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A Mean Field Model With Two Order Parameters for Three-Phase Coexistence Near the Tricritical Point

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Abstract

This study gives a mean field model with two order parameters for three-phase coexistence near the multicritical point. The critical exponents calculated from our model are the tricritical exponents for the order parameters, susceptibility and the specific heat. Hence, our mean field model describes adequately the tricritical behaviour of a system in the region of three-phase coexistence.

1. Introduction

The mean field theory of symmetrical and unsymmetrical tricritical points with one order parameter for three-phase coexistence has been studied by Griffiths [1]. He applied his theory to fluid mixtures [1] and liquid mixtures [2]. Similar treatment with one order parameter has been applied to liquid crystals for the nematic-smectic A tricritical phase transition [3,5].

It has been suggested in the literature that one can study a system exhibiting three different phases with two order parameters. Those systems are, for example, liquid crystals with the nematic, smectic A and smectic C phases near the NAC point [6-9]; a ferroelectric system such as sodium nitrite with the paraelectric, incommensurate and ferroelectric phases [10]; ferroelectric liquid crystals with the paraelectric and anti-ferroelectric phases [11], and the ammonium halides with the disordered β, anti-ferroordered γ and ferro-ordered δ phases [12].

In this study we have extended Griffiths theory with one order parameter to a mean field model with two order parameters for three-phase coexistence near the multicritical point. The free energy we give in this study has the most general form that one can obtain from a system with three different phases. From this form of free energy we calculate the critical exponents for the order parameters, susceptibility and the specific heat. It turns out that these critical exponents are the tricritical exponents. In Section 2 we give the form of the free energy of our model and we calculate the critical exponents. In Section 3 we give a brief discussion of our calculations. Finally in Section 4 we give our conclusions.

2. Theory

In this section we study a mean field model for three-phase coexistence near the multicritical point. We assume that the system is characterized with two order parameters Ψ and η.

The free energy of our model has three components. The contribution due to the first order parameter Ψ is given by

$$\begin{aligned} F_1 = &(\Psi-\Psi_\alpha)(\Psi-\Psi_\beta)(\Psi-\Psi_\gamma)^4 \\ &+(\Psi-\Psi_\alpha)(\Psi-\Psi_\beta)^2(\Psi-\Psi_\gamma)^3 \\ &+(\Psi-\Psi_\alpha)(\Psi-\Psi_\beta)^3(\Psi-\Psi_\gamma)^2 \\ &+(\Psi-\Psi_\alpha)(\Psi-\Psi_\beta)^4(\Psi-\Psi_\gamma) \\ &+(\Psi-\Psi_\alpha)^2(\Psi-\Psi_\beta)(\Psi-\Psi_\gamma)^3 \\ &+(\Psi-\Psi_\alpha)^2(\Psi-\Psi_\beta)^2(\Psi-\Psi_\gamma)^2 \\ &+(\Psi-\Psi_\alpha)^2(\Psi-\Psi_\beta)^3(\Psi-\Psi_\gamma) \\ &+(\Psi-\Psi_\alpha)^3(\Psi-\Psi_\beta)(\Psi-\Psi_\gamma)^2 \\ &+(\Psi-\Psi_\alpha)^3(\Psi-\Psi_\beta)^2(\Psi-\Psi_\gamma) \\ &+(\Psi-\Psi_\alpha)^4(\Psi-\Psi_\beta)(\Psi-\Psi_\gamma) \end{aligned} \tag{1a}$$

