xi \boldsymbol{\omega}, \; \boldsymbol{j}_A^{LPE} = \xi_5 \boldsymbol{\omega}.$$

where

$$\xi = \frac{\mu\mu_5}{\pi^2\hbar^2}, \quad \xi_5 = \frac{T^2}{6\hbar^2} + \frac{\mu^2 + {\mu_5}^2}{2\pi^2\hbar^2}.$$

They coincide with the ones presented in [15].

6. Conclusions

We reviewed how the fermionic fluid in the presence of the Coriolis force due to vorticity is represented by the scalar field b and the vector-field a_μ. We studied the equations of motion resulting from the action (3.1) which are shown to be equivalent to relativistic Euler equations. Within this method we expressed the field strength tensor of the field a_α in terms of the vorticity and enthalpy.

Then we showed that the invariance of the action of Dirac spinors under the gauge transformations yields the QKE satisfied by the Wigner function, (4.6). Hence, the original QKE [2] should be modified adequately to take into account the vorticity of fermionic fluids.

Chiral kinetic equation which results from the modified QKE is presented. We obtain the 3D CKT by integrating it over p_0. The resulting theory generates the anomalous effects correctly. It is consistent with the chiral anomaly.

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