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$\pi^{\pm}$-, π^{0}-meson electroweak decays within relativistic quark model based on point form of Poincaré-invariant quantum mechanics

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Abstract: In the course of work the integral representation of π^0-meson form-factor of $\pi^0 \to \gamma\gamma$ process has been obtained within relativistic quark model based on point form of Poincaré-invariant quantum mechanics taking into account anomalous magnetic moments of κ_u and κ_d quarks. The parameters obtained using pseudoscalar density constant $g_{\pi^\pm}$, current masses of u- and d-quarks as well as decay constant $\pi^\pm \to \ell^\pm \nu_\ell$ were used for numerical modeling of $F_{\pi^0\gamma}(t)$ form-factor in time-like region. Similar calculations for $\pi^0 \to \gamma\gamma$ decay characteristics with reference to quark structure in point form of Poincaré-invariant quantum mechanics have been carried out for the first time.

Keywords: Point form, form-factor, meson, quark, electroweak decay

1. Introduction

Due to the accuracy of modern experimental collaborations we can study electromagnetic characteristics of various short-lived mesons. Of particular interest are the mesons of the light sector. Such systems are relativistic [1], which makes it possible to study the mechanism of u- and d-quark interaction. Studying electroweak properties of light mesons in low-energy areas where perturbative QCD calculations are impossible plays an important role in bound quark systems research. The significance of π-mesons characteristics research is due to the wide range of highly accurate experimental data.

Among a number of approaches to describe two-particle quark systems that can be found in recent works [2–7], we have emphasized the models based on Poincaré group [8]. Despite the fact that Poincaré-invariant quantum mechanics (further PiQM) can be interpreted as an independent phenomenology, it can also be used in QCD research. The key feature of this approach is including the interaction operator in Poincaré algebra. Dirac has shown [9] that there is no definite operator division of the group and suggested three variants of creating kinematic (without interaction) and dynamic (with interaction) subgroup. That way in instant form of dynamics 3-momentum operator $\mathbf{P}$ and angular momentum $\mathbf{J}$ operator do not involve interaction, however, interaction is included in energy operator P^0, and as a result, in boosts operator. The latter complicates transition from one frame of reference to another. Kinematic subgroup contains maximum number of operators in light-

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front dynamics [10, 11] however, the interaction in angular momentum operator leads to rotational invariance violation. Interaction operator in point form of dynamics can be found only in 4-velocity $\mathbb{V}^\mu$ operator. The above-noted feature makes this form of dynamics the most obvious for compound quark systems description.

Currently light-front dynamics is mostly used for bound systems description. In this way in work [12] form-factor $\pi^0 \to \gamma\gamma^*$ decay study in different reference frames $q^+ = 0$ and $q^+ \neq 0$ has been carried out. In particular, it has been shown that in the system $q^+ \neq 0$ the model describes form-factor behavior most effectively in time-like region. However, existence of zero mode in this form of dynamics complicates forward calculation and requires involving additional relations.

Studying light-sector mesons in instant form of dynamics has been carried out in works [13, 14]. The authors of these works have shown that form-factors of $\pi^\pm$- and $\rho^\pm$-mesons can be calculated with different types of way functions including oscillatory. The distinctive feature of the calculations is using anomalous quarks magnetic moments. It should also be noted that expressions for form-factors obtained in the above-noted PiQM forms are in good agreement with equations of dispersion analysis whose research algorithms are also used for observable quark bound systems [4].

Quark compound systems research in point form of dynamics is represented to a lesser extent. Despite the advanced mathematical apparatus [15], the existence of mass operator in 4-velocity of bound system complicates forward calculation of model parameters significantly as QCD potential depends on a number of parameters (see for example [16, 17]). Theoretical calculations in the work [18] differ from the experimental data for different values of constituent quark masses. The authors have also shown that there is a discrepancy between the calculations of point form and Dirac point form of PiQM [19]. In the above-noted works, the absence of quark structure is assumed despite the fact that the approach is purely relativistic. Quark structure was assumed earlier in other models, including in the analysis of baryon magnetic moments [20, 21].

It should also be mentioned that the question of the order of basic model parameters is determined as significant in three forms of dynamics. For instance, variational principle for constituent quark masses determination was used in [12] for $\pi^0 \to \gamma\gamma^*$ form-factor calculation; in [22] model parameters were obtained on condition of $\pi^\pm$- and $K^\pm$-mesons root-mean-square radius compliance; in [23] various schemes of calculating model parameters from leptonic $\pi^\pm$- and $\rho^\pm$-meson decays were used. It follows that the development of models based on Poincaré group is associated with search for new calculation methods for basic model parameters.