The contribution to the free energy from the order parameter η is given by

$$\begin{aligned} F_2 = &(\eta-\eta_\alpha)(\eta-\eta_\beta)(\eta-\eta_\gamma)^4 \\ &+(\eta-\eta_\alpha)(\eta-\eta_\beta)^2(\eta-\eta_\gamma)^3 \\ &+(\eta-\eta_\alpha)(\eta-\eta_\beta)^3(\eta-\eta_\gamma)^2 \\ &+(\eta-\eta_\alpha)(\eta-\eta_\beta)^4(\eta-\eta_\gamma) \\ &+(\eta-\eta_\alpha)^2(\eta-\eta_\beta)(\eta-\eta_\gamma)^3 \\ &+(\eta-\eta_\alpha)^2(\eta-\eta_\beta)^2(\eta-\eta_\gamma)^2 \\ &+(\eta-\eta_\alpha)^2(\eta-\eta_\beta)^3(\eta-\eta_\gamma) \\ &+(\eta-\eta_\alpha)^3(\eta-\eta_\beta)(\eta-\eta_\gamma)^2 \\ &+(\eta-\eta_\alpha)^3(\eta-\eta_\beta)^2(\eta-\eta_\gamma) \\ &+(\eta-\eta_\alpha)^4(\eta-\eta_\beta)(\eta-\eta_\gamma). \end{aligned} \tag{1b}$$

The third contribution to the free energy due to the coupling between two order parameters Ψ and η is given by

$$F_3 = (\Psi-\Psi_\alpha)(\Psi-\Psi_\beta)(\Psi-\Psi_\gamma)(\eta-\eta_\alpha)(\eta-\eta_\beta)(\eta-\eta_\gamma). \tag{1c}$$

Hence, the free energy of our model is

$$F = F_1 + F_2 + F_3. \tag{1d}$$

Here we denote the three phases as α, β and γ. Ψ_α, Ψ_β and Ψ_γ are the values of the order parameter Ψ in the phases α, β and γ, respectively. Also η_α, η_β and η_γ are the values of the order parameter η in the phases α, β and γ, respectively. This free energy is equal to zero when $\Psi = \Psi_\alpha$ and $\eta = \eta_\alpha$ for the α phase; $\Psi = \psi_\beta$ and $\eta = \eta_\beta$ for the β phase; and $\Psi = \Psi_\gamma$ and $\eta = \eta_\gamma$ for the γ phase. Therefore, this free energy is valid in the region of three-phase coexistence.

This free energy given in Eq.(1d) can be rewritten as

$$\begin{aligned} F = {} & a_0 + a_1\Psi + a_2\Psi^2 + a_3\Psi^3 + a_4\Psi^4 + a_5\Psi^5 + a_6\Psi^6 \\ & + b_1\eta + b_2\eta^2 + b_3\eta^3 + b_4\eta^4 + b_5\eta^5 + b_6\eta^6 \\ & + c_1\Psi\eta + c_2\Psi^2\eta + c_3\Psi\eta^2 + c_4\Psi^2\eta^2 + c_5\Psi^3\eta \\ & + c_6\Psi\eta^3 + c_7\Psi^2\eta^3 + c_8\Psi^3\eta^2 + c_9\Psi^3\eta^3. \end{aligned} \tag{2}$$

Here, the coefficients $a_i (i = 1, 2, 3, 4, 5, 6)$, $b_i (i = 1, 2, 3, 4, 5, 6)$ and $c_i (i = 1, 2, 3, 4, 5, 6, 7, 8, 9)$ in terms of Ψ_i, η_i $(i = \alpha, \beta, \gamma)$ are given in Appendix A. As we see from Eq.(A5), a_5 is linearly proportional to Ψ_i. We assume that the temperature dependence of a_5 is

$$a_5 = a_{50}(T - T_c)^x.$$

Here, T_c denotes the critical temperature. From the Landau theory, the temperature dependence of a_2 should be

$$a_2 = a_{20}(T - T_c).$$

Therefore, from Eqs.(A2) and (A5) we conclude that the power x should be 1/4. Hence, using Eqs.(A0-A6) the temperature dependence of the a_i's should be as follows:

$$\begin{aligned} a_0 &\sim |T - T_c|^{3/2}, \\ a_1 &\sim |T - T_c|^{5/4}, \\ a_2 &\sim |T - T_c|, \\ a_3 &\sim |T - T_c|^{3/4}, \\ a_4 &\sim |T - T_c|^{1/2}, \\ a_5 &\sim |T - T_c|^{1/4}. \end{aligned} \tag{3}$$

