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High-Power Diffusion Cooling Planar Waveguide CO_2 Laser

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Abstract

A diffusion-cooled CO_2 laser using a coaxial waveguide is investigated theoretically. The resonator extracting the laser beam consits of two annular plane mirrors enclosing the two ends of the waveguide. A theoretical resonator model based on the vector modes of propagation in a dielectric coaxial waveguide is described.

1. Introduction

Laser action of carbon dioxide (CO_2) was reported for the first time by Patel in 1964 [1]. The principles of a waveguide gas laser were first discussed by Marcatili and Schmeltzer (1964) and waveguide operation of He- Ne was reported by Smith (1971) [2]. Ordinarily, operation of a laser in an extremely small bore ($\leq$ 1mm) tube greatly enhances the radiation loss due to diffraction.

However, when the tube is constructed in the form of a dielectric waveguide diffraction losses are minimized and advantage may be taken of the proximity of the walls of the discharge tube to reduce the gas temperature and to facilitate deexcitation of molecular species through collision with the walls. These factors result in the possibility of operating at high pressure with an attendant increase in gain, power output per unit volume, linewidth and saturation intensity [3,4,5]. CO_2 waveguide gas laser has the high specific output power that can be obtained from a very small device. A maximum specific power of 0.85 W/cm^2 obtained so far has been reported by the group of Hall [6,7,8].

2. Theory and Results

High-power diffusion-cooled planar waveguide lasers have recently been investigated [9,10]. This work presents a theoretical analysis of a coaxial waveguide laser. A Schematic view of the laser resonator is shown in Figure 1.

An annular beam is guided within the coaxial waveguide and reflected by two annular mirrors attached close to the waveguide ends [11].

For a theoretical analysis of this resonator the modes of propagation in a coaxial waveguide are essential. Metallic guide materials and dielectrics are covered by the same mathematical approach because they are both described by their complex refraction index. Due to vector boundary conditions at the guide surfaces the radiation has to be treated with all its field compenents. Earlier investigations of dielectric coaxial waveguides were conducted in order to use these waveguides for optical transmission line purposes. Marcatili gives approximative solutions for the lowest azimuthal order mode [12]. V. A. Pruzhanovskii obtained solutions for modes of radial and azimuthal orders less than 2 using rather complex analytical approximations [13].

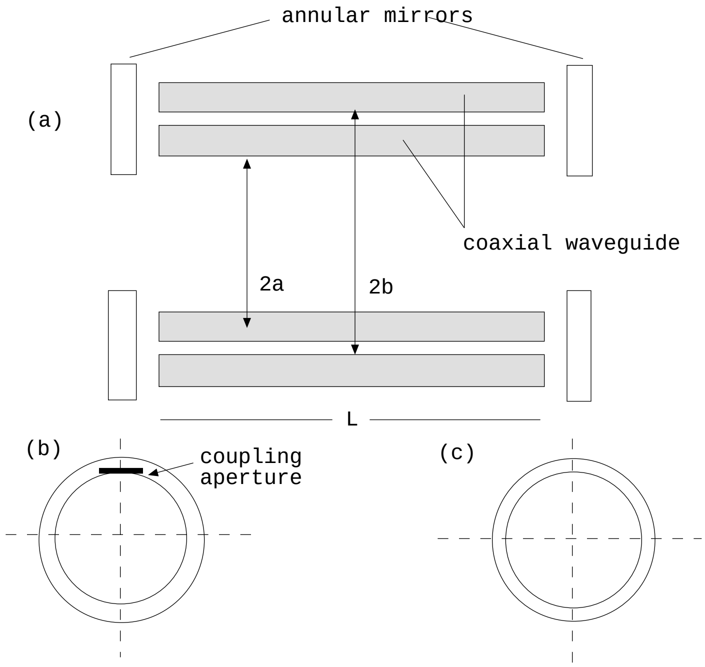

Figure 1. Schematic of waveguide resonator. Two annular plane mirrors enclose the coaxial waveguide (a). The two mirrors (b, c) are attached close to the guide ends in order to avoid free space propagation losses. One mirror (b) is provided with a coupling aperture [10].

