Tr. J. of Physics
22 (1998) , 395 – 400.

$\tau \to \rho\nu$ Decay and Leptonic Decay Constants of Vector Mesons

Kh. ABLAKULOV, B.N. KURANOV, T.Z. NASYROV
Institute of Nuclear Physics of,
Uzbekistan Academy of Sciences, Ulugbek 702132
Tashkent - UZBEKISTAN
e-mail: Nasirt@suninp.tashkent.su

Received 14.01.1998

Abstract

The new Salpeter equation for vector mesons is proposed and the representation for their leptonic decay constants is obtained.

1. Introduction

At present, spectroscopy of pseudoscalar and vector mesons is mainly described in the framework of phenomenological models based on the ideas of the quantum chromodynamics (QCD). The bilocal relativistic potential model (BRPM) [1], on the basis of which there are two principles - *the minimal quantization of gauge fields* and *a choice of the quantization axis* [2], also belongs to these of models.

In papers [3, 4, 5], within the BRPM, the description of spectroscopy of pseudoscalar mesons, including pion, had been obtained on a satisfactorily level with their experimental values. However, the estimates for the leptonic decay constant of pion f_π were considerably smaller than its experimental value [4, 5]. In paper [6] the value of constant f_π also could be reproduced by means of modification of the Schwinger-Dyson equation (SDE) in this model.

Using decomposition of the vector meson wave function over its structure components and considering that its polarization vectors ϵ_μ^λ ($\lambda = 1,\ 2,\ 3$ are the polarization indices) depend on relative momentum q in papers [1, 3] the Salpeter equation (SE) for vector mesons had been obtained. However, it is known that vector particles with nonzero mass having internal structure is described by three orthogonal spacelike vector ϵ_μ^λ, which depend on total momentum $\mathcal{P}^\mu$ and obey the orthogonality condition $\mathcal{P}^\mu \epsilon_\mu^\lambda = 0$. Hence it follows that $\mathcal{P}^\mu$ plays role of vector giving some direction to which the vector fields are referred [7].

In the present paper we proceed from the idea that ϵ_μ^λ are functions of $\mathcal{P}^\mu$ and propose a new SE which describes a mass spectrum and wave functions of vector mesons. The $\tau \to \rho\nu$ decay is considered and the representation for leptonic decay constant of ρ meson f_ρ is given. Choosing the interquark interaction potential as sum of the oscillator and Coulomb type potentials and using solutions of the SDE and SE proposed, we obtain the ρ meson mass and constant f_ρ, together with masses of other charge vector mesons consisting of "up" and "down" quarks with different flavours as well on a qualitative level. The values for their leptonic decay constants are also predicted.

2. The New Salpeter Equation for Vector Mesons

In order to calculate the matrix elements of the processes with presence of a vector meson as a $\overline{q}q$-bound state, which is described by the effective QCD action for the bilocal meson field $\mathcal{M}(x,y)$ proposed in Refs. [1, 3], we decompose $\mathcal{M}(x,y)$ over the creation and annihilation operators of mesons $a_i^\pm(\mathcal{P})$, with eigenvalues $\sqrt{\mathcal{P}^2} = M$, $\omega = (\vec{\mathcal{P}}^2 + M^2)^{1/2}$ and

$$\begin{aligned}\mathcal{M}(x,y) &= \mathcal{M}(z|X) = \int \frac{d^3\vec{\mathcal{P}}}{(2\pi)^{3/2}\sqrt{2\omega}} \int \frac{d^4q}{(2\pi)^4} \exp(iqz) \\ &\times [\exp(i\mathcal{P}X)\Gamma_i(q|\mathcal{P})a_i^+(\mathcal{P}) + \exp(-i\mathcal{P}X)\overline{\Gamma}_i(q|\mathcal{P})a_i^-(\mathcal{P})] \end{aligned} \tag{1}$$

