

# Testing Deca-TeV Unified Compositeness at the 4 TeV $\mu^+\mu^-$ Collider\*

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## Abstract

In the framework of the unified compositeness of leptons, quarks and Higgs bosons, the hidden local symmetry  $\hat{H}_{loc} = SU(2)_L \times U(1)_Y$  with the heavy composite vector bosons, in addition to the SM gauge bosons, is briefly described. Supplementary hypothesis of the vector boson dominance (VBD) of the SM gauge interactions is considered. It is argued that this should produce the universal dominant residual interactions of the SM composite particles, i.e., all of the fermions and Higgs bosons. Restrictions on the universal residual fermion-fermion, fermion-boson and boson-boson interactions due to the VBD are investigated. Manifestations of the residual interactions at the 4 TeV  $\mu^+\mu^-$  collider are studied. It is shown that at 95% C.L. the unified substructure could be investigated at the collider in the processes  $\mu^+\mu^- \rightarrow \bar{f}f$  up to the compositeness scale  $\mathcal{O}(150 \text{ TeV})$ , in the processes  $\mu^+\mu^- \rightarrow ZH, W^+W^-$  up to  $\mathcal{O}(100 \text{ TeV})$  and in the process  $\mu^+\mu^- \rightarrow ZHH$  up to  $\mathcal{O}(40 \text{ TeV})$ , which lie in the naturally preferable Deca-TeV region.

## Introduction

The scheme of the unified compositeness of leptons, quarks and Higgs bosons, with constituents in common, provides one of the promising ways to go beyond the Standard

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Model (SM) (for a short review, see [1]). Treating the SM Higgs doublet as Goldstone boson in the scheme, one can solve, in particular, the naturalness problem of the Higgs sector in the SM without supersymmetry. A nonlinear model was investigated in the lines described above by one of the present authors (Yu.F.P.) in refs. [2, 3]. Here the SM is to be considered just a renormalizable part of the “low energy” effective field theory caused by the unified compositeness.

The effective “low energy” theory of the unified compositeness is based on some rather general assumptions about symmetry properties. Let the hypothetical hyperstrong interactions responsible for the internal binding of the SM composite particles possess a global chiral symmetry  $G$ . Under the hyperstrong confinement, the symmetry  $G$  breaks down to some of its subgroup  $H \subset G$  at the scale  $\mathcal{F}$ . In this, the true Goldstone bosons appear which are ultimately identified, in particular, as the Higgs doublet. The unbroken symmetry  $H$  must contain the SM symmetry  $SU(2)_L \times U(1)_Y$ . Thus at the first stage, the electroweak symmetry remains unbroken. Ultimate taking into account the gauge quantum corrections, corresponding to some extended electroweak symmetry  $I_{loc} \subset G$ , results in the SM electroweak symmetry breaking at the Fermi scale  $v \ll \mathcal{F}$ . If this breaking happens only under two-loop corrections, the naturalness relation between the scales  $v$  and  $\mathcal{F}$  takes place:  $\mathcal{F} = \mathcal{O}(2m_W/\alpha_W)$ . So  $\mathcal{F}$  is expected to lie naturally in the Deca-TeV region:  $\mathcal{F} = \mathcal{O}(10 \text{ TeV})$ . The minimal extension of the SM symmetry to implement such a scenario is given by the choice  $G = SU(3)_L \times U(1)$  and  $H = SU(2)_L \times U(1)_Y$ , the intrinsic local subgroup being  $I_{loc} = SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ . The corresponding nonlinear model  $G/H$  may be called the Minimal Nonlinear Standard Model (MNSM).

In what follows, we describe in short the linearization of the model via the phenomenon of the hidden local symmetry. Then we present the crucial phenomenological consequences of the unified compositeness at the future 4 TeV  $\mu^+ \mu^-$  collider (see, e. g., refs. [4]–[7]).

## 1. Universal Residual Interactions

As the nonlinear model, the MNSM is built on the nonlinear realization of  $G$  that becomes linear when restricted to  $H$  [8]. Such a model is equivalent, at least at the classical level, to the model with linearly realized symmetry  $G \times \hat{H}_{loc}$  [9]. Here  $\hat{H}_{loc}$  is the hidden local symmetry with the appropriate auxiliary gauge bosons. In the context of the MNSM the phenomenon of the hidden local symmetry was studied in ref. [3]. The essence of the latter one is as follows.

In the linear model, the field variable is the element of the whole group  $G$  which can be parametrized as:

$$\hat{\xi} = \xi h, \quad h \in H \quad (1)$$

and

$$\xi = e^{i\phi' Y'/\mathcal{F}'} e^{i(\phi_\alpha X^{\dagger\alpha} + h.c.)/\mathcal{F}} \in G/H. \quad (2)$$

Here  $\phi$  is the Higgs-Goldstone doublet,  $\phi'$  is the Goldstone boson corresponding to the broken hypercharge  $Y'$ , with  $\mathcal{F}$  and  $\mathcal{F}'$  being the symmetry breaking mass scales. The

following transformation law under  $\gamma \times \hat{h}(x) \in G \times \hat{H}_{loc}$  takes place:

$$\gamma \times \hat{h}(x) : \hat{\xi} \rightarrow \gamma \hat{\xi} \hat{h}^\dagger(x). \quad (3)$$

The linear model describes spontaneous/dynamical symmetry breaking  $G \times \hat{H}_{loc} \rightarrow H$ , with the total local symmetry being broken as  $I_{loc} \times \hat{H}_{loc} \rightarrow H_{loc} = SU(2)_L \times U(1)_Y$ .