The electromagnetic modes in a coaxial waveguide are solutions of the Maxwell equations satisfying boundary conditions at the guide surfaces. The dielectric properties of the guide walls are given by their complex refraction index $n = \sqrt{\varepsilon\mu/\varepsilon_0\mu_0}$. The waveguide consists of an inner and outer guide wall with refraction index n_1 and n_2. As the thickness of the guide wall is much larger than the optical penetration depth the outer wall is assumed to be of infinite thickness and the inner wall is treated like a massive cylinder. Laser gas with refraction index n_3 fills the gap.

The electromagnetic modes are assumed to be monochromatic and to be eigenfunctions of the propagation in z-direction with fixed azimuthal symmetry L:

$$E(r, \varnothing, z, t) = E(r)\exp[i(Bz + L\varnothing - wt)] \tag{1}$$

$$E(r, \varnothing, z, t) = H(r)\exp[i(Bz + L\varnothing - wt)]. \tag{2}$$

Waves traveling in positive z-direction have a positive real part of the axial propagation constant B while damping is given by a positive imaginary part [10, 14]. The field components are coupled by the Maxwell equation and a convenient approach in waveguide theory is to express the fields E and H by their independent z components. With $n^2k^2 = w^2$, we have the components

$$E_r = \frac{iB}{n^2K^2 - B^2}\left[\frac{\partial E_z}{\partial r} + \frac{Mw}{B}\frac{1}{r}\frac{\partial Hz}{\partial \varnothing}\right] \tag{3}$$

$$E_\varnothing = \frac{iB}{n^2K^2 - B^2}\left[\frac{1}{r}\frac{E_z}{\partial \varnothing} - \frac{Mw}{B}\frac{\partial Hz}{\partial r}\right] \tag{4}$$

$$-H_r = \frac{iB}{n^2K^2 - B^2}\left[\frac{n^2K^2}{MwB}\frac{1}{r}\frac{\partial E_z}{\partial \varnothing} + \frac{\partial Hz}{\partial r}\right] \tag{5}$$

$$H_\varnothing = \frac{iB}{n^2K^2 - B^2}\left[\frac{n^2K^2}{MwB}\frac{\partial E_z}{\partial r} - \frac{1}{r}\frac{\partial Hz}{\partial \varnothing}\right]. \tag{6}$$

The Maxwell equations or, equivalently, the wave equation for the field components E_z and H_z must be solved [10].

Using representation (1)-(2) the wave equation in cylindrical coordinates for the z-components reads

$$\left[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})\frac{L^2}{r^2} + n^2K^2 - B^2\right]\Psi = O. \tag{7}$$

The solution of (7) consists of two linearly independent parts:

$$\Psi(r) = AJ_L(K_r) + BN_L(K_r r) \tag{8}$$

with

$$K_r^2 = n^2K^2 - B^2, \tag{9}$$

where J_L is the Bessel function whereas N_L is the Neumann function.

An alternative set of linearly independent solutions is given by the Hankel functions:

$$H_L^{(1)} = J_L + iN_L \tag{10}$$

$$H_L^{(2)} = J_L - iN_L. \tag{11}$$

The asymptotic behavior of these functions for $Z = K_r r \to \infty$ is helpful for dropping physically meaningless solutions:

$$J_L(z) \to \sqrt{\frac{2}{\pi Z}} \cos(z - L\pi/2 - \pi/4) \tag{12}$$

$$N_L(z) \to \sqrt{\frac{2}{\pi Z}} \sin(z - L\pi/2 - \pi/4) \tag{13}$$

$$H_L^{(1)} \to \sqrt{\frac{2}{\pi Z}} \exp[i(z - L\pi/2 - \pi/4)] \tag{14}$$

$$H_L^{(2)} \to \sqrt{\frac{2}{\pi Z}} \exp[-i(z - L\pi/2 - \pi/4)]. \tag{15}$$

J_L and N_L asymptotically represent standing waves while $H_L^{(2)}$ and $M_L^{(2)}$ are out ward and inward traveling waves, respectively. Since there is no radiation traveling from outer sources towards the origin the only possible solution for the outer area is $H_L^{(1)}$. Apart from J_L all these functions are singular at the origin $r = o$. The only acceptable solution for the inner areas is therefore J_L. The fields in the three areas of the waveguide must have the general form [10].