where

$$z = x - y, \qquad X = \frac{x+y}{2}.$$

Here, M is the mass of the $\overline{q}q$-bound state (meson), and $\Gamma_i(q|\mathcal{P})$ and $\overline{\Gamma}_i(q|\mathcal{P})$ are vertex functions of the vector meson, which are defined from Bethe-Salpeter equation

$$\Gamma_{i(a,b)}(p|\mathcal{P}) = -i \int \frac{d^4q}{(2\pi)^4} V(p^\perp - q^\perp)\, \not{\eta} G_{\Sigma_a}\left(q + \frac{\mathcal{P}}{2}\right) \Gamma_{i(a,b)}(q|\mathcal{P}) G_{\Sigma_a}\left(q - \frac{\mathcal{P}}{2}\right) \not{\eta}, \tag{2}$$

where

$$p^\perp = p - p^{||}, \qquad p^{||} = \not{\eta}(p\eta), \qquad \not{\eta} = \eta_\mu \gamma^\mu, \qquad \eta_\mu = \mathcal{P}_\mu / \sqrt{\mathcal{P}}^2.$$

Here, $V(p^\perp - q^\perp)$ is the phenomenological interquark interaction potential, $G_{\Sigma_{a,b}}$ is the quark Green function and a, b are the quark and antiquark flavours.

After integrating of Eq. (2) over the longitudinal momentum $q^{||}$ and introducing the "dressed" vector meson wave function $\Psi_{i(a,b)}(q^\perp|\mathcal{P})$, we have

$$\begin{aligned}\Psi_{i(a,b)}(q^\perp|\mathcal{P}) &= S_a(q^\perp)\Psi^0_{i(a,b)}(q^\perp|\mathcal{P})S_b(q^\perp) \\ &= i\int \frac{dq^{||}}{2\pi} G_{\Sigma_a}\left(q + \frac{\mathcal{P}}{2}\right) \Gamma_{i(a,b)}(q^\perp|\mathcal{P}) G_{\Sigma_b}\left(q - \frac{\mathcal{P}}{2}\right),\end{aligned} \tag{3}$$

where

$$S_{a,b}(q^{\perp}) = \exp\left[\frac{\not{q}^{\perp}}{|q^{\perp}|}\vartheta_{a,b}(q^{\perp})\right] \tag{4}$$

is the Foldy-Vauthoysen type transformation matrix [1], and

$$\Psi^{0}_{i(a,b)}(q^{\perp}|\mathcal{P}) = \not{\epsilon}_i(\mathcal{P})(N_1(q^{\perp}) - \not{\eta}N_2(q^{\perp})) \tag{5}$$

is the "undressed" vector meson wave function.

We obtain the SE for vector mesons:

$$\begin{aligned} MN_{\binom{2}{1}}(p^{\perp}) &= E_t(p^{\perp})N_{\binom{1}{2}}(p^{\perp}) - \frac{1}{3}\int \frac{d^3q^{\perp}}{(2\pi)^3}V(p^{\perp}-q^{\perp})[c_p^{\mp}c_q^{\mp} + c_p^{+}c_q^{-} + c_p^{-}c_q^{+} \\ & -\xi(2s_p^{\mp}s_q^{\mp} + s_p^{\pm}s_q^{\pm}) + \xi^2(c_p^{-}c_q^{-} + c_p^{+}c_q^{-} + c_p^{-}c_q^{+} + c_p^{+}c_q^{+})] \\ & N_{\binom{2}{1}}(q^{\perp}), \end{aligned} \tag{6}$$

where

$$c_p^{\pm} = \cos[\vartheta_a(p^{\perp}) \pm \vartheta_a(p^{\perp}) \pm \vartheta_b(p^{\perp})], \quad s_p^{\pm} = \sin[\vartheta_a(p^{\perp}) \pm \vartheta_b(p^{\perp})], \quad \xi = \frac{p^{\perp}q^{\perp}}{|p^{\perp}||q^{\perp}|},$$

$$E_t(p^{\perp}) = E_a(p^{\perp}) + E_b(p^{\perp}).$$

Here, $\vartheta_{a,b}(p^{\perp})$ and $E_{a,b}(p^{\perp})$ are the single-particle phase functions and energies of quark and antiquark inside the meson, which are found by solving the SDE [1, 3].