The presented article is devoted to studying electromagnetic $\pi^\pm$- and π^0-meson form-factor in point form of PiQM. Basic relations of the model based on the constituent quark model and on point form of dynamics are presented in Section 2. The original technique for determining the model parameters is proposed in Section 3 using $f_{\pi^\pm}$ constant of pseudoscalar $\pi^\pm$-meson and pseudoscalar density constant $g_{\pi^\pm}$. The key feature of the proposed technique is using current quark masses for the determination of constituent masses u- and d–quark values. The obtained parameters were compared with the models based on instant form of PiQM and light-front dynamics, and their comparative analysis has been carried out as a result.

The procedure for calculating neutral pseudoscalar meson constant in a pair of photons is presented in Section 4. The proposed method for calculating is shown to lead to numerical evaluations of observables of $\pi^0 \to \gamma\gamma$ decay correlated with modern experimental data. As a result, the study of form-factor behavior $F_{\pi^0\gamma}(t)$ has been carried out in Section 5 taking into account anomalous magnetic moments of κ_u and κ_d quarks. The obtained results have been compared with experimental data of A2 [24] and N6A2 [25] collaboration in small transmitted momenta area, where the constituent quark model gives more reliable predictions [26].

2. Model description

The state vector of meson mass M, spin J and 4-momentum $Q=\{\omega_M(\mathrm{Q}),\mathbf{Q}\}$, $Q^2=M^2$ in point form of PiQM is defined below. For this purpose we shall define two-particle quark basis $|\mathbf{p}_1,\lambda_1,\mathbf{p}_2,\lambda_2\rangle$ with m_q, $m_{\bar{Q}}$ masses and $\mathbf{p}_1$, $\mathbf{p}_2$ momentums and spin projections λ_1, λ_2 respectively. Meson state vector in such basis is determined using relative momentum $\mathbf{k}$ [1] as

$$|\mathbf{Q},J\mu,M\rangle=\sum_{\lambda_1,\lambda_2}\sum_{\nu_1,\nu_2}\int \mathrm{d}\mathbf{k}\ \Phi^J_{\ell S}\left(\mathrm{k},\beta^I_{q\bar{Q}}\right)\sqrt{\frac{\omega_{m_q}(\mathrm{p}_1)\,\omega_{m_{\bar{Q}}}(\mathrm{p}_2)}{\omega_{m_q}(\mathrm{k})\,\omega_{m_{\bar{Q}}}(\mathrm{k})\,\mathbb{V}_0}}\times$$

$$\Omega\left(\begin{smallmatrix}\ell & S & J\\ \nu_1,\nu_2,\mu\end{smallmatrix}\right)(\theta_k,\varphi_k)\ D^{1/2}_{\lambda_1,\nu_1}(\mathbf{n}_{W_1})\ D^{1/2}_{\lambda_2,\nu_2}(\mathbf{n}_{W_2})\,|\mathbf{p_1},\lambda_1,\mathbf{p_2},\lambda_2\rangle\,. \tag{2.1}$$

For brevity, in expression (2.1) we use the following notation

$$\Omega\left(\begin{smallmatrix}\ell & S & J\\ \nu_1,\nu_2,\mu\end{smallmatrix}\right)(\theta_\mathrm{k},\varphi_\mathrm{k})=Y_{\ell m}(\theta_\mathrm{k},\varphi_\mathrm{k})\mathrm{C}\begin{pmatrix}s_1 & s_2 & S\\ \nu_1 & \nu_2 & \lambda\end{pmatrix}\mathrm{C}\left(\begin{smallmatrix}\ell & S & J\\ m & \lambda & \mu\end{smallmatrix}\right), \tag{2.2}$$

where $Y_{\ell m}(\theta_\mathrm{k},\varphi_\mathrm{k})$ are spherical functions defined be vector $\mathbf{k}$ angles, $\mathrm{C}\begin{pmatrix}s_1 & s_2 & S\\ \nu_1 & \nu_2 & \lambda\end{pmatrix}$, $\mathrm{C}\left(\begin{smallmatrix}\ell & S & J\\ m & \lambda & \mu\end{smallmatrix}\right)$ are Clebsch-Gordan coefficients of SU(2) group and $D^{1/2}_{\lambda,\nu}(\mathbf{n}_W)$ are Wigner rotations functions [8]. The wave function in (2.1) is subject to normalization condition as

$$\sum_{\ell,S}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\left|\Phi^J_{\ell S}\left(\mathrm{k},\beta^I_{q\bar{Q}}\right)\right|^2=1. \tag{2.3}$$