$$E_z^{(1)} = AJ_L(K_r 1r) \exp[i(B_1 z + L\varnothing - wt] \tag{16}$$

$$H_z^{(1)} = BJ_L(K_r 1r) \exp[i(B_1 z + L\varnothing - wt] \tag{17}$$

$$E_z^{(3)} = [(J_L(K3r) + DN_L(kr^3)] \exp[i(B_3 z + L\varnothing - wt)] \tag{18}$$

$$H_z^{(3)} = [FJ_L(K_r 3r) + GN_L(kr^3)] \exp[i(B_3 z + L\varnothing - wt)] \tag{19}$$

$$E_z^{(2)} = IH_L^{(1)}(K_r 2r) \exp[i(B_2 z + L\varnothing - wt)] \tag{20}$$

$$H_z = KH_K^{(1)}(K_r 2r) \exp[i(B_2 z + L\varnothing - wt)], \tag{21}$$

with unknown coefficients A, B, C, D, F, G, I and K [12].

The tangential components E and H have to be continuous in the absence of free currents and changes at the boundaries of two different media.

These boundary conditions determine the 8 unknown coefficients A, B, C, D, F, G, I and K. Since the boundary conditions have to be satisfied for all axial coordinates z the equality $B_1 = B_2 = B_3 = B$ follows. The wave equation (7) provides relations between the different propagation constants

$$K_{r^1}^2 = n_1^2 K^2 - B^2 \tag{22}$$

$$K_{r^2}^2 = n_2^2 K^2 - B^2 \tag{23}$$

$$K_{r^3}^2 = n_3^2 K^2 - B^2. \tag{24}$$

The axial propagation constant B determines all the radial propagation constants K_r. Application of the boundary conditions leads to a linear set of equations for the 8 unknown coefficients A, B, C, D, F, G, I and K. The coefficients of the linear equations thus form an 8X8 matrix M(B).

A nontrivial solution exists when the determinant of this matrix M(B) is zero. The zeros of the determinant [M (B)] determine discrete propagation constants B. If there is no degeneracy, each B corresponds to a certain transversal field distribution given by E_z and H_z and A_z through (3)-(6). The determinant will be different for each azimuthal order L and the zeros represent the propagation constants of different radial modes for fixed L. The matrix and determinant are explicitely derived in what follows. The coefficients of (16)-(21) (A, B, C, D, F, G, I, K) are determined by the continuity of the tangential components of H and E at the boundaries. Continuity of E_z in a and b gives:

$$AJ_L(K_r 1a) = CJ_L(K_r 3a) + DN_L(K_r 3a) \tag{25}$$

$$IH_L^{(1)}(K_r 2b) = CJ_L(K_r 3b) + DN_L(K_r 3b). \tag{26}$$

Continuity of H_z in a and b gives:

$$BJ_L(K_r 1a) = FJ_L(K_r 3a) + GN_L(K_r 3a) \tag{27}$$

$$KH_L^{(1)}(K_r 2b) = FJ_L(K_r 3b). \tag{28}$$

Continuity of $E_\varnothing$ in a and b gives:

$$\frac{1}{K_{r^1}^2}\left[A\frac{iL}{a}(J_L K\frac{1a}{a}) - \frac{Mw}{B}B\frac{\partial J_L(K_r,1a)}{\partial r}\right] = \frac{1}{K_{r^3}^2}[\frac{iL}{a}(C(J_L(K_{r^3}a) + DN_L(K_r^3 a)) - \frac{Mw}{B}(F\frac{\partial J_L(K_r^3 a)}{\partial r} + (G\frac{\partial N_L}{\partial r}(K_r^3 a)], \tag{29}$$

$$\frac{1}{K_{r^2}^2}\left[I\frac{iL}{b}H_L^{(1)}(K_r^3 b) - \frac{Mw}{B}K\frac{\partial H_L^{(1)}(K_r^2 b)}{\partial r}\right] = \frac{1}{K_{r^3}^2}\left[\frac{iL}{b}(C(J_L(K_r^3 b) + \frac{Mw}{B} DN_L(K_r^3 b)) - \frac{Mw}{B}(F\frac{\partial J_L(K_r^3 b)}{\partial r} + G\frac{\partial N_L(K_r^3 b)}{\partial r})\right]. \tag{30}$$