In Exp. (5) $N_1(q^{\perp})$ and $N_2(q^{\perp})$ are the structure form factors of "undressed" meson and $\epsilon^{\mu}_{\lambda}(\mathcal{P})$ are polarization vectors which satisfy the following respective conditions,

$$\frac{4N_c}{M}\int \frac{d^3q^{\perp}}{(2\pi)^3}N_1(q^{\perp})N_2(q^{\perp}) = 1,$$

$$\epsilon^{\alpha}_{\lambda}(\mathcal{P})\epsilon^{\beta}_{\lambda}(\mathcal{P}) = -\left(g^{\alpha\beta} - \frac{\mathcal{P}^{\alpha}\mathcal{P}^{\beta}}{M^2}\right).$$

It is to be noted that unlike works [1, 3], where N_1 and N_2 are the vector quantities, in our scheme they are divided to the polarization vectors matrix and structure scalar formfactors (5), which are decomposed over spherical functions. This circumstance simplifies to find the masses spectrum and wave functions of vector mesons, and leads to the only leptonic decay constant.

3. $\tau \to \rho\nu$-Decay

Now we consider $\tau \to \rho\nu$ decay, the study of which gives information about the hadronization of the intermediate charge weak vector W boson to the vector meson, allowing calculation of the constant f_p.

For the matrix elements of the decay we have

$$\langle \rho\nu | W_i^{int} | \tau \rangle = \frac{(2\pi)^4 \delta^4(k_r - k_\nu - \mathcal{P})}{[(2\pi)^9 \cdot 2\omega_\tau \cdot 2\omega_\nu \cdot 2\omega]^{1/2}} \mathcal{L}_\mu(k_r, k_\nu) \mathcal{H}_i^\mu(\mathcal{P}), \tag{7}$$

where $\mathcal{L}_\mu(k_r, k_\nu)$ is the leptonic part of the matrix elements [8], $\mathcal{H}_i^\mu(\mathcal{P})$ is its hadronic part, which in the BRPM has form

$$\mathcal{H}_i^\mu(\mathcal{P}) = N_c \int \frac{d^3 q^\perp}{(2\pi)^3} tr[S_u(q^\perp)\Psi_i^0(q^\perp|\mathcal{P}) S_d(q^\perp)\gamma^\mu(1-\gamma_5)]. \tag{8}$$

Hence taking into accout Exps. (4) and (5) we obtain

$$\mathcal{H}_i^\mu(\mathcal{P}) = f_\rho \epsilon_i^\mu(\mathcal{P}), \tag{9}$$

where

$$f_\rho = \frac{4N_c}{3} \int \frac{d^3 q^\perp}{(2\pi)^3} N_1(q^\perp)\{2\cos[\vartheta_u(q^\perp) - \vartheta_d(q^\perp)] + \cos[\vartheta_u(q^\perp) + \vartheta_d(q^\perp)]\} \tag{10}$$

is the leptonic decay constant of ρ meson.

It is seen from (10) that the constant f_p is expressed in terms of solutions of the SDE for the single-particle phase functions of u and d quarks inside ρ meson and the SE for the wave function itself.

It should be noted that representation (10) can also be used to define leptonic decay constants of other charge vector mesons consisting of “up” and “down” quarks.

4. Numerical Results and Conclusion

Proceeding from the fact that the oscillator potential leads to spontaneous breakdown of the chiral symmetry, which describes on a qualitative level the large difference of masses between π and ρ mesons [9], to combine Coulomb asymptotic freedom contributions [10], we use the interquark interaction potential

$$V(r) = \frac{4}{3}\left(V_0 r^2 - \frac{\alpha_s}{r}\right), \tag{11}$$

where V_0 and α_s are the parameters of their potentials.

In order to remove ultraviolet divergences in the SDE, arising due to the Coulomb part of the potential, the standard renormalization scheme [11] is used.

