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Ordinary and coherent bremsstrahlung at linac-ring ep colliders

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Abstract

The ordinary bremsstrahlung $ep \to ep\gamma$ can be used at the linac-ring ep colliders for luminosity measurement. It is known that at high energies this process has a large correction due to beam-size effect. We have calculated this effect for ep colliders of the linac-ring type. For the LHC+CLIC collider the correction exceeds 10% for $E_\gamma < 0.95\, E_e$.

As a rule, the bremsstrahlung of protons in the ep scattering is not considered due to its small cross section. However, if the photon energy E_γ becomes small enough, the number of produced photons become large because in this case the radiation is determined by the interaction of a proton with the collective electromagnetic field of the electron bunch. It is coherent bremsstrahlung (CBS). We present the main characteristics of CBS calculated for linac-ring ep colliders. At the LHC+CLIC collider it should be about 1700 dE_γ/E_γ photons for a single collision of bunches at $E_\gamma \lesssim 0.2$ MeV. It seems that CBS can be a potential tool for fast control over collisions and for measuring beam parameters. Indeed, the electron bunch length l_e can be found from the critical energy $E_c \propto 1/l_e$, the transverse bunch size $\sigma_\perp$ is related to the photon rate $dN_\gamma \propto 1/\sigma_\perp^2$. A specific dependence of dN_γ on the impact parameters between the beams allows for a fast control over beam displacement.

1. Parameters

In this paper we restrict ourselves to two typical examples of linac-ring ep colliders

$$\mathrm{LHC} + \mathrm{CLIC}: \ E_p = 7 \text{ TeV}, \ E_e = 0.5 \text{ TeV}, \ N_p = 10^{11}, \ N_e = 0.8 \cdot 10^{10},$$

$$\sigma_{px} = \sigma_{py} = 16\ \mu\text{m}, \ \sigma_{ex} = 0.25\ \mu\text{m}, \ \sigma_{ey} = 0.0075\ \mu\text{m}, \ l_e = \sigma_{ez} = 200\ \mu\text{m}, \qquad (1)$$

and

$$\mathrm{HERA} + \mathrm{TESLA}: \ E_p = 0.82 \text{ TeV}, \ E_e = 0.8 \text{ TeV}, \ N_p = 10^{11}, \ N_e = 1.8 \cdot 10^{10},$$

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$$\sigma_{px} = 265\ \mu\text{m},\ \sigma_{py} = 50\ \mu\text{m},\ \sigma_{ex} = 0.4755\ \mu\text{m},\ \sigma_{ey} = 0.0037\ \mu\text{m},\ l_e = 700\ \mu\text{m}\,. \quad (2)$$

Here E_i and N_i are the energy of a particle and the number of particles in the i-th bunch, l_e is the longitudinal, σ_{ix} and σ_{iy} are the horizontal and vertical transverse bunch sizes. Besides, we use notations: $\gamma_i = E_i/(m_i c^2)$ is the Lorentz factor and $r_i = e^2/(m_i c^2)$ is the classical radius of the i-th particle.

2. Ordinary bremsstrahlung

At ep collider HERA one of the important parameters, the luminosity, is measured by detecting the photons of the $ep \to ep\gamma$ reaction (Fig. 1). This reaction has a large cross

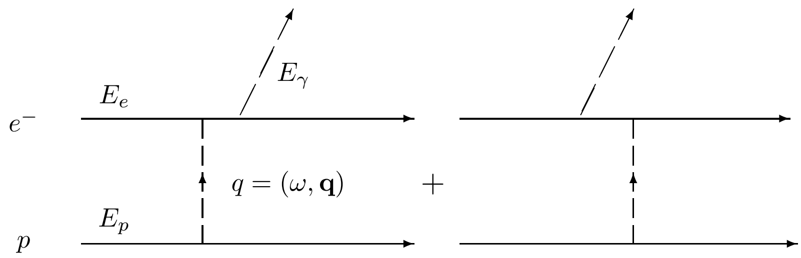

Figure 1. Feynman diagrams for the $ep \to ep\gamma$ process (diagrams for the photon emission by an electron).

section and it is convenient for registration. We may suppose that it is the reaction that will be convenient for luminosity measurement of linac-ring ep colliders as well. The standard cross section of this process for unpolarized beams has a form

$$d\sigma = \frac{16}{3}\,\alpha\, r_e^2\, \frac{dy}{y}\left(1 - y + \frac{3}{4}\,y^2\right)\left(\ln\frac{4\gamma_e\gamma_p(1-y)}{y} - \frac{1}{2}\right)\,,\ \ y = \frac{E_\gamma}{E_e}\,, \quad (3)$$

and it is of the order of 0.1 barn at $y \sim 0.1$ for examples (1) and (2). This corresponds to the rate $\dot{N}_\gamma \sim 10^4$ photons per second for luminosity about 10^{29} cm^{-2} s^{-1}.