To construct the Lagrangian of the linear model one has to introduce the modified differential 1-form  $\hat{\omega}_\mu = 1/i \hat{\xi}^\dagger \hat{D}_\mu \hat{\xi}$ , with  $\hat{D}_\mu$  being the derivative covariant both under the intrinsic gauge symmetry  $I_{loc}$  and the hidden local symmetry  $\hat{H}_{loc}$ . Let us divide  $\hat{\omega}_\mu$  into two parts:  $\hat{\omega}_{\parallel\mu}$  which is parallel to  $G/H$  and  $\hat{\omega}_{\perp\mu}$  orthogonal to it. Under  $G \times \hat{H}_{loc}$  the parallel part  $\hat{\omega}_{\parallel\mu}$  transforms homogeneously as in the original nonlinear model, and so does now the orthogonal part  $\hat{\omega}_{\perp\mu}$ . It is precisely introducing the auxiliary vector fields  $\hat{W}_\mu^i$  and  $\hat{S}_\mu$ , corresponding to  $\hat{H}_{loc}$ , that makes the transformation of  $\hat{\omega}_\perp$  homogeneous. In the unitary under  $\hat{H}_{loc}$  gauge, i.e., at  $h \equiv 1$  in Eq. 1, the modified 1-form looks like

$$\begin{aligned} \hat{\omega}_{\parallel\mu} &= \omega_{\parallel\mu}, \\ \hat{\omega}_{\perp\mu}^i &= \omega_{\perp\mu}^i - \hat{g} \hat{W}_\mu^i, \\ \hat{\omega}_{\perp\mu}^0 &= \omega_{\perp\mu}^0 - \hat{g}_1 \hat{S}_\mu, \end{aligned} \quad (4)$$

where  $\omega_\mu$  is the 1-form present in the original MNSM,  $\hat{g}$  and  $\hat{g}_1$  being some new strong coupling constants (expectedly,  $\hat{g}^2/4\pi = \mathcal{O}(1)$  and similarly for  $\hat{g}_1$ ).

In the Lagrangian of the linear model, the new terms appear. They are related with the orthogonal part of the modified 1-form. Here are some of the appropriate terms in the gauge sector:

$$\frac{\lambda \mathcal{F}^2}{2} (\hat{\omega}_{\perp\mu}^i)^2 + \frac{\lambda_1 \mathcal{F}^2}{2} (\hat{\omega}_{\perp\mu}^0)^2 + \dots, \quad (5)$$

and for fermions they are

$$\begin{aligned} &\bar{\psi} \gamma_\mu i (\partial_\mu + i \hat{g} \hat{W}_\mu^i T^i + i \hat{g}_1 \hat{S}_\mu) \psi \\ &+ \kappa \bar{\psi} \gamma_\mu T^i \psi \hat{\omega}_{\perp\mu}^i + \kappa_1 \bar{\psi} \gamma_\mu Y \psi \hat{\omega}_{\perp\mu}^0 + \dots \end{aligned} \quad (6)$$

Here  $\lambda$ 's and  $\kappa$ 's are free parameters. It's to be noted that the matter fields  $\psi$  transform now only under  $\hat{H}_{loc}$ . The modified covariant derivative for them contains only the composite  $\hat{W}_\mu$  and  $\hat{S}_\mu$ , but not the elementary  $W_\mu$  and  $S_\mu$ , the latter ones entering only through the nonminimal interactions.

Introducing the vector fields in such a way without kinetic terms is just a formal procedure. But we believe that the required kinetic terms are developed by the quantum effects, and the new composite vector bosons become physical. This takes place, e.g., in 2- and 3-dimensional nonlinear  $\sigma$ -models [10], as well as in the hadron physics as accomplished fact.

From the Lagrangian of the linear model, one can read off the Lagrangian terms of the vector boson-current interactions:

$$\mathcal{L}_{int} = -g W_\mu^i \left( (1 - \lambda) J_\mu^i(\phi) + \kappa J_\mu^i(\psi) \right)$$

$$-\hat{g}\hat{W}_\mu^i\left(\lambda J_\mu^i(\phi) + (1 - \kappa)J_\mu^i(\psi)\right). \quad (7)$$

Here  $J_\mu^i(\psi) = \bar{\psi}\gamma_\mu T^i\psi$  and  $J_\mu^i(\phi) = \phi^\dagger i\tau^i/2 \overleftrightarrow{D}_\mu \phi$  are the usual SM isotriplet currents, with  $D_\mu$  being the SM covariant derivative. To these isospin terms, one has to add the similar hypercharge isosinglet terms. Impose now the natural requirement that all the composite particles  $\phi$  and  $\psi$  interact directly only with the composite vector bosons  $\hat{W}$  and  $\hat{S}$ , but not with the elementary ones  $W$  and  $S$ . In other words, this is the well-known hypothesis of the vector boson dominance (VBD). This requirement allows one to fix the free parameters:  $\lambda = 1$ ,  $\kappa = 0$  and similarly for the isosinglet parameters.

The terms  $(\hat{\omega}_\perp^i)^2$  and  $(\hat{\omega}_\perp^0)^2$  describe the mass mixing of the elementary and composite gauge bosons, namely,  $W$  with  $\hat{W}$  and  $S$  with  $\hat{S}$ . Diagonalizing these terms one gets two sets of physical vector bosons: the massless isotriplet and isosinglet physical bosons  $\bar{W}^i$  and  $\bar{S}$ , as well as the massive ones  $\bar{W}^i$  and  $\bar{S}$  with masses of order  $\mathcal{F}$ . Due to the heavy physical vector boson exchange, the new low energy effective current-current interactions appear in addition to that of the SM:

$$\begin{aligned} \mathcal{L}_{int}^{(VBD)} &= -\frac{1}{2\mathcal{F}^2}\left(J_\mu^i(\psi)J_\mu^i(\psi) + \eta_1 J_\mu^0(\psi)J_\mu^0(\psi)\right) \\ &\quad -\frac{1}{\mathcal{F}^2}\left(J_\mu^i(\psi)J_\mu^i(\phi) + \eta_1 J_\mu^0(\psi)J_\mu^0(\phi)\right). \end{aligned} \quad (8)$$

Here  $\eta_1$  is a free parameter, related to the original MNSM. Note that the VBD does not affect the low energy Higgs boson self-interactions, the latter ones being determined by the original MNSM alone:

$$\mathcal{L}_{int}(\phi) = -\frac{1}{\mathcal{F}^2}\left(\frac{1}{3}J_\mu^i(\phi)J_\mu^i(\phi) + J_\mu^0(\phi)J_\mu^0(\phi)\right) \quad (9)$$

(up to the Fierz rearrangement). All these expressions are valid only at energies  $\sqrt{s} \ll \mathcal{F}$ .