Using the continuous analogy of the Newton method [12] and fitting values of the input parameters of model by masses of π, ρ, K, D and B mesons, we obtain:

$$(4V_0/3)^{1/3} = 299MeV, \qquad \alpha_s = 0.2, \qquad m^0_{u,d} = 2.3MeV,$$

$$m^0_s = 68MeV, \qquad m^0_c = 1273MeV, \qquad m^0_b = 4720MeV.$$

When taking into account these parameters, the masses and leptonic decay constants of charge vector mesons can be calculated and are given in Table, from which it is seen that the BRPM with potential (11) describes the constant f_ρ as well as masses of other vector mesons on a satisfactory level.

Table 1. Values of the masses of vector mesons and their leptonic decay constants.

Vector	Masses, MeV		f_V, $\times 10^5$ MeV^2	
meson	model	exp. [13]	model	exp.
ρ	<u>770</u>	770	1.03	1.57 ± 0.09 [14]
K^*	807	892	1.17	
D^*	1880	2010	3.21	
D^*_s	1894	2110	3.51	
B^*	5281	5325	6.61	
B^*_c	6203		15.28	

Authors thank N.A. Sarikov and B.S. Yuldashev for stimulating discussions of the results, and the Foundation for Fundamental Investigations of Uzbekistan State Committee of Science and Technics (Grant 2/95) for the support of the present investigations.

References

[1] Yu L. Kalinovsky, W. Kallies, B.N. Kuranov et.al., *Yadernaya Fizika*, **49** (1989) 1709; Yu L. Kalinovsky, W. Kallies, L. Kaschluhn et.al., *Fortschr. Phys.*, **38** (1990) 333; Yu L. Kalinovsky, W. Kallies, L. Münchow et.al., *Few Body Systems*, **10** (1991) 87.

[2] Nguyen Suan Han, V.N. Pervushin, *Fortschr. Phys.*, **37** (1989) 8; N.P. Iliyeva, Nguyen Suan Han, V.N. Pervushin, *Yadernaya Fizika*, **45** (197) 1169.

[3] Yu L. Kalinovsky, C. Weiss, *Z. Phys.*, **C63** (1994) 275.

[4] R. Horvat, D. Kekez, D. Klabucar and D. Palle, *Phys. Rev.*, **D44** (1991) 1585.

[5] R. Horvat, D. Kekez, D. Palle, D. Klabucar, *Z. Phys.*, **C68** (1995) 303.

[6] T.Z. Nasyrov, N.A. Sarikov, *Yadernaya Fizika*, **59** (1996).

[7] J. Schwinger, *Particles, Sources, and Fields* (Addison-Wesley Publishing Company, Ontario 1970).

[8] L.B. Okun, *Leptons and Quarks*, (Nauka, Moscow 1990, in Russian).

[9] A. Le Yaouanc, L. Oliver, O. Pene, J.-C. Raynal, *Phys. Rev.*, **D31** (1985) 137.

[10] H.D. Politzer, *Phys. Rev. Lett.*, **30** (1973) 1346; D.J. Gross, F. Wilczek, *Phys. Rev. Lett.*, **30** (1973) 1343; *Phys. Rev.*, **D8** (1973) 3633; *Phys. Rev.*, **D9** (1974) 980.

[11] S.L. Adler and A.C. Davis, *Nucl. Phys.*, **B244** (1984) 469;
R. Alkofer and P.A. Amundsen, *Nucl. Phys.*, **B306** (1988) 305; A. Trzupek, *Acta. Phys. Pol.*, **B20** (1989) 93.

[12] I.V. Amirkhanov, O.M. Juraev, V.N. Pervushin, et.al., *JINR Preprint*, E11-91-108, Dubna, 1991; I.V. Amirkhanov, O.M. Juraev, V.N. Pervushin, et.al., *JINR Preprint*, P11-91-111, Dubna 1991.

[13] Particle Data Properties, *Phys. Rev.*, **D50** (1994) 1443.

[14] J. Gasser, H. Leutwyler, *Phys. Reports*, **87** (1982) 77.