Continuity of $H_\varnothing$ in a and b gives:

$$\frac{1}{K_{r^1}^2}[A\frac{n_1^2 K_L^2}{MwB}\frac{\partial J_L(K_r^1 a)}{\partial r} + B\frac{iL}{a}J_L(K_r^1 a)] = \frac{1}{K_{r^3}^2}\frac{n_3^2 K^2}{MwB}(C\frac{\partial J_L(K_r^3 a)}{\partial r} + D\frac{\partial N_L(K_r^3 a)}{\partial r}) + \frac{iL}{a}(FJ_L(K_r^3 a) + GN_L(K_r^3 a))], \tag{31}$$

$$\frac{1}{K_{r^2}^2}[I\frac{n_2^2K^2}{MwB}\frac{\partial H_L^{(1)}(K_r^{(2)}b)}{\partial r}+K\frac{iL}{b}H_L^{(1)}(K_r^2b)]=\frac{1}{K_{r^3}^2}\frac{n_3^2K^2}{KwB}((\frac{\partial J_L(K_r^3b)}{\partial r}+D\frac{\partial N_L(K_r^3b)}{\partial r}$$
$$+\frac{iL}{b}(FJ_L(K_r^3b)+GN_L(K_r^3b))] \tag{32}$$

where $\frac{\partial J_L(Kr)}{\partial r}$ has been replaced by $\frac{\partial J_L(Ka)}{\partial r}$.

The notation is further simplified by setting $u = K_r1a$, $W = K_r3a$, $S = K_r3b$, $t = K_r2b$, $J_L = J$, $N_L = N$, and $H_L^{(H)} = H$ denotes a derivative with respect to U, W, S and t, respectively.

The set of equations (24)-(31) yields the coefficient matrix

$$M(B)=\begin{vmatrix} J(U) & 0 & -J(W) & -N(W) & 0 & 0 & 0 & 0 \\ 0 & 0 & -J(S) & -N(S) & 0 & 0 & H_{(t)} & 0 \\ 0 & -J(U) & 0 & 0 & -J(W) & -N(W) & 0 & 0 \\ 0 & 0 & 0 & 0 & -J(S) & -N(S) & 0 & H(t) \\ \frac{iIB}{U^2}J(U) & \frac{-Mw}{U}J(U) & \frac{iIB}{W^2}J'(W) & \frac{-iIB}{W^2}N(W) & \frac{Mw}{W}J^1(W) & \frac{Mw}{W}N^1(W) & 0 & 0 \end{vmatrix}$$

determining the coefficients A, B, C, D, F, G, I, K via

$$M(B).\begin{vmatrix} A \\ B \\ C \\ D \\ F \\ G \\ I \\ K \end{vmatrix}=0. \tag{33}$$

The calculation of the determinant of this matrix is very sensitive to round off errors giving rise to numerical instabilities. This is due to the large imaginary part of K_{r^2} and K_{r^1} representing the strong and nearly exponential damping of radiation that penetrates the guide surfaces. This imaginary part leads either to extremely large or extremely small values of $J_L(K_r1a)$, H_L (K_r2b) and their derivatives. Errors due to the corresponding extreme coefficients are removed by an asymptotical evaluation of the ratio

$$\frac{H'(K_r2b)}{H(K_r2b)}=\frac{i\exp[i(K_r2b-\frac{L\pi}{2}-\frac{\pi}{4})]}{\exp[i(K_r2b-\frac{L\pi}{2}-\frac{\pi}{4})]}=i \tag{34}$$

This approximation was used earlier by Marcatili deriving the waveguide modes in a hollow bore waveguide [12, 10].

Since the guide material is absorbing K_{r^1} is complex with a positive imaginary part, thus $J_L(K_r1a)$ splits into an exponentially decreasing ($H_L^{(1)}$) and increasing ($H_L^{(2)}$) part:

$$J_L(K_r1a)=\frac{1}{2}\left[H_L^{(1)}K_r^{(1)}+H_L^{(2)}(K_L1a)\right]. \tag{35}$$

Neglecting the decreasing part one can likewise evaluate the ratio

$$\frac{J^1(K_r^1 a)}{J(K_r^1 a)} = \frac{H_L^{(2)}(K_r^1 a)}{H_L^{(2)}(K_r^1 a)} = \frac{-i\exp[-i(K_r^1 a - \frac{L\pi}{2} - \frac{\pi}{4})]}{\exp[-i(K_r^1 a - \frac{L\pi}{2} - \pi/4)]} = -i. \tag{36}$$