It is known since 1982 [1] that at high energies the bremsstahlung cross section has a large correction to the standard expression due to MD or beam-size effect (for review see Ref. [2]). For the HERA collider this effect has been predicted in Refs. [3] and has been observed in 1995 [4]. Let us give the qualitative description of this effect. The discussed process is defined by the diagram of Fig. 1. The main contribution to the cross section is given by the region of small momentum transfer $q_\perp$ and small energy of the virtual (or equivalent) photon produced by a proton. The equivalent photon with energy $\hbar\omega$ and the electron with energy $E_e \gg \hbar\omega$ move toward each other and perform a Compton scattering. The main contribution to the Compton scattering is given by the region where the scattered photons go in a direction opposite to that of the initial photons. For such a backward scattering the energies of the equivalent photon $\hbar\omega$ and of the scattered photon E_γ are related by

$$\hbar\omega \sim \frac{E_\gamma}{4(1-y)\gamma_e^2}\,. \tag{4}$$

The equivalent photons with energy $\hbar\omega$ form a "disk" of radius

$$\varrho_m \sim \frac{c\gamma_p}{\omega} \sim \lambda_e \frac{4\gamma_e\gamma_p(1-y)}{y}\,,\quad \lambda_e = \frac{\hbar}{m_e c} = 3.86\cdot 10^{-11}\text{ cm.} \tag{5}$$

For the discussed colliders this radius is macroscopically large:

$$\varrho_m \gtrsim 1\text{ cm}\quad\text{for}\quad E_\gamma \lesssim 0.1\,E_e\,, \tag{6}$$

and it is by several orders of magnitude larger than the transverse beam sizes.

The standard calculation corresponds to the interaction of the photons forming the "disk" with the unbounded flux of electrons. However, the particle beams at the *ep* colliders have finite transverse beam sizes $\sigma_\perp \lesssim 0.01$ cm. Therefore, the equivalent photons from the region

$$\sigma_\perp \lesssim \varrho \lesssim \varrho_m \tag{7}$$

cannot interact with the electrons from the oncoming beam. This leads to the decreasing number of the observed photons, and the "observed cross section" $d\sigma_{\rm obs}$ is smaller than the standard cross section $d\sigma$ (3) calculated for an infinite transverse extension of the electron beam

$$d\sigma_{\rm obs} = d\sigma - d\sigma_{\rm cor}. \tag{8}$$

Here the correction $d\sigma_{\rm cor}$ is given in the form

$$d\sigma_{\rm cor} = d\sigma_{\rm Compt}(E_\gamma,\omega)\; dn_{\rm mis}(\omega) \tag{9}$$

where $dn_{\rm mis}(\omega)$ denotes the number of "missing" equivalent photons and $d\sigma_{\rm Compt}$ is the cross section of the Compton scattering. In logarithmic approximation this number is of the form

$$dn_{\rm mis} = \frac{\alpha}{\pi}\frac{d\omega}{\omega}\frac{dq_\perp^2}{q_\perp^2}\,. \tag{10}$$

Since $q_\perp \sim \hbar/\varrho$ we have

$$dn_{\rm mis} = \frac{\alpha}{\pi}\frac{d\omega}{\omega}\frac{d\varrho^2}{\varrho^2}$$

with the integration region in ϱ given by Eq. (7). It gives

$$dn_{\rm mis}(\omega) = 2\frac{\alpha}{\pi}\frac{d\omega}{\omega}\ln\frac{c\gamma_p}{\omega\sigma_\perp}\,. \tag{11}$$

Integrating Eq. (9) over ω using the result (11) we obtain the correction to the standard cross section with logarithmic accuracy.