Using relation

$$\langle 0\left|\hat{J}^\mu\right|\mathbf{Q},M_P\rangle=\frac{i}{(2\pi)^{3/2}}\frac{Q^\mu}{\sqrt{2\ \omega_{M_P}(\mathrm{Q})}}f_P, \tag{2.4}$$

for pseudoscalar $P(q\bar{Q})$ meson and

$$\langle 0\left|\hat{J}^\mu\right|\mathbf{Q},\lambda_V,M_V\rangle=\frac{i}{(2\pi)^{3/2}}\frac{\varepsilon^\mu(\lambda_V)M_V}{\sqrt{2\ \omega_{M_V}(\mathrm{Q})}}f_V \tag{2.5}$$

for vector $V(q\bar{Q})$ meson respectively let us define the integral representation of leptonic decay constants in meson rest frame. Spinor part calculation of electroweak (ew) quark current

$$\langle 0|\hat{J}^\mu_{\mathrm{ew}}|\mathbf{k},\nu_1,-\mathbf{k},\nu_2\rangle=\frac{\bar{\upsilon}_{\nu_2}(-\mathbf{k},m_{\bar{Q}})\Gamma^\mu_{\mathrm{ew}}u_{\nu_1}(\mathbf{k},m_q)}{(2\pi)^3\sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{k})\ 2\omega_{m_q}(\mathrm{k})}} \tag{2.6}$$

with further integration over the solid angle of vector $\mathbf{k}$ leads to an integral representation of leptonic decay constant for pseudoscalar ($I = P$) and vector ($I = V$) meson in point form of PiQM (definitions and calculations are given in [27–29]):

$$f_I(m_q, m_{\bar{Q}}, \beta^I_{q\bar{Q}}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\ \Phi(\mathrm{k}, \beta^I_{q\bar{Q}})\times \tag{2.7}$$

$$\sqrt{\frac{W^+_{m_q}(\mathrm{k})\ W^+_{m_{\bar{Q}}}(\mathrm{k})}{M_0\ \omega_{m_q}(\mathrm{k})\ \omega_{m_{\bar{Q}}}(\mathrm{k})}}\left(1 + a_I\frac{\mathrm{k}^2}{W^+_{m_q}(\mathrm{k})W^+_{m_{\bar{Q}}}(\mathrm{k})}\right),\ a_P = -1,\ a_V = 1/3.$$

The following notations are used in the article

$$W^\pm_m(\mathrm{k}) = \omega_m(\mathrm{k}) \pm m,\ \omega_m(\mathrm{k}) = \sqrt{\mathrm{k}^2 + m^2},\ \mathrm{k} = |\mathbf{k}|\,,$$

with the determination of the invariant mass of two constituent quarks $M_0 = \omega_{m_q}(\mathrm{k}) + \omega_{m_{\bar{Q}}}(\mathrm{k})$ [1, 8].

Model parameters determination will be conducted using pseudoscalar density constant [28]

$$\left\langle 0 \left| \hat{J}_5 \right| \mathbf{Q}, M_P\right\rangle = -\frac{i}{(2\pi)^{3/2}}\frac{g_P}{\sqrt{2\ \omega_{M_P}(\mathrm{Q})}}, \quad \langle 0|\hat{J}_5|\mathbf{k}, \nu_1, -\mathbf{k}, \nu_2\rangle = \frac{\bar{\upsilon}_{\nu_2}(-\mathbf{k}, m_{\bar{Q}})\gamma_5 u_{\nu_1}(\mathbf{k}, m_q)}{(2\pi)^3\sqrt{2\omega_{m_{\bar{Q}}}(\mathrm{k})\ 2\omega_{m_q}(\mathrm{k})}}. \tag{2.8}$$

After similar spinor part calculations of (2.8) one can obtain an integral g_P representation in point form of PiQM [29]

$$g_P(m_q, m_{\bar{Q}}, \beta^P_{q\bar{Q}}) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^\infty \mathrm{dk}\ \mathrm{k}^2\frac{\Phi(\mathrm{k}, \beta^P_{q\bar{Q}})}{\sqrt{\omega_{m_q}(\mathrm{k})\ \omega_{m_{\bar{Q}}}(\mathrm{k})}}\times \tag{2.9}$$

$$\sqrt{M_0}\left(\sqrt{W^+_{m_q}(\mathrm{k})W^+_{m_{\bar{Q}}}(\mathrm{k})} + \sqrt{W^-_{m_q}(\mathrm{k})W^-_{m_{\bar{Q}}}(\mathrm{k})}\right).$$

The integral representations obtained in the section will be used to calculate the parameters of the model based on point form of dynamics.