Similarly, using Eqs.(A7-A12) the temperature dependence of the b_i's are given below:

$$
\begin{aligned}
b_1 &\sim |T - T_c|^{5/4} \\
b_2 &\sim |T - T_c| \\
b_3 &\sim |T - T_c|^{3/4} \\
b_4 &\sim |T - T_c|^{1/2} \\
b_5 &\sim |T - T_c|^{1/4}
\end{aligned} \tag{4}
$$

Using Eqs.(A13-A21) the temperature dependence of the coefficient c_i's can be found as

$$
\begin{aligned}
c_1 &\sim |T - T_c| \\
c_2 &\sim |T - T_c|^{3/4} \\
c_3 &\sim |T - T_c|^{3/4} \\
c_4 &\sim |T - T_c|^{1/2} \\
c_5 &\sim |T - T_c|^{1/2} \\
c_6 &\sim |T - T_c|^{1/2} \\
c_7 &\sim |T - T_c|^{1/4} \\
c_8 &\sim |T - T_c|^{1/4} .
\end{aligned} \tag{5}
$$

Since Ψ_i and η_i are linearly dependent on a_5 and b_5, respectively, the temperature dependences of the order parameter Ψ_i and η_i are given as

$$\Psi_i \sim |T - T_c|^{\beta} \tag{6}$$

and

$$\eta_i \sim |T - T_c|^{\beta} , \tag{7}$$

where the critical exponent for the order parameter is $\beta = 1/4$.

Now, we want to find the critical exponent γ for the susceptibility χ. For this purpose we write the definition of the susceptibility χ_i $(i = \alpha, \beta, \gamma)$ as

$$\chi_i^{-1} = \left(\frac{\partial^2 F}{\partial \Psi^2}\right)_{\substack{\Psi = \Psi_i \\ \eta=\eta_i}} \tag{8}$$

From Eqs.(2) and (9), we find χ_i as

$$
\begin{aligned}
\chi_i^{-1} = {} & 2a_2 + 6a_3\Psi_i + 12a_4\Psi_i^2 + 20a_5\Psi_i^3 \\
& + 30a_6\Psi_i^4 + 2c_2\eta_i + 2c_4\eta_i^2 + 6c_5\Psi_i\eta_i \\
& + 2c_7\eta_i^3 + 6c_8\Psi_i\eta_i^2 + 6c_9\Psi_i\eta_i^3 .
\end{aligned} \tag{9}
$$

Using Eqs.(4), (5), (6), (7) and (8), each term in Eq.(10) has the temperature dependence as

$$\chi_i^{-1} \sim (T - T_c)$$

or

$$\chi_i \sim (T - T_c)^{-1} \tag{10}$$

Since the susceptibility behaves as

$$\chi_i \sim (T - T_c)^{-\gamma}$$

we have the critical exponent for the susceptibility as $\gamma = 1$.

The susceptibility $\chi_i'(i = \alpha, \beta, \gamma)$ can also be defined as

$$\chi_i'^{-1} = (\frac{\partial^2 F}{\partial \eta^2})_{\substack{\Psi = \Psi_i \\ \eta = \eta_i}}. \tag{11}$$

From Eqs.(2) and (12), we find χ_i' as

$$\begin{aligned} \chi'^{-1} &= 2b_2 + 6b_3\eta_i + 12b_4\eta_i^2 + 20b_5\eta_i^3 \\ &+ 30b_6\eta_i^4 + 2c_3\Psi_i + 2c_4\Psi_i^2 \\ &+ 6c_6\Psi_i\eta_i + 2c_8\Psi_i^3 + 6c_7\eta_i\Psi_i^2 \\ &+ 6c_9\eta_i\Psi_i^3. \end{aligned} \tag{12}$$

Using Eqs. (4), (5), (6), (7) and (8), each term in Eq.(13) has the temperature dependence as

$$\chi_i'^{-1} \sim (T - T_c). \tag{13}$$

Therefore, the critical exponent for this susceptibility is also $\gamma = 1$.