To resume, the unified compositeness plus the VBD prescribe the two-parameter set of the universal residual fermion-fermion, fermion-boson and boson-boson interactions, with their space-time and internal structure being fixed, sign including. The unified compositeness scale  $\mathcal{F}$  is expected to lie in the Deca-TeV region. Hence the TeV energies are required to probe these new contact interactions.

## 2. Manifestations of VBD at $\mu^+\mu^-$ Collider

In a series of papers we have investigated the possibility to test the hypothesis of the VBD of electroweak interaction at the future 2 TeV  $e^+e^-$  linear collider via the processes  $e^+e^- \rightarrow f\bar{f}$ , where  $f = e^-, \mu^-, \tau^-, u, d, s, c, b$  [11],  $e^+e^- \rightarrow ZH, W^+W^-$  [12] and  $ZHH$  [13]<sup>1</sup>. In this report we have reconsidered the results for the future  $\mu^+\mu^-$  collider

<sup>1</sup>This process was investigated with the CompHEP package for the symbolical and numerical calculations in the high energy physics [14].

with the total energy 4 TeV and the integrated luminosity  $10^3 \text{ fb}^{-1}$  [15], and found that this collider could present the definite answer about the existence (or opposite) of the Deca-TeV unified compositeness.

To illustrate the dependence of the observables on the parameters  $\eta_1$  and  $\mathcal{F}$ , we present in what follow the simple approximate formulas for differential cross-sections for some of the processes.

$$\mu^+ \mu^- \rightarrow e^+ e^- (\tau^+ \tau^-):$$

$$\begin{aligned} \frac{d\sigma(\mu_L^-)}{d\cos\theta} &= \frac{\pi\bar{\alpha}^2}{4s} \left( \kappa_1^2 \frac{1}{16\bar{c}^4\bar{s}^4} (1 + \cos\theta)^2 + \kappa_2^2 \frac{1}{4\bar{c}^4} (1 - \cos\theta)^2 \right), \\ \frac{d\sigma(\mu_R^-)}{d\cos\theta} &= \frac{\pi\bar{\alpha}^2}{4s} \kappa_2^2 \left( \frac{1}{\bar{c}^4} (1 + \cos\theta)^2 + \frac{1}{4\bar{c}^4} (1 - \cos\theta)^2 \right), \end{aligned} \quad (10)$$

here and in what follows  $\bar{c} \equiv \cos\bar{\theta}_W$ ,  $\bar{s} \equiv \sin\bar{\theta}_W$  and  $\bar{\theta}_W$  is the effective weak mixing angle and  $\bar{\alpha}$  is the effective fine structure constant at energies under consideration. Scattering angle  $\theta$  is that between  $e^-$  and  $\mu^-$ .

$$\mu^+ \mu^- \rightarrow W^+ W^-:$$

$$\begin{aligned} \frac{d\sigma(\mu_L^-)}{d\cos\theta} &= \frac{\pi\bar{\alpha}^2}{4s} \left( \kappa_1^2 \frac{1}{16\bar{c}^4\bar{s}^4} + \frac{1}{4\bar{s}^4} \frac{u^2 + t^2}{t^2} \right) (1 - \cos^2\theta), \\ \frac{d\sigma(\mu_R^-)}{d\cos\theta} &= \frac{\pi\bar{\alpha}^2}{4s} \kappa_2^2 \frac{1}{4\bar{c}^4} (1 - \cos^2\theta), \end{aligned} \quad (11)$$

here  $\theta$  is the scattering angle between  $W^-$  and  $\mu^-$  and  $s$ ,  $t$ ,  $u$  are the usual invariant kinematical variables.

$$\mu^+ \mu^- \rightarrow ZH:$$

$$\begin{aligned} \frac{d\sigma(\mu_L^-)}{d\cos\theta} &= \frac{\pi\bar{\alpha}^2}{2s} \frac{E_Z}{\sqrt{s}} \kappa_3^2 \frac{(\bar{s}^2 - \bar{c}^2)^2}{4\bar{s}^4\bar{c}^4} \frac{E_Z^2}{s} (1 - \cos^2\theta), \\ \frac{d\sigma(\mu_R^-)}{d\cos\theta} &= \frac{\pi\bar{\alpha}^2}{2s} \frac{E_Z}{\sqrt{s}} \kappa_2^2 \frac{1}{\bar{c}^4} \frac{E_Z^2}{s} (1 - \cos^2\theta), \end{aligned} \quad (12)$$

$\theta$  is the scattering angle between  $Z$  and  $\mu^-$  and  $E_Z$  is the c.m.  $Z$  boson energy.