3. Original technique for model parameters determination

The original method of model parameters determination in point form of dynamics is presented below. Using the experimental values of the decay constant $f^{(\mathrm{exp.})}_{\pi^\pm}$, pseudoscalar $\pi^\pm$-meson $M_{\pi^\pm}$ mass as well as the relations in (2.7) and (2.9) one has

$$\begin{cases} 1/2\,(\hat{m}_u + \hat{m}_d) = (3.45 \pm 0.4)\ \mathrm{MeV}, \\ f_P(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\pi^\pm}, \\ (\hat{m}_u + \hat{m}_d)\ \ g_P(m_u, m_d, \beta^P_{u\bar{d}}) = f^{(\mathrm{exp.})}_{\pi^\pm} M^2_{\pi^\pm}, \end{cases} \tag{3.1}$$

where $\hat{m}_u, \hat{m}_d$ are current quark masses [30]. Using oscillatory wave function

$$\Phi(\mathrm{k}, \beta^P_{q\bar{Q}}) = \frac{2}{\pi^{1/4}\left(\beta^P_{q\bar{Q}}\right)^{3/2}}\exp\left[-\frac{\mathrm{k}^2}{2\left(\beta^P_{q\bar{Q}}\right)^2}\right] \tag{3.2}$$

on condition of quality of constituent u-, d-quarks quark masses [12, 13, 18, 20, 23] lead to the following values of basic model parameters based on point form of PiQM:

$$m_{u,d} = 221.52 \pm 2.82 \text{ MeV}, \;\; \beta^P_{u\bar{u}} = \beta^P_{d\bar{d}} = \beta^P_{u\bar{d}} = 374.51 \pm 4.28 \text{ MeV}. \tag{3.3}$$

It should be noted that the reverse procedure was used in the work [23]: constituent quark masses values derived from leptonic $\pi^{\pm}$- and $\rho^{\pm}$-meson decays lead to current quark masses values obtained from MS-scheme calculations [30]. Let us compare the parameters in the proposed approach and the values in models based on instant and light-front forms of dynamics:

Table 1. Model parameters in different models, based on PiQM.

	Light-front dynamics [12]	Instant form of dynamics [13, 14]	This model
m_u, MeV	220	250 ± 5	221.52 ± 2.82
m_d, MeV	220	250 ± 5	221.52 ± 2.82
$\beta^P_{u\bar{d}}$, MeV	450	370 ± 20	374.51 ± 4.28

The analysis of Table 1 shows that the proposed procedure can be used to determine basic parameters of the model. The proposed model is used below to calculate the observables of $\pi^0 \to \gamma\gamma$ decay.

4. $\pi^0 \to \gamma\gamma$ decay in point form of PiQM

Parametrization of neutral pseudoscalar meson decay $P \to \gamma\gamma^*$ is presented with [12]

$$\langle \gamma\gamma^* \left| \hat{J}^\mu \right| \mathbf{Q}, M_P \rangle = e^2 \; F_{P\gamma}(t) \; \frac{i}{(2\pi)^{3/2}} \; \frac{\varepsilon^{\mu\nu\rho\sigma} \; Q_\nu \; \epsilon^*_\rho(\lambda^{real}) \; q^{virt.}_\sigma}{\sqrt{2\, \omega_{M_{P^0}}(\mathrm{Q})}}, \tag{4.1}$$

where $q^{real/virt.}$ are 4-momentums of real $(q^{real} \cdot q^{real}) = 0$ and virtual $(q^{virt.} \cdot q^{virt.}) = t$ photons ($\sqrt{t}$ is the transmitted to the leptonic pair momentum), and $\epsilon(\lambda^{real/virt.})$ correspond to polarization vectors. Since 4-velocity in point form of PiQM with and without interaction coincides, relation (4.1) in terms of meson velocity $\mathbb{V}^\nu = Q^\nu / M_0$ can be written as

$$\langle \gamma\gamma^* \left| \hat{J}^\mu \right| \mathbf{Q}, M_P \rangle = e^2 \; F_{P\gamma}(t) \; \frac{i}{(2\pi)^{3/2}} \; \frac{K^\mu(\lambda^{real})}{\sqrt{2\, \mathbb{V}_0}}, \tag{4.2}$$

where

$$K^\mu(\lambda^{real}) = \varepsilon^{\mu\nu\rho\sigma} \; \mathbb{V}_\nu \, \epsilon^*_\rho(\lambda^{real}) \, q^{virt.}_\sigma \, \sqrt{M_0}. \tag{4.3}$$

In our approach we consider the following mechanisms [31] for the decay of a neutral pseudoscalar meson into a pair of photons (see Figure 1).