Now, we want to find the critical exponent α for the specific heat C. Using Eqs. (4), (5), (6), (7) and (8), each term in the free energy given by Eq.(2) has the temperature dependence as

$$F = A(T - T_c)^{3/2}, \tag{14}$$

where A is a constant. Hence, we have

$$C = T(\frac{\partial^2 F}{\partial T^2})|_{T=T_c} = \frac{3}{4}AT_c(T - T_c)^{-1/2}. \tag{15}$$

Since the temperature dependence of the specific heat can be expressed using the power law as

$$C \sim (T - T_c)^{-\alpha} \tag{16}$$

from Eq.(16) we have $\alpha = \frac{1}{2}$.

3. Discussion

In this study we give the form of the free energy (Eq.1d) with two order parameters Ψ and η to describe the multicritical behaviour of a system in the region of three-phase coexistence. The form of the free energy we give in Eq(1d) is the most general form that one can obtain for a system of three-phase coexistence. As in the Landau theory, the temperature dependences of a_2 and b_2, which are the coefficients of Ψ^2 and η^2, respectively, as given in Eq.(2), were taken as

$$a_2 = a_{20}(T - T_c)$$

and

$$b_2 = b_{20}(T - T_c).$$

Using these temperature dependences of the coefficients, we predicted the critical behaviour of the order parameters Ψ and η, susceptibility χ and the specific heat C. The critical exponents that we calculated from our model turned out to be the tricritical exponents. This shows that our mean field model which has the free energy given by Eq.(1d), describes the critical behaviour of those systems exhibiting the three-phase coexistence near the tricritical point. As an example, we have used this mean field model to describe the tricritical behaviour of nematic, smectic A and smectic C phases of liquid crystals near the NAC point [13]. Our mean field model can also be used to describe the tricritical behaviour of those systems such as ferroelectric systems, ferroelectric liquid crystals and ammonium halides, which exhibit three-phase coexistence.

Conclusions

In this study we have developed a mean field model with two order parameters for those systems exhibiting three-phase coexistence near the multicritical point. Using our model we have calculated the critical exponents for the order parameters, susceptibility and the specific heat. And these exponents are the tricritical exponents. Therefore, our mean field model can be used to describe the tricritical behaviour of a system in the region of three-phase coexistence.

Appendix A

If we expand Eq.(1d) we obtain Eq.(2) with the coefficients a_i, b_i, and c_i as follows:

$$\begin{aligned} a_0 = {} & \Psi_\alpha^4\Psi_\beta\Psi_\gamma + \Psi_\alpha^3\Psi_\beta^2\Psi_\gamma + \Psi_\alpha^2\Psi_\beta^3\Psi_\gamma \\ & + \Psi_\alpha\Psi_\beta^4\Psi_\gamma + \Psi_\alpha^3\Psi_\beta\Psi_\gamma^2 + \Psi_\alpha^2\Psi_\beta^2\Psi_\gamma^2 \\ & + \Psi_\alpha\Psi_\beta^3\Psi_\gamma^2 + \Psi_\alpha^2\Psi_\beta\Psi_\gamma^3 + \Psi_\alpha\Psi_\beta^2\Psi_\gamma^3 \\ & + \Psi_\alpha\Psi_\beta\Psi_\gamma^4 + \Psi_\alpha\Psi_\beta\Psi_\gamma\eta_\alpha\eta_\beta\eta_\gamma \end{aligned} \tag{A0}$$