The structure of these expressions is rather simple, namely, the appropriate SM contributions to the cross-sections are rescaled by factors

$$\begin{aligned} \kappa_1 &= 1 - (1 + \eta_1) \frac{\bar{c}^2 \bar{s}^2}{\bar{e}^2} \frac{s}{\mathcal{F}^2}, \\ \kappa_2 &= 1 - \eta_1 \frac{\bar{c}^2}{\bar{e}^2} \frac{s}{\mathcal{F}^2}, \end{aligned} \quad (13)$$

$$\kappa_3 = 1 - (\eta_1 - 1) \frac{\bar{s}^2 \bar{c}^2}{\bar{e}^2 (\bar{s}^2 - \bar{c}^2)} \frac{s}{\mathcal{F}^2}. \quad (14)$$

All these expressions are valid in the kinematical region  $m_W^2, m_Z^2 \ll s, |t| \ll \mathcal{F}^2$  and are obtained in the high energy limit by neglecting the terms  $\mathcal{O}(m^2/\mathcal{F}^2, m^2/s)$  relative to these  $\mathcal{O}(s/\mathcal{F}^2)^2$ . All the leading terms in this limit come from the Lagrangian of Eq. 8, i.e., from the VBD interactions. Note that the cross-sections for all the processes  $\mu^+\mu^- \rightarrow \bar{f}f$  (except for  $\mu^+\mu^- \rightarrow \mu^+\mu^-$ ) have the same structure as that for  $\mu^+\mu^- \rightarrow e^+e^-$  (Eq. 10) with the same rescaling factors. Similarly for the process  $\mu^+\mu^- \rightarrow ZHH$  relative to that  $\mu^+\mu^- \rightarrow ZH$ , but here  $(1 - \cos^2 \theta)$  in Eq. 12 should be replaced by a more complicated function of kinematical variables.

The differential cross-section is the most sensitive observable for detecting any kind of contact interactions via the deviation from the SM. But the parameter dependence of the angular distributions is quite involved. To unravel it without calculating a lot of angular distributions we chose as more illuminative a set of integral characteristics. They are: the relative deviation in the total cross-sections from the SM values

$$\Delta(P_\mu) = \frac{\sigma(P_\mu) - \sigma_{SM}(P_\mu)}{\sigma_{SM}(P_\mu)}, \quad (15)$$

with  $\sigma(P_\mu)$  being the polarized cross-section  $\sigma(P_\mu) = (1 - P_\mu)/2 \cdot \sigma(\mu_L^-) + (1 + P_\mu)/2 \cdot \sigma(\mu_R^-)$ , the forward-backward charge asymmetry

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \quad (16)$$

the left-right polarization asymmetry

$$A_{LR} = \frac{\sigma(\mu_L^-) - \sigma(\mu_R^-)}{\sigma(\mu_L^-) + \sigma(\mu_R^-)} \quad (17)$$

and the polarized charge asymmetry

$$A_{LR}^{FB} = \frac{\sigma_F(\mu_L^-) + \sigma_B(\mu_R^-) - \sigma_F(\mu_R^-) - \sigma_B(\mu_L^-)}{\sigma_F(\mu_L^-) + \sigma_B(\mu_R^-) + \sigma_F(\mu_R^-) + \sigma_B(\mu_L^-)}. \quad (18)$$

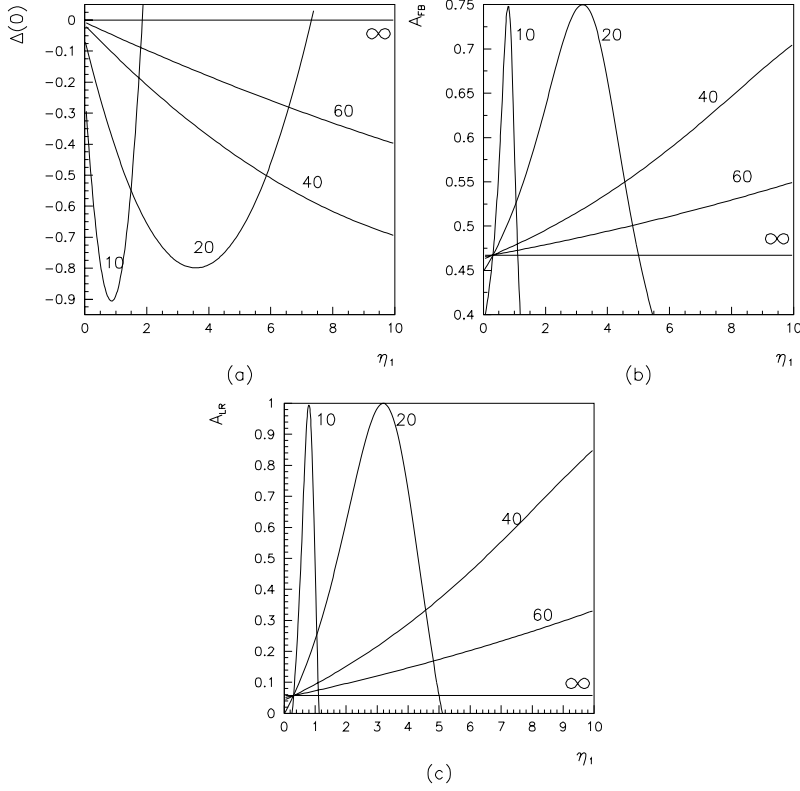
We have calculated these observables (if not trivial) for the processes  $\mu^+\mu^- \rightarrow e^+e^-$ ,  $\mu^+\mu^-$ ,  $\tau^+\tau^-$ ,  $\bar{b}b$ ,  $\bar{c}c$ , *jet jet* and  $\mu^+\mu^- \rightarrow W^+W^-$ ,  $ZH$ ,  $ZHH$  as functions of the parameter  $\eta_1$  for the various values of  $\mathcal{F}$ . Under “jets” we mean only these of the light and charmed hadrons. Fig. 1 is a typical example of such a calculation for the process  $\mu^+\mu^- \rightarrow e^+e^-$ . Note that all the numerical results have been obtained using the exact Born expressions for differential cross-sections. Nevertheless Eqs. 10–12 give good approximations for both the qualitative and quantitative conclusions. For all the processes (except for  $\mu^+\mu^- \rightarrow \mu^+\mu^-$  and the  $W$  pair production) all the asymmetries have the

<sup>2</sup>The net effect of the  $\mu\mu H$  coupling ( $\sim m_\mu$ ) in the total cross-sections of the processes  $\mu^+\mu^- \rightarrow ZH$  and  $ZHH$  proved to be numerically negligible at the energy under consideration.