Matrix elements in two-particle quark basis $|\mathbf{k}, \lambda_1, -\mathbf{k}, \lambda_2\rangle$ corresponding to Figure 1 taking into account pseudoscalar meson state vector (2.1) in meson rest system $\mathbf{Q} = \mathbf{0}$ will take the following

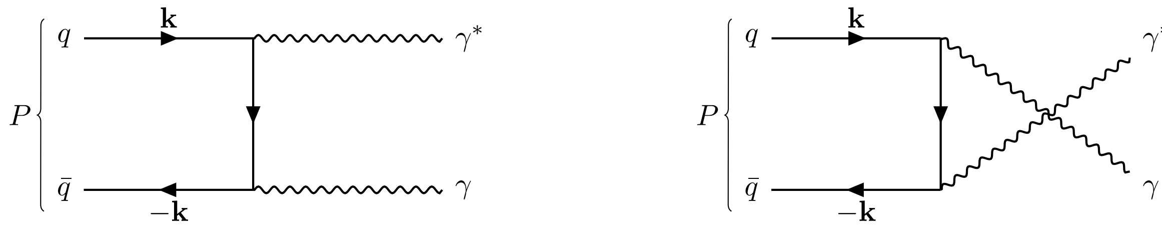

Figure 1. $P(q\bar{q}) \to \gamma\gamma^*$ decay mechanism in proposed model.

form

$$\langle \gamma\gamma^* \left| \hat{J}^{\mu}_{\text{ew}} \right| \mathbf{0}, M_P \rangle = \sum_{\lambda_1,\lambda_2} \int \mathbf{dk}\, \Omega \left({}^{0}_{\lambda_1,} {}^{0}_{\lambda_2,} {}^{0}_{0} \right)(\theta_{\mathrm{k}}, \varphi_{\mathrm{k}})\ \ \Phi(\mathrm{k}, \beta^{P}_{q\bar{q}}) \frac{1}{\sqrt{\mathbb{V}_0}} \left(M^{\mu}_{I}(\lambda_1, \lambda_2) + M^{\mu}_{II}(\lambda_1, \lambda_2) \right) \quad (4.4)$$

where

$$M^{\mu}_{I}(\lambda_1, \lambda_2) = \frac{1}{(2\pi)^3} \frac{\bar{\upsilon}_{\lambda_2}(-\mathbf{k}, m_{\bar{q}})}{\sqrt{2\,\omega_{m_{\bar{q}}}(\mathbf{k})}} \left(\Gamma_{e_{\bar{q}}} \cdot \epsilon^*(\lambda^{real}) \right) \frac{\hat{k} - \hat{q}^{virt.} + m_q}{(k - q^{virt.})^2 - m_q^2} \Gamma^{\mu}_{e_q} \frac{u_{\lambda_1}(\mathbf{k}, m_q)}{\sqrt{2\,\omega_{m_q}(\mathbf{k})}}, \quad (4.5)$$

$$M^{\mu}_{II}(\lambda_1, \lambda_2) = \frac{1}{(2\pi)^3} \frac{\bar{\upsilon}_{\lambda_2}(-\mathbf{k}, m_{\bar{q}})}{\sqrt{2\,\omega_{m_{\bar{q}}}(\mathbf{k})}} \Gamma^{\mu}_{e_{\bar{q}}} \frac{\hat{k} - \hat{q}^{real} + m_q}{(k - q^{real})^2 - m_q^2} \left(\Gamma_{e_q} \cdot \epsilon^*(\lambda^{real}) \right) \frac{u_{\lambda_1}(\mathbf{k}, m_q)}{\sqrt{2\,\omega_{m_q}(\mathbf{k})}}. \quad (4.6)$$