$$+ \eta_\alpha^4\eta_\beta\eta_\gamma + \eta_\alpha^3\eta_\beta\eta_\gamma^2 + \eta_\alpha^2\eta_\beta^3\eta_\gamma$$
$$+ \eta_\alpha\eta_\beta^4\eta_\gamma + \eta_\alpha^3\eta_\beta\eta_\gamma^2 + \eta_\alpha^2\eta_\beta^2\eta_\gamma^2$$
$$+ \eta_\alpha\eta_\beta^3\eta_\gamma^2 + \eta_\alpha^2\eta_\beta\eta_\gamma^3 + \eta_\alpha\eta_\beta^2\eta_\gamma^3$$
$$+ \eta_\alpha\eta_\beta\eta_\gamma^4$$

$$\begin{aligned} a_1 = &-\Psi_\alpha^4\Psi_\beta - \Psi_\alpha^3\Psi_\beta^2 - \Psi_\alpha^2\Psi_\beta^3 - \Psi_\alpha\Psi_\beta^4 \\ &- \Psi_\alpha^4\Psi_\gamma - \Psi_\alpha^3\Psi_\gamma^2 - \Psi_\alpha^2\Psi_\gamma^3 - \Psi_\alpha\Psi_\gamma^4 \\ &- \Psi_\beta^4\Psi_\gamma - \Psi_\beta^3\Psi_\gamma^2 - \Psi_\beta^2\Psi_\gamma^3 - \Psi_\beta\Psi_\gamma^4 \\ &- 8\Psi_\alpha\Psi_\beta^2\Psi_\gamma^2 - 8\Psi_\alpha\Psi_\beta\Psi_\gamma^3 - 8\Psi_\alpha\Psi_\beta^3\Psi_\gamma \\ &- 8\Psi_\alpha^2\Psi_\beta\Psi_\gamma^2 - 8\Psi_\alpha^2\Psi_\beta^2\Psi_\gamma - 8\Psi_\alpha^3\Psi_\beta\Psi_\gamma \\ &- \Psi_\alpha\Psi_\beta\eta_\alpha\eta_\beta\eta_\gamma - \Psi_\alpha\Psi_\gamma\eta_\alpha\eta_\beta\eta_\gamma \\ &- \Psi_\beta\Psi_\gamma\eta_\alpha\eta_\beta\eta_\gamma \end{aligned} \tag{A1}$$

$$\begin{aligned} a_2 = &\Psi_\alpha^4 + \Psi_\beta^4 + \Psi_\gamma^4 + 7\Psi_\alpha\Psi_\beta^3 + 7\Psi_\alpha\Psi_\gamma^3 \\ &+ 7\Psi_\beta\Psi_\gamma^3 + 7\Psi_\alpha^2\Psi_\beta^2 + 7\Psi_\beta^2\Psi_\gamma^2 + 7\Psi_\alpha^2\Psi_\gamma^2 \\ &+ 7\Psi_\alpha^3\Psi_\beta + 7\Psi_\alpha^3\Psi_\gamma + 7\Psi_\beta^3\Psi_\gamma \\ &+ 28\Psi_\alpha\Psi_\beta\Psi_\gamma^2 + 28\Psi_\alpha\Psi_\beta^2\Psi_\gamma + 28\Psi_\alpha^2\Psi_\beta\Psi_\gamma \\ &+ \Psi_\alpha\eta_\alpha\eta_\beta\eta_\gamma + \Psi_\beta\eta_\alpha\eta_\beta\eta_\gamma + \Psi_\gamma\eta_\alpha\eta_\beta\eta_\gamma \end{aligned} \tag{A2}$$

$$\begin{aligned} a_3 = &-6\Psi_\alpha^3 - 6\Psi_\beta^3 - 6\Psi_\gamma^3 - 21\Psi_\alpha\Psi_\beta^2 - 21\Psi_\alpha^2\Psi_\beta \\ &-21\Psi_\alpha\Psi_\gamma^2 - 21\Psi_\alpha^2\Psi_\gamma \\ &-21\Psi_\beta\Psi_\gamma^2 - 21\Psi_\beta^2\Psi_\gamma - 56\Psi_\alpha\Psi_\beta\Psi_\gamma \\ &-\eta_\alpha\eta_\beta\eta_\gamma \end{aligned} \tag{A3}$$