similar behaviour. First of all, there exists a particular value of  $\eta_1 = \bar{s}^2/\bar{c}^2 \simeq 0.3$  when all the rescaling factors coincide with each other

$$\kappa_1 = \kappa_2 = \kappa_3 = 1 - \frac{\bar{s}^2}{\bar{c}^2} \frac{s}{\mathcal{F}^2}, \quad (19)$$

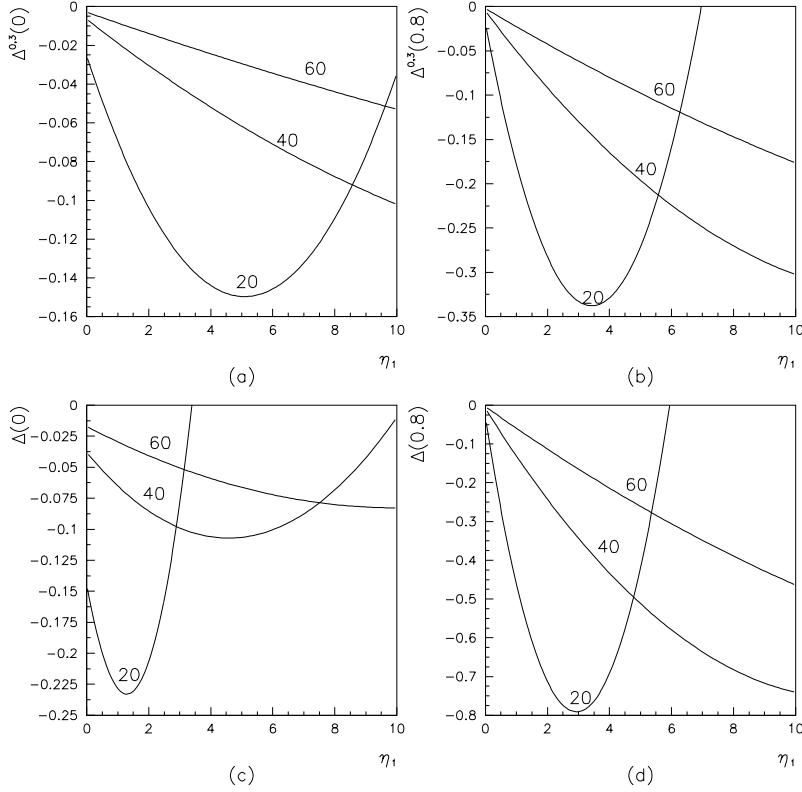
and thus all the asymmetries coincide with those of the SM. The only way to unravel the contact interactions in this particular case is to study directly the total cross-sections. But there should be strong natural reasons for this exceptional case to be realized. Another particular value of  $\eta_1 = g_1^2 \mathcal{F}^2/s$  provides the best case for studying the contact interactions, when all the asymmetries in all the processes saturate their maximal values.



**Figure 1.** Process  $\mu^+\mu^- \rightarrow e^+e^-$ : (a) relative deviations in the total unpolarized cross-section marked with the values of the compositeness scale  $\mathcal{F}$  in TeV; (b) forward-backward asymmetry; (c) left-right asymmetry.

It is of no importance whether muon beam is polarized or not in the case of fermion pair production. But it is not so for the processes  $\mu^+\mu^- \rightarrow W^+W^-, ZH$  and  $ZHH$ .

In all the cases of bosons production one has  $|\Delta(-1)| \ll |\Delta(+1)|$ . Hence one is lead to conclude that it is preferable to work with the maximally right-handedly polarized muons to observe as large deviations in the total cross-sections from the SM as possible. Fig. 2 presents the deviations in the total cross-sections for the unpolarized muons, as well as for the right-handed muons with  $P_\mu = 0.8$ . Here the Higgs boson mass is taken to be  $m_H = 200$  GeV. The results are quite insensitive to it for the light and intermediate Higgs boson. One can see that the deviations for the right-handed polarizations are at least three times as large as these for the unpolarized muon beam.



**Figure 2.** Relative deviations in the total cross-sections: (a) process  $\mu^+\mu^- \rightarrow W^+W^-$  with the  $\mu^-$  polarization  $P_\mu = 0$ ; (b) the same process with  $P_\mu = 0.8$ ; (c) processes  $\mu^+\mu^- \rightarrow ZH$  and  $ZHH$  with  $P_\mu = 0$ ; (d) the same processes with  $P_\mu = 0.8$ .

To evaluate the statistical significance of the observed deviations consider, e.g., the total cross-sections. Taking into account only statistical errors, let us introduce the quantity  $n_\sigma = \Delta N / \sqrt{N_{SM}} = (\Delta\sigma / \sigma_{SM}) \sqrt{\sigma_{SM} \int \mathcal{L} dt}$  that shows the number of the



standard deviations from the SM predictions. We take the integrated luminosity  $\int \mathcal{L} dt$  expectedly to be  $10^3 \text{ fb}^{-1}$  [15]. Fig. 3 and Fig. 4 present the reach for the scale  $\mathcal{F}$  at  $2\sigma$  statistical level (95% C.L.) via the total cross-sections in the various channels. Note that the calculation for the  $W^+W^-$  pair production has been made supposing the instrumental cut-off  $-0.8 \leq \cos \theta \leq 0.8$ . In the cases of both the  $\mu^+\mu^- \rightarrow W^+W^-$  and  $\mu^+\mu^- \rightarrow \mu^+\mu^-$  optimal values of cut-offs, equal to  $-0.8 \leq \cos \theta \leq 0.3$  and  $|\cos \theta| \leq 0.8$ , respectively, have been chosen at the given instrumental ones. Here the reach is maximal due to the maximal suppression of the  $t$  channel peak, at the statistics being still high enough. We see that the VBD can be tested for the unified substructure scale  $\mathcal{F}$  up to  $\mathcal{O}(150 \text{ TeV})$  in the processes  $\mu^+\mu^- \rightarrow ff$ , up to  $\mathcal{O}(100 \text{ TeV})$  in the  $\mu^+\mu^-$  annihilation into boson pairs and up to  $\mathcal{O}(40 \text{ TeV})$  in the process  $\mu^+\mu^- \rightarrow ZHH$  (with the right-handedly polarized muon beam). For comparison we present also the reach for the scale  $\mathcal{F}$  at the  $3\sigma$  statistical level (99% C.L.). We see that it is not much lower, except for the channel  $\mu^+\mu^- \rightarrow ZH$  with the unpolarized muon beam <sup>3</sup>.