A more accurate calculation at $\varrho_m \gg \sigma_\perp$ or at

$$\Lambda = 2\sqrt{2}\gamma_e\gamma_p \,\frac{1-y}{y}\, \frac{(a_x+a_y)\lambda_e}{a_x a_y} \gg 1\,, \quad a_x = \sqrt{\sigma_{px}^2+\sigma_{ex}^2}\,,\; a_y = \sqrt{\sigma_{py}^2+\sigma_{ey}^2} \tag{12}$$

gives (see Refs. [3])

$$d\sigma_{\rm cor} = \frac{16}{3}\,\alpha r_e^2\,\frac{dy}{y}\,\left\{\left(1-y+\frac{3}{4}y^2\right)\left(\ln\Lambda - \frac{2+C}{2}\right) - \frac{1-y}{12}\right\}\,, \quad C = 0.577\ldots. \tag{13}$$

Fig. 2 presents the ration

$$\frac{d\sigma_{\rm cor}/dy}{d\sigma/dy} \equiv \text{correction} \tag{14}$$

of the correction cross section (13) to the standard QED

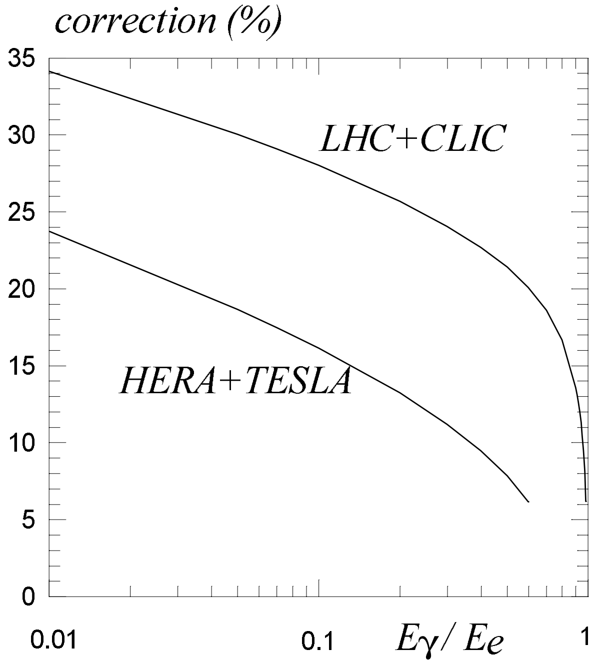

Figure 2. Ratio of the correction cross section (13) to the standard QED cross section (3) in dependence on $y = E_\gamma/E_e$.

distribution (3) in percentages as a function of the photon energy fraction $y = E_\gamma/E_e$ for unpolarized beams. We observed that the correction exceeds 10 % at $E_\gamma < 0.95\,E_e$ for the LHC+CLIC collider (1) and at $E_\gamma < 0.3\,E_e$ for the HERA+TESLA collider (2).

3. Coherent bremsstrahlung of protons

As a rule, bremsstrahlung of protons at *ep* colliders is not consider because its cross section has an additional very small factor $\sim m_e^2/m_p^2 \sim 10^{-7}$ as compare with the cross section (3) for the bremsstrahlung of electrons. However, for small enough photon energies the radiation of a proton becomes coherent and the number of produced photons becomes large. It is known (see, e.g. §77 in [5]) that the properties of this coherent radiation are quite different for proton deflection angles θ_d much larger or much smaller than the typical radiation angle $\sim 1/\gamma_p$. It is not difficult to estimate that

$$\theta_d \sim \frac{\eta}{\gamma_p}; \quad \eta = \frac{r_p N_e}{\sigma_x + \sigma_y}.$$

We call an electron bunch *long* if $\eta \gg 1$. The radiation in this case is usually called *beamstrahlung.*

An electron bunch is *short* if $\eta \ll 1$. In some respect, the radiation in the short bunch fields is similar to the ordinary bremsstrahlung, therefore we call it *coherent bremsstrahlung (CBS).* It differs substantially from the beamstrahlung. In most of the colliders the ratio η is either much smaller than one (all the *pp*, $\bar{p}p$ and relativistic heavy-ion colliders, some e^+e^- colliders and B–factories) or of the order of one (e.g. LEP, TRISTAN). Only linear e^+e^- colliders have $\eta \gg 1$. Therefore, the CBS has a very wide region of use.

A classical approach to CBS was given in [6]. A quantum treatment of CBS based on the rigorous concept of colliding wave packets and some applications of CBS to modern colliders were considered in [7, 8, 9]. A new simple method to calculate CBS based on the equivalent photon approximation for the collective electromagnetic field of the oncoming bunch is presented in [9].