In relations (4.5) and (4.6) gauge-invariant electromagnetic vertex was defined taking into consideration constituent quark anomalous magnetic moment [32], namely

$$\Gamma^{\mu}_{e} = e \left(\gamma^{\mu} + i\,\kappa \frac{\sigma^{\mu\nu}}{2m}\, q_{\nu} \right),\ \ \sigma^{\mu\nu} = \frac{i}{2}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}) \quad (4.7)$$

where κ is quark anomalous magnetic moment defined from $\mu = e/2m\ (1+\kappa)$, and q_{ν} is vertex transmitted momentum. The substitution of (4.4) with (4.2) taking into account relation (4.5) and (4.6) leads to integral representation of pseudoscalar meson form-factor decay

$$F_{P\gamma}(t) = \frac{\sqrt{2}}{(2\pi)^{3/2}} \sum_{\lambda^{real}} \sum_{\lambda_1,\lambda_2} \int \mathbf{dk}\, \Omega \left({}^{0}_{\lambda_1,} {}^{0}_{\lambda_2,} {}^{0}_{0} \right)(\theta_k, \varphi_k)\ \ \Phi(\mathrm{k}, \beta^{P}_{q\bar{q}}) \left(K(\lambda^{real}) \cdot K^*(\lambda^{real}) \right)^{-1} \times \quad (4.8)$$

$$\left(\frac{\bar{\upsilon}_{\lambda_2}(-\mathbf{k}, m_{\bar{q}})}{\sqrt{2\,\omega_{m_{\bar{q}}}(\mathbf{k})}} \left(\Gamma_{e_{\bar{q}}} \cdot \epsilon^*(\lambda^{real}) \right) \frac{\hat{k} - \hat{q}^{virt.} + m_q}{(k - q^{virt.})^2 - m_q^2} \left(\Gamma_{e_q} \cdot K^*(\lambda^{real}) \right) \frac{u_{\lambda_1}(\mathbf{k}, m_q)}{\sqrt{2\,\omega_{m_q}(\mathbf{k})}} + \right.$$

$$\left. + \frac{\bar{\upsilon}_{\lambda_2}(-\mathbf{k}, m_{\bar{q}})}{\sqrt{2\,\omega_{m_{\bar{q}}}(\mathbf{k})}} \left(\Gamma_{e_{\bar{q}}} \cdot K^*(\lambda^{real}) \right) \frac{\hat{k} - \hat{q}^{real} + m_q}{(k - q^{real})^2 - m_q^2} \left(\Gamma_{e_q} \cdot \epsilon^*(\lambda^{real}) \right) \frac{u_{\lambda_1}(\mathbf{k}, m_q)}{\sqrt{2\,\omega_{m_q}(\mathbf{k})}} \right),$$

where

$$\left| \mathbf{q}^{virt.} \right| = \left| \mathbf{q}^{real} \right| = \frac{M_0^2 - t}{2M_0},\ \ K^{\mu}(\lambda^{real}) = \frac{M_0^2 - t}{2\sqrt{2}\sqrt{M_0}}\ \{0, i\,\lambda_{real}, 1, 0\}. \quad (4.9)$$

The relations that were obtained in the section allow us to get integral representation of $g_{\pi^0\gamma} = F_{\pi^0\gamma}(t \to 0)$ constant $\pi^0 \to \gamma\gamma$ decay. The calculation of spinor part in (4.8) taking into account kinematics decay (21) at $t \to 0$ after solid $\mathbf{k}$-angle integration leads to

$$g_{\pi^0\gamma} = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^\infty \mathrm{dk}\,\mathrm{k}^2\ \Phi(\mathrm{k},\beta^P_{u\bar{d}})\left(e_u^2\ \left(f_1(\mathrm{k},m_u)+\frac{\kappa_u}{2\ m_u}\ f_2(\mathrm{k},m_u)\right)-\right. \tag{4.10}$$

$$\left.-\ e_d^2\ \left(f_1(\mathrm{k},m_d)+\frac{\kappa_d}{2\ m_d}\ f_2(\mathrm{k},m_d)\right)\right),$$

where auxiliary functions are introduced

$$f_1(\mathrm{k},m)=\frac{m}{2\ \mathrm{k}\ \omega_m^{5/2}}\ \log\left(\frac{\omega_m(\mathrm{k})+\mathrm{k}}{\omega_m(\mathrm{k})-\mathrm{k}}\right), \tag{4.11}$$

$$f_2(\mathrm{k},m)=-\frac{1}{\mathrm{k}\ \omega_m^{5/2}}\left(2\ \mathrm{k}\ \omega_m(\mathrm{k})+m^2\log\left(\frac{\omega_m(\mathrm{k})+\mathrm{k}}{\omega_m(\mathrm{k})-\mathrm{k}}\right)\right)$$

along with $\pi^0 = (1/\sqrt{2})\ (u\bar{u}-d\bar{d})$ quark structure [30]. In expressions (4.10), (4.11) we neglect proportional summand κ_q^2 in view of smallness of constituent quarks anomalous magnetic moments [32]. Additionally, expression $f_1(\mathrm{k},m)$ correlates with calculations in quasipotential approach [31].