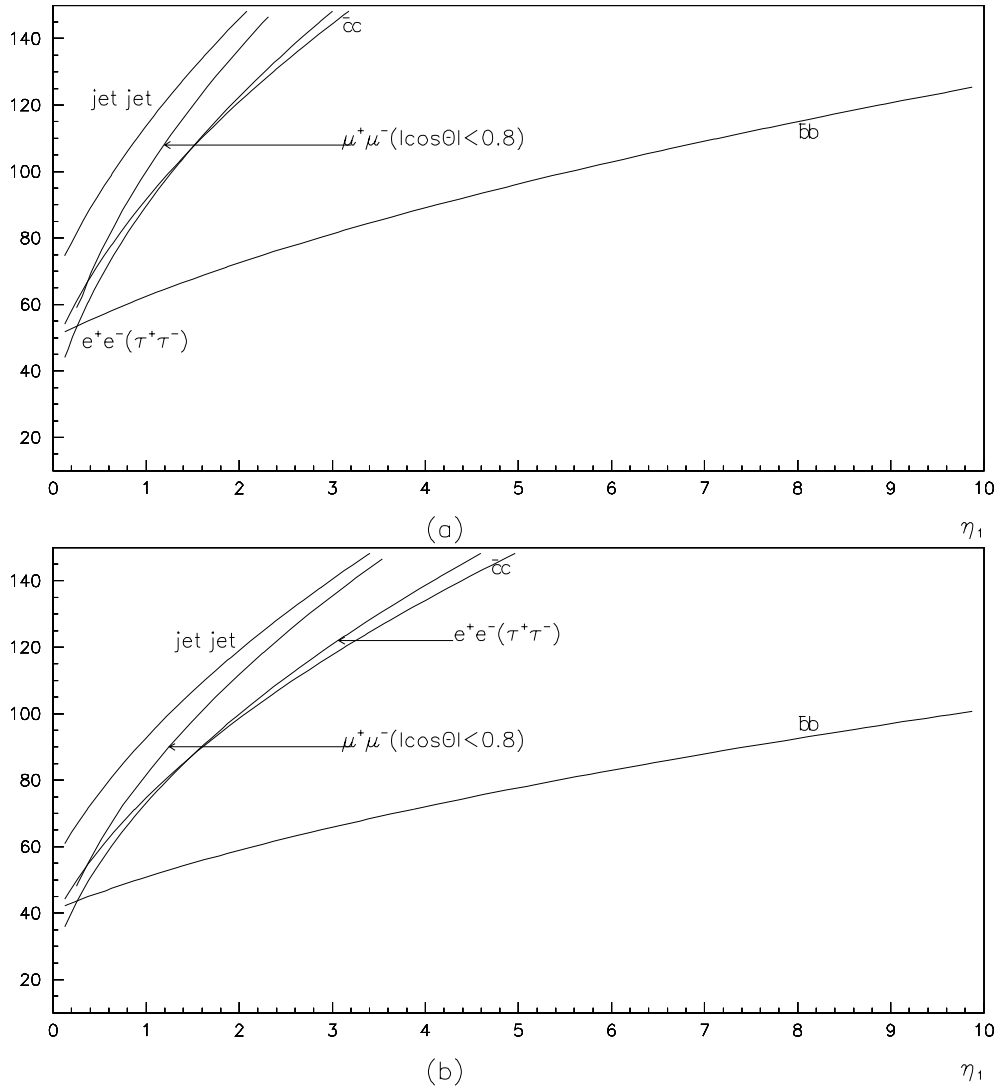
One can estimate the energy and luminosity dependence of the attainable scale  $\mathcal{F}$  by equating the statistical uncertainty in the event number  $\Delta N \sim \sqrt{\int \mathcal{L} dt/s}$  and the expected number of additional events due to the contact interactions  $\Delta N \sim \int \mathcal{L} dt/\mathcal{F}^2$ . This gives  $\mathcal{F} \sim \sqrt[4]{s \int \mathcal{L} dt}$ . Thus the decrease of energy down to 2 TeV at the fixed luminosity would result in  $\sqrt{2}$  decrease in the attainable scale  $\mathcal{F}$ . Hence 2 TeV collider is able to unravel the Deca-TeV substructure, too.

**Anomalous Triple Gauge Interactions** In addition to the VBD interactions, a lot of other “low energy” residual interactions is allowed in the scheme of the unified compositeness. In particular, the exotic triple gauge interactions (TGI) [16] are conceivable too and can contribute to the  $W^+W^-$  pair production. The question arises as to what extent the two types of new interactions could imitate each other.