3.1. Distinctions of CBS from the usual bremsstrahlung and from the beamstrahlung

In the usual bremsstrahlung the number of photons emitted by protons is proportional to the number of electrons and protons:

$$dN_\gamma \;\propto\; N_e\; N_p\, \frac{dE_\gamma}{E_\gamma}\;. \tag{15}$$

With decreasing photon energies the coherence length $\sim 4\gamma_p^2 \hbar c/E_\gamma$ becomes comparable to the length of the electron bunch l_e. At photon energies

$$E_\gamma \lesssim E_c = 4\,\gamma_p^2\,\frac{\hbar c}{l_e} \tag{16}$$

the radiation arises from the interaction of the proton with the electron bunch as a whole, but not with each electron separately. The quantity E_c is called the *critical photon energy.*

Therefore the electron bunch is similar to a "particle" with the huge charge $(-e\,N_e)$ and with an internal structure described by the form factor of the bunch. The radiation probability is proportional to N_e^2 and the number of the emitted photons is given by

$$dN_\gamma \;\propto\; N_p\, N_e^2\, \frac{dE_\gamma}{E_\gamma}\;. \tag{17}$$

The CBS differs strongly from the beamstrahlung in the soft part of its spectrum. As one can see from (17) the total number of CBS photons diverges in contrast to the beamstrahlung for which (as well as for the synchrotron radiation) the total number of photons is finite.

The beamstrahlung has been observed in a single experiment at SLC [10], and it has been demonstrated that it can be used for measuring the transverse bunch size of the order of $\sim 5\ \mu$m.

The main characteristics of the CBS have been calculated only recently and an experiment for its observation is now under preparation at VEPP-2M.

3.2. Qualitative description of CBS

Just as in the standard calculations we can estimate the number of CBS photons using equivalent photon approximation. Taking into account that the number of equivalent photons (EP) increases by a factor $\sim N_e$ compared to the ordinary bremsstrahlung we get (using $d(q_\perp^2) \to d^2q_\perp/\pi$)

$$dn_{EP} \sim N_e\, \frac{\alpha}{\pi^2}\, \frac{d\omega}{\omega}\, \frac{d^2q_\perp}{q_\perp^2}. \tag{18}$$

Since the impact parameter $\boldsymbol{\varrho}$ is of the order of $\varrho_x \sim \hbar/q_x,\ \varrho_y \sim \hbar/q_y$, we can rewrite this expression in another form

$$dn_{EP} \sim N_e\, \frac{\alpha}{\pi^2}\, \frac{d\omega}{\omega}\, \frac{d^2\varrho}{\varrho^2}\;. \tag{19}$$

It is not difficult to estimate the region which gives the main contribution

$$|\varrho_x| \sim |\hbar/q_x| \sim \sigma_x, \quad |\varrho_y| \sim |\hbar/q_y| \sim \sigma_y\;. \tag{20}$$

Integrating over this region we obtain

$$dn_{EP} \sim N_e\, \frac{\alpha}{\pi}\, \frac{d\omega}{\omega}\, \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\;. \tag{21}$$

As a result the "effective cross section" for CBS is of the order of

$$d\sigma_{\text{eff}} \sim N_e\, \alpha\, r_p^2\, \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\frac{dE_\gamma}{E_\gamma}. \tag{22}$$

3.3. Results of calculations

The number of the CBS photons for *a single collision* of the beams is (for detail see Refs. [7])

$$dN_\gamma = N_0\, \Phi(E_\gamma/E_c)\, \frac{dE_\gamma}{E_\gamma}.$$

The function

$$\Phi(x) = 1 \ \text{ at } \ x \ll 1; \quad \Phi(x) = (0.75/x^2) \cdot e^{-x^2} \ \text{ at } \ x \gg 1$$

is given explicitly in [7]. Using formulae from Refs. [7] we have calculated quantities E_c and N_0 for colliders (1), (2)

$$\text{LHC} + \text{CLIC}: \ N_0 = 1700\,, \ \ E_c = 0.22 \ \text{ MeV} \tag{23}$$

and

$$\text{HERA} + \text{TESLA}: \ N_0 = 200\,, \ \ E_c = 0.86 \ \text{ keV}\,. \tag{24}$$

Specific features of CBS:

a sharp dependence of spectrum on the electron bunch length, $E_c \propto 1/l_e$;

an azimuthal asymmetry and polarization of photons;

an unusual behavior of CBS photon rate in dependence on the impact parameter between axes of the colliding bunches.