Division of the columns of a determinant does not change its zeros and the asymptotic approximations (33) and (35) are used to eliminate the undesired coefficients giving the approximate determinant equation:

$$0=\begin{vmatrix} 1 & 0 & -J(W) & -N(W) & 0 & 0 & 0 & 0 \\ 0 & 0 & -J(S) & -N(S) & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -J(W) & -N(W) & 0 & 0 \\ 0 & 0 & 0 & 0 & -J(S) & -N(S) & 0 & 1 \\ \frac{iLB}{U^2} & \frac{iMW}{U} & \frac{iLB}{W^2}J(W) & \frac{-iLB}{W^2}N(W) & \frac{MW}{W}W^1(W) & \frac{MW}{W}N^1(W) & 0 & 0 \\ 0 & 0 & \frac{-iLB}{W^2}J(S) & \frac{-iLB}{S^2}N(S) & \frac{MW}{S}J^1(S) & \frac{iLB}{W^2}N(S) & \frac{iLB}{t^2} & \frac{-iMW}{t} \\ \frac{in_L^2K^2}{MWU} & \frac{iLB}{U^2} & \frac{n^2K^2}{MWW}J^1(W) & \frac{n_3^2}{MWW}N^1(W) & \frac{iLB}{W^2}J(W) & \frac{iLB}{W^2}N(W) & 0 & 0 \\ 0 & 0 & \frac{n_3^2K^2}{MWS}J^1(S) & \frac{n_3^2K^2}{MWS}N^1(S) & \frac{iLB}{S^2}J(S) & \frac{iLB}{S^2}N(S) & \frac{n_3^2K^2}{MWt} & \frac{-iLB}{t^2} \end{vmatrix}. \tag{37}$$

3. Discussion and Conclusion

In the present work have investigated a theoretical resonator model based on the vector modes of propagation in a dielectric coaxial waveguide. High-power diffusion cooling planar waveguide CO_2 laser is described theoretically.

References

[1] C. K. N. Patel, *Phys. Rev.,***136** (1964) 1787.

[2] W. W. Duley, “CO_2 Lasers”, York University, Canada., (1976) 20-25.

[3] W. J. Witteman, *Appl. Phys. Lett.,***11** (1967) 337-8.

[4] W. J. Witteman and H. W. Werner, *Phys. Lett.,* **11** (1968) 454-5.

[5] R. Malz, “Numerical Studies of the CO_2 Waveguide Laser: Application of a Numerical Model to Explain Experimental Results and Conclusions for Laser Efficiency and Lifetime” J. Phys D: Appl. Phys., **27**, (1994) 698-706.

[6] D. He and D. R. Hall, *Appl. Phys., Lett.,* **43** (1983) 726.

[7] D. R. Hall, K. M. Abramski, and A. D. Colley, *Appl. Phys. Lett.,***54** (1989) 1833.

[8] W. J. Witteman, M. B. Heeman-IIıeva, “RF Excited 1.1 W/cm^2 Waveguide CO_2 Laser”, *J. Appl. Phys.,* **74** (1993) 7.

[9] P. E. Jackson, H. J. Baker and D. R. Hall, “CO_2 Large-Area Discharge Laser Using an Unstable-Waveguide Hybrid Resonator.” *Appl. Phys. Lett.,* **54** (1989) 1950-1952.

[10] D. Ehrlichmann, U. Habich, “High-Power CO_2 Laser with Coaxial Waveguide and Diffusion Cooling”. IEEE Journal of Quantum Electronics, **29** (1993) 2211-2219.

[11] J. J. Chang and W. H. Christiansen, “Waveguide CO_2 Lasers Pumped by Broad-Band Radiation”. IEEE Journal of Quantum Electronics, Vol. 29, No: **5**, (1993) 1412-20.

[12] E. A. Marcatili, “Light Transmission in a Multiple Dielectric Guide”. Bell. Syst. Tech. J., **45** (1966) 97-103.

[13] V. A. Pruzhavovskii, “Asymmetric Modes in a Metal Coaxial Waveguide”, **Sov. Phys-Tech. Phys., 15** (1970) 897-900.

[14] J. A. Stratton, “Electromagnetic Theory”, (New York: McGraw-Hill, 1991).