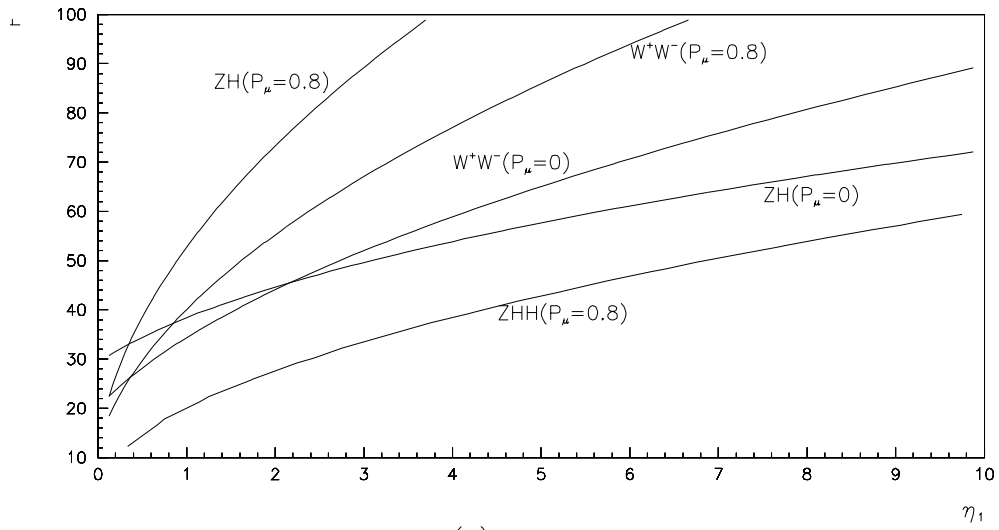
The anomalous TGI should originate from a kind of the SM extension. Here, the SM symmetry  $SU(2)_L \times U(1)_Y$  could be realized either linearly or nonlinearly. In the case of the nonlinear realization (being still linear on the unbroken  $U(1)_{em}$  subgroup), the nonlinearity scale  $\Lambda$  is just the SM v.e.v.  $v$ . Thus this kind of extension, in general, has nothing to do with the unified compositeness we consider. On the other hand, for the linear SM symmetry realization the scale  $\Lambda$  is not directly related with  $v$  and could be as high as desired. Thus we chose it to be the unified compositeness scale  $\mathcal{F} = \mathcal{O}(10 \text{ TeV})$ .

All the conceivable linearly realized residual interactions are described by the  $SU(2)_L \times U(1)_Y$  invariant operators built of the SM fields [17, 18]. All the operators which are relevant to the anomalous TGI vertices are naturally expected to be  $\mathcal{O}(g)$  or less in the gauge couplings, but one exception  $\mathcal{O}_{WS}$ . The latter stems from the nonlinear generalization of the field strengths in the NMSM. The similar gauge kinetic terms of the isotriplet  $W$  and isosinglet  $S$  bosons have no gauge couplings. So the same must naturally happen for

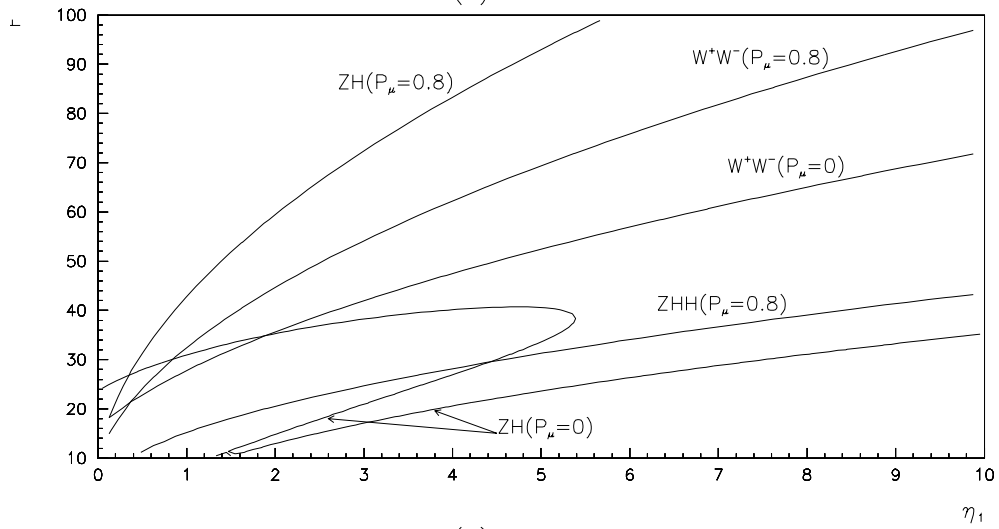
<sup>3</sup>It is to be studied to what extent the  $P_\mu \neq 0$  effect could overwhelm an induced reduction in luminosity.



**Figure 3.** (a) The reach at 95% C.L. ( $2\sigma$  statistical level) for the compositeness scale  $\mathcal{F}$ , vs. the parameter  $\eta_1$ , via studying the total cross-sections of the processes  $\mu^+\mu^- \rightarrow f\bar{f}$  with  $P_\mu = 0$ , where  $f = e, \mu, \tau, u, d, c, b$ ; (b) the same at the 99% C.L. ( $3\sigma$  statistical level).



(a)



(b)

**Figure 4.** (a) The same as in Fig. 3 (a) for the processes  $\mu^+\mu^- \rightarrow ZH$ ,  $W^+W^-$  and  $ZHH$  with two various polarizations  $P_\mu$ ; (b) the same at the 99% C.L. ( $3\sigma$  statistical level).

$\mathcal{O}_{WS}$ , for its origin is of the same nature.

Thus we have retained the  $\mathcal{O}_{WS}$  operator alone and have chosen the proper effective Lagrangian to be

$$\mathcal{L}_{eff} = \frac{C}{2} \frac{1}{\mathcal{F}^2} \mathcal{O}_{WS} \equiv \frac{C}{2} \frac{1}{\mathcal{F}^2} \phi^\dagger \frac{\tau_i}{2} \phi W_{\mu\nu}^i S_{\mu\nu}, \quad (20)$$

where  $C = \mathcal{O}(1)$ . With account for all the contributions from this operator we have found that the deviations from the SM predictions even in this most enhanced TGI case are much smaller than these in the VBD case. So the VBD is in fact dominant.