Let us consider in more detail the last property. If the proton bunch axis is shifted in the horizontal (vertical) direction by a distance R_x (R_y) from the electron bunch axis, the luminosity decreases exponentionally

$$L(\mathbf{R}) \ = \ L(0)\, \exp\left(-\frac{R_x^2}{2a_x^2} - \frac{R_y^2}{2a_y^2}\right)\,, \tag{25}$$

where quantities a_x and a_y are defined in (12). But dN_γ for the considered cases decreased much more slowly (see Fig. 3) in accordance with the slow decrease of the electromagnetic fields of the electron bunch on distances of the order of σ_{py}.

Such an unusual dependence of the CBS photon rate on $\mathbf{R}$ can be used for a fast control over impact parameters between beams and over transverse beam sizes. For the case of long round bunches, such an experiment has already been successfully performed at the SLC collider [10].

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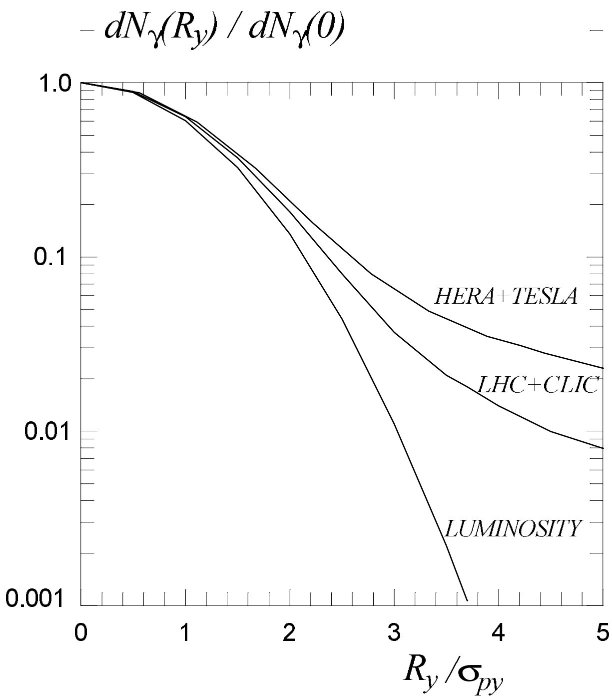

Figure 3. Normalized photon emission rate of CBS in dependence on the displacement R_y of the bunch axes in the vertical direction.

References

[1] A.E. Blinov et al., *Phys. Lett.*, **B 113** (1982) 423; V.N. Baier, V.M. Katkov and V.M. Strakhovenko, *Sov. Yad. Fiz.*, **36** (1982) 163; A.I. Burov and Ya.S. Derbenyev, preprint INP 82-07 (Novosibirsk, 1982).

[2] G.L. Kotkin, V.G. Serbo and A. Schiller, *Inter. J. Modern Phys.*, **A 7** (1992) 4707.

[3] G.L. Kotkin, S.I. Polityko and V.G. Serbo, *Sov. Yad. Fiz.*, **42** (1985) 925; G.L. Kotkin, S.I. Polityko, A. Schiller and V.G. Serbo, *Zeit. Physik*, **C 39** (1988) 61.

[4] K. Piotrzkowski, *Zeit. Physik*, **C 67** (1995) 577.

[5] L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, Nauka, Moscow, 1988

[6] M. Bassetti, J. Bosser, M. Gygi-Hanney, A. Hoffmann, E. Keil and R. Coisson, *IEEE Trans. on Nucl. Science*, **NS-30** (1983) 2182.

[7] I.F. Ginzburg, G.L. Kotkin, S.I. Polityko and V.G. Serbo, *Phys. Rev. Lett.*, **68** (1992) 788; *Phys. Lett.*, **B 286** (1992) 392; *Zeit. Phys.* **C 60** (1993) 737; *Yadernaya Fizika (in Russian)*, **55** (1992) 3310 and 3324

[8] S.I. Polityko and V.G. Serbo, *Phys. Rev.*, **E 51** (1995) 2493; R. Engel, A. Schiller and V.G. Serbo, *Part. Accelerators*, **56** (1996) 1.

[9] R. Engel, A. Schiller and V.G. Serbo, *Zeit. Phys.*, **C 71** (1996) 651.

[10] G. Bonvicini et al. *Phys. Rev. Letters*, **62** (1989) 2381