Let us compare the theoretical predictions of expression (4.10) and the experimental data, using model parameters from Section 3, the oscillatory wave function (3.2) and general expression for width decay $\pi^0 \to \gamma\gamma$

$$\Gamma = \frac{\pi}{4}\alpha^2\left|g_{\pi^0\gamma}\right|^2 M^3_{\pi^0}. \tag{4.12}$$

Table 2. Comparing model calculation with experimental $g_{\pi^0\gamma}$ data.

	Calculation by (4.10) without κ_q terms	Calculation by (4.10) with κ_u, κ_d terms	PDG [30]
$\lvert g_{\pi^0\gamma}\rvert$, GeV^{-1}	0.238 ± 0.005	0.272 ± 0.007	0.272 ± 0.003

The analysis of Table 2 shows that using quark anomalous magnetic moments leads to good agreement with the experimental data. Comparison of u- and d- quark magnetic moments obtained in the work with the values that were used to describe $V(P) \to P(V)\gamma$ decays and other models is given below:

Table 3. Quarks magnetic moments comparison.

	[27, 33]	[20]	[21]	This work
μ_u in nuclear magneton units μ_N	2.497	2.262	1.852	2.664
μ_d in nuclear magneton units μ_N	−1.326	−1.187	−0.972	−1.262

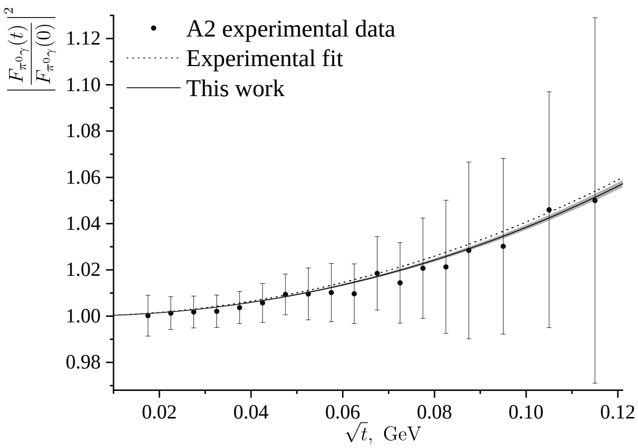

Figure 2a. The comparison of model calculation with A2 collaboration data [24].

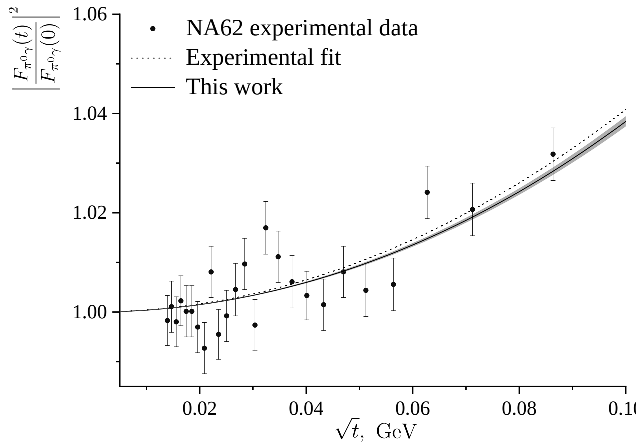

Figure 2b. The comparison of model calculation with NA62 collaboration data [25].

The results that are given in Table 3 show that anomalous quark κ_u and κ_u magnetic moments values that were evaluated from $\mu_{u,d} = e_{u,d}/2m_{u,d}\ (1+\kappa_{u,d})$ correlate with the values that were obtained from calculations of radiative decays in the proposed model [27, 33], as well as with the values obtained during baryonic magnetic moments analysis.

5. Form-factor $F_{\pi^0\gamma}(t)$ investigation

Studying form-factor decay $\pi^0 \to \gamma\gamma^*$ for small transmitted momenta will be conducted in this section. It should be noted that similar calculations were carried out in the light-front dynamics: in [12] it is shown that in system $q^+ \neq 0$ the experimental data fit well into small transmitted momenta area. There is no dependence on frame of reference choice in PiQM point form. That is why form-factor $F_{\pi^0\gamma}(t)$ investigation will be carried out in meson rest system.