## Conclusions

The main results of our study are as follows:

- VBD of the SM gauge interactions is expected to be the universal dominant low energy feature of the unified compositeness of leptons, quarks and Higgs bosons.
- VBD of the SM electroweak interactions can be tested at the 4 TeV  $\mu^+\mu^-$  collider for the unified compositeness scale  $\mathcal{F}$  up to  $\mathcal{O}(150 \text{ TeV})$  in  $\mu^+\mu^- \rightarrow \bar{f}f$ , up to  $\mathcal{O}(100 \text{ TeV})$  in  $\mu^+\mu^- \rightarrow ZH, W^+W^-$  and up to  $\mathcal{O}(40 \text{ TeV})$  in the process  $\mu^+\mu^- \rightarrow ZHH$ .
- Processes  $\mu^+\mu^- \rightarrow \bar{f}f$  with various final fermions and  $\mu^+\mu^- \rightarrow W^+W^-, ZH, ZHH$  are mutually complimentary. I.e., at any values of compositeness scale  $\mathcal{F}$  and parameter  $\eta_1$  (but for  $\eta_1 \simeq 0.3$ ) one can choose the environments where the deviations from the SM are not zero. More than that, these deviations are tightly correlated.
- For  $\mu^+\mu^- \rightarrow W^+W^-, ZH$  and  $ZHH$  it is of importance to operate with the right-handed muons to observe as large deviations in the total cross-sections as possible.
- For  $\mu^+\mu^- \rightarrow \mu^+\mu^-$  and  $W^+W^-$  there exist the optimal angular cut-offs  $|\cos\theta| \leq 0.8$  and  $-0.8 \leq \cos\theta \leq 0.3$ , respectively, at which the attainable compositeness scale  $\mathcal{F}$  is maximal.

We conclude that the future  $\mu^+\mu^-$  collider with the total energy 4 TeV and the integrated luminosity  $10^3 \text{ fb}^{-1}$  could present the definite answer about the existence of the Deca-TeV unified compositeness, or v.v.

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## Note added

After the report was submitted, we became aware of ref. [19] where the process  $\mu^+ \mu^- \rightarrow ZH$  was studied in the framework of the SM.

## References

- [1] V. V. Kabachenko and Yu. F. Pirogov, submitted to the 28th Int. Conf. on High Energy Physics, Warsaw, 1996; hep-ph/9612275, to be published in *Yad. Fiz.*
- [2] Yu. F. Pirogov, *Int. J. Mod. Phys. A* **7**, 6473 (1992); in *Proc. of the Int. Conf. "Quarks '92"*, Zvenigorod, 1992, eds. D. Yu. Grigoriev *et al.* (World Scientific, Singapore, 1993) 375.
- [3] Yu. F. Pirogov, *Mod. Phys. Lett. A* **8**, 3129 (1993); in *Proc. of the Int. School "Particles and Cosmology"*, Baksan Valley, 1993, eds. E. N. Alexeev *et al.* (World Scientific, Singapore, 1994) p. 151.
- [4] *Proc. of the First Workshop on the Physics Potential and Development of  $\mu^+ \mu^-$  Collider*, Napa, 1992, *Nucl. Instr. Meth. A* **350**, 24 (1994).
- [5] V. Barger *et al.*, MADPH-95-873 (1995), in *Proc. of the Second Workshop on the Physics Potential and Development of  $\mu^+ \mu^-$  Collider*, Sausalito, 1994.
- [6] J. F. Gunion, UCD-95-35 (1995), in *Proc. of the Intern. Europhysics Conf. on High Energy Physics*, Brussels, 1995.
- [7] V. Barger, M. S. Berger, J. F. Gunion and T. Han, hep-ph/9704290, in *Proc. of the Third Workshop on the Physics Potential and Development of  $\mu^+ \mu^-$  Collider*, Santa Barbara, 1996.
- [8] S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177**, 2239 (1969); C. G. Callan, S. Coleman, J. Wess and B. Zumino, *ibid.*, 2247.
- [9] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, *Phys. Rev. Lett.* **54**, 1215 (1985); M. Bando, T. Kugo and K. Yamawaki, *Nucl. Phys. B* **259**, 493 (1985); *Prog. Theor. Phys.* **73**, 1541 (1985); *Phys. Rep.* **164**, 217 (1988).
- [10] A. D'Adda, P. di Vecchia and M. Lüscher, *Nucl. Phys. B* **146**, 63 (1978); *Nucl. Phys. B* **152**, 125 (1979); A. M. Polyakov, Gauge Fields and Strings, *Contemporary Concepts in Physics*, v. 3 (Harwood Academic Publishers, 1987) p. 139.
- [11] V. V. Kabachenko and Yu. F. Pirogov, *Int. J. Mod. Phys. A* **10**, 3187 (1995).
- [12] V. V. Kabachenko and Yu. F. Pirogov, *Int. J. Mod. Phys. A* **11**, 2293 (1996).
- [13] D. Guryev, V. A. Ilyin, V. V. Kabachenko and Yu. F. Pirogov, in preparation.
- [14] E. E. Boos, M. N. Dubinin, V. A. Ilyin, A. E. Pukhov and V. I. Savrin, preprint SNUTP-94-116, hep-ph/9503280; E. E. Boos *et al.*, *Int. J. Mod. Phys. C* **5** 615 (1994).
- [15] M. Atac, report at this Workshop.
- [16] K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, *Nucl. Phys. B* **282**, 253 (1987); M. A. Samuel *et al.*, *Phys. Rev. Lett.* **67**, 9 (1991).

- [17] C. N. Leung, S. L. Love and S. Z. Rao, *Z. Phys. C – Part. and Fields* **31**, 433 (1986);  
W. Buchmuller and D. Wyler, *Nucl. Phys.* **B236**, 621 (1986).
- [18] A. de Rujula, M. B. Gavela, P. Hernandez and E. Masso, *Nucl. Phys.* **B384**, 3 (1992).
- [19] V. A. Litvin and F. F. Tikhonin, IHEP 97-24 (1997).