$$a_4 = 15\Psi_\alpha^2 + 15\Psi_\beta^2 + 15\Psi_\gamma^2 + 35\Psi_\alpha\Psi_\beta + 35\Psi_\alpha\Psi_\gamma + 35\Psi_\beta\Psi_\gamma \tag{A4}$$

$$a_5 = -20\Psi_\alpha - 20\Psi_\beta - 20\Psi_\gamma \tag{A5}$$

$$a_6 = 10 \tag{A6}$$

$$\begin{aligned}b_1 = &-\eta_\alpha^4\eta_\beta - \eta_\alpha^3\eta_\beta^2 - \eta_\alpha^2\eta_\beta^3 - \eta_\alpha\eta_\beta^4\\ &- \eta_\alpha^4\eta_\gamma - \eta_\alpha^3\eta_\gamma^2 - \eta_\alpha^2\eta_\gamma^3 - \eta_\alpha\eta_\gamma^4\\ &- \eta_\beta^4\eta_\gamma - \eta_\beta^3\eta_\gamma^2 - \eta_\beta^2\eta_\gamma^3 - \eta_\beta\eta_\gamma^4\\ &- 8\eta_\alpha\eta_\beta^2\eta_\gamma^2 - 8\eta_\alpha\eta_\beta\eta_\gamma^3 - 8\eta_\alpha\eta_\beta^3\eta_\gamma\\ &- 8\eta_\alpha^2\eta_\beta\eta_\gamma^2 - 8\eta_\alpha^2\eta_\beta^2\eta_\gamma - 8\eta_\alpha^3\eta_\beta\eta_\gamma\\ &- \eta_\alpha\eta_\beta\Psi_\alpha\Psi_\beta\Psi_\gamma - \eta_\alpha\eta_\gamma\Psi_\alpha\Psi_\beta\Psi_\gamma\\ &- \eta_\beta\eta_\gamma\Psi_\alpha\Psi_\beta\Psi_\gamma\end{aligned} \tag{A7}$$

$$\begin{aligned}b_2 = &\,\eta_\alpha^4 + \eta_\beta^4 + \eta_\gamma^4 + 7\eta_\alpha\eta_\beta^3 + 7\eta_\alpha\eta_\gamma^3\\ &+ 7\eta_\beta\eta_\gamma^3 + 7\eta_\alpha^2\eta_\beta^2 + 7\eta_\alpha^2\eta_\gamma^2 + 7\eta_\beta^2\eta_\gamma^2\\ &+ 7\eta_\alpha^3\eta_\beta + 7\eta_\alpha^3\eta_\gamma + 7\eta_\beta^3\eta_\gamma\\ &+ 28\eta_\alpha\eta_\beta\eta_\gamma^2 + 28\eta_\alpha\eta_\beta^2\eta_\gamma + 28\eta_\alpha^2\eta_\beta\eta_\gamma\\ &+ \eta_\alpha\Psi_\alpha\Psi_\beta\Psi_\gamma + \eta_\beta\Psi_\alpha\Psi_\beta\Psi_\gamma + \eta_\gamma\Psi_\alpha\Psi_\beta\Psi_\gamma\end{aligned} \tag{A8}$$

$$\begin{aligned}b_3 = &-6\eta_\alpha^3 - 6\eta_\beta^3 - 6\eta_\gamma^3 - 21\eta_\alpha\eta_\beta^2 - 21\eta_\alpha^2\eta_\beta\\ &- 21\eta_\alpha\eta_\gamma^2 - 21\eta_\alpha^2\eta_\gamma - 21\eta_\beta\eta_\gamma^2 - 21\eta_\beta^2\eta_\gamma\\ &- 56\eta_\alpha\eta_\beta\eta_\gamma - \Psi_\alpha\Psi_\beta\Psi_\gamma\end{aligned} \tag{A9}$$