Spinor part calculation of (4.8) taking into account kinematics of decay (4.9) leads to

$$F_{\pi^0\gamma}(t) = \sqrt{\frac{3}{2}}\frac{1}{\pi}\int_0^\infty \mathrm{dk\ k}^2\ \Phi(\mathrm{k}, \beta^P_{u\bar{d}})\left(e_u^2\ \left(\tilde{f}_1(\mathrm{k}, m_u, t) + \frac{\kappa_u}{2\ m_u}\ \tilde{f}_2(\mathrm{k}, m_u, t)\right) - \right. \tag{5.1}$$

$$\left. - \ e_d^2\ \left(\tilde{f}_1(\mathrm{k}, m_d, t) + \frac{\kappa_d}{2\ m_d}\ \tilde{f}_2(\mathrm{k}, m_d, t)\right)\right),$$

where

$$\tilde{f}_1(\mathrm{k}, m, t) = \frac{2\,m}{\mathrm{k}\ \omega_m^{1/2}(\mathrm{k})\ (4\,\omega_m^2(\mathrm{k}) - t)}\ \log\left(\frac{\omega_m(\mathrm{k}) + \mathrm{k}}{\omega_m(\mathrm{k}) - \mathrm{k}}\right), \tag{5.2}$$

$$\tilde{f}_2(\mathrm{k}, m, t) = -\frac{8}{\mathrm{k}\ \omega_m^{1/2}(\mathrm{k})\ (4\,\omega_m^2(\mathrm{k}) - t)}\ \left(\mathrm{k}\ \omega_m(\mathrm{k}) + \frac{1}{2}\ m^2\ \log\left(\frac{\omega_m(\mathrm{k}) + \mathrm{k}}{\omega_m(\mathrm{k}) - \mathrm{k}}\right)\right).$$

Relations (5.2) coincide with (4.11) at $t \to 0$. Substitution of model parameters from Section 3 in relation (5.1) with constituent quarks anomalous magnetic moments leads to obtaining form-factor $F_{\pi^0\gamma}(t)$ behaviour for different transmitted momenta. The analysis of Figure 2a and Figure 2b shows that the proposed model describes the experimental data of A2 and N6A2 collaborations successfully.

The results that were obtained in this section can be generalized in preparation for large transmitted momenta. The values of $t\, F_{\pi^0\gamma}(t)$ are shown to coincide for both time-like and space-like transmitted momentum at $t \to \infty$ in the work [12]. As a result, Brodsky-Lepage limit calculation $\lim_{t\to\infty} t\, F_{\pi^0\gamma}(t) = \sqrt{2}\, f_{\pi^\pm} = 0.185$ GeV [34] can be possible in the proposed approach. It has also been confirmed experimentally by Belle collaboration in $t > 15$ GeV^2 area [35]. Naïve calculation from relations (5.1) and (5.2) without involving dispersion analysis relations [4] leads to theoretical prediction of $\lim_{t\to\infty} t\, F_{\pi^0\gamma}(t) = 0.161$ GeV, which can be considered a good result from the authors' point of view due to the simplicity of the given decay mechanism (see Figure 1). It should be noted that the result is sensitive to quark structure: calculation of relation (5.1) without $\kappa_{u,d}$ proportional summands leads to the following result: $\lim_{t\to\infty} t\, F_{\pi^0\gamma}(t) = 0.2$ GeV, which confirms the existence of constituent quarks structure indirectly.

6. Conclusion

This work is dedicated to light $\pi^\pm$- and π^0-mesons electromagnetic decays studying in point form of PiQM. In the course of work the authors determine the values of quarks constituent masses and the parameters of the wave functions with the following neutral π^0-meson decay constant calculation. The original method for calculating model parameters was used. It has been shown that taking into account quarks structural characteristics leads to constant $g_{\pi^0\gamma}$ numerical value that coincides with the experimental data. The calculation technique is applied to the case of various transmitted momenta to virtual γ^*-quantum. It has been demonstrated that at small transmitted momenta area where rescattering effects are insignificant [26] the proposed model, based on point form of dynamics describes the experimental data of A2 and NA62 collaborations satisfactorily.

In conclusion it should be noted that the presented model describes form-factors of radiative $V(P) \to P(V)\gamma$ decays, $\rho^\pm$-meson magnetic momentum, and electromagnetic characteristics of light sector mesons well [33]. The calculation technique can also be used for η/η'-mesons form-factors study, as well as for pseudoscalar meson $P^\pm(q\bar{Q}) \to \ell^\pm \nu_\ell \gamma$ axial and vector form-factor calculation.

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