$$\begin{aligned}b_4 = &\,15\eta_\alpha^2 + 15\eta_\beta^2 + 15\eta_\gamma^2 + 35\eta_\alpha\eta_\beta\\ &+ 35\eta_\alpha\eta_\gamma + 35\eta_\beta\eta_\gamma\end{aligned} \tag{A10}$$

$$b_5 = -20\eta_\alpha - 20\eta_\beta - 20\eta_\gamma \tag{A11}$$

$$b_6 = 10 \tag{A12}$$

$$\begin{aligned}c_1 = &\,\Psi_\alpha\Psi_\beta\eta_\alpha\eta_\beta + \Psi_\alpha\Psi_\beta\eta_\alpha\eta_\gamma\\ &+ \Psi_\alpha\Psi_\beta\eta_\beta\eta_\gamma + \Psi_\alpha\Psi_\gamma\eta_\alpha\eta_\beta\\ &+ \Psi_\alpha\Psi_\gamma\eta_\alpha\eta_\gamma + \Psi_\alpha\Psi_\gamma\eta_\beta\eta_\gamma\\ &+ \Psi_\beta\Psi_\gamma\eta_\alpha\eta_\beta + \Psi_\beta\Psi_\gamma\eta_\alpha\eta_\gamma\\ &+ \Psi_\beta\Psi_\gamma\eta_\beta\eta_\gamma\end{aligned} \tag{A13}$$

$$
\begin{aligned}
c_2 = &-\Psi_\alpha\eta_\alpha\eta_\beta - \Psi_\alpha\eta_\alpha\eta_\gamma - \Psi_\alpha\eta_\beta\eta_\gamma \\
&- \Psi_\beta\eta_\alpha\eta_\beta - \Psi_\beta\eta_\alpha\eta_\gamma - \Psi_\beta\eta_\beta\eta_\gamma \\
&- \Psi_\gamma\eta_\alpha\eta_\beta - \Psi_\gamma\eta_\alpha\eta_\gamma - \Psi_\gamma\eta_\beta\eta_\gamma
\end{aligned} \tag{A14}
$$

$$
\begin{aligned}
c_3 = &-\eta_\alpha\Psi_\alpha\Psi_\beta - \eta_\alpha\Psi_\alpha\Psi_\gamma - \eta_\alpha\Psi_\beta\Psi_\gamma \\
&- \eta_\beta\Psi_\alpha\Psi_\beta - \eta_\beta\Psi_\alpha\Psi_\gamma - \eta_\beta\Psi_\beta\Psi_\gamma \\
&- \eta_\gamma\Psi_\alpha\Psi_\beta - \eta_\gamma\Psi_\alpha\Psi_\gamma - \eta_\gamma\Psi_\beta\Psi_\gamma
\end{aligned} \tag{A15}
$$

$$
\begin{aligned}
c_4 = &\Psi_\alpha\eta_\alpha + \Psi_\alpha\eta_\beta + \Psi_\alpha\eta_\gamma + \Psi_\beta\eta_\alpha + \Psi_\beta\eta_\beta \\
&+ \Psi_\beta\eta_\gamma + \Psi_\gamma\eta_\alpha + \Psi_\gamma\eta_\beta + \Psi_\gamma\eta_\gamma
\end{aligned} \tag{A16}
$$

$$c_5 = \eta_\alpha\eta_\beta + \eta_\alpha\eta_\gamma + \eta_\beta\eta_\gamma \tag{A17}$$

$$c_6 = \Psi_\alpha\Psi_\beta + \Psi_\alpha\Psi_\gamma + \Psi_\beta\Psi_\gamma \tag{A18}$$

$$c_7 = -\Psi_\alpha - \Psi_\beta - \Psi_\gamma \tag{A19}$$

$$c_7 = -\eta_\alpha - \eta_\beta - \eta_\gamma \tag{A20}$$

$$c_9 = 1. \tag{A21}$$

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