Tr. J. of Physics
22 (1998) , 719 – 724.


International Workshop on Linac-Ring
Type $ep$ and $\gamma p$ Colliders

# The Heavy Higgs Effects in the Framework of SM

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The Higgs mechanism of spontaneous symmetry breaking is the one of the most intriguing parts of the Standard Model ($SM$) and it is still remains to be understood. On the one hand, the Higgs boson is of utmost necessity for the renormalizability of $SM$, and this principle has proven to be a successful guideline. On the other hand there are no generally accepted theoretical estimates on the Higgs mass up to now. Modern experiments at relatively low effective energies, about 100 $GeV$, still leaves a large room for theoretical speculations. Studying of the processes $e^+e^- \to Hf\bar{f}$ at LEP1 gives the lower bound on the Higgs mass $m_H \geq 63.9\, GeV/c^2$ at the 95% CL [1]. The global fit to the electroweak data from LEP, SLC and FNAL gives a preference to a light Higgs boson $m_H = 149^{+148}_{-82}\, GeV/c^2$, $m_H \leq 450\, GeV/c^2$ [2]. But one can not exclude the possibility of heavy Higgs scenario, since the results on $A_{LR}$, differing from the predictions of the $SM$, dominate in the last estimate. Without $A_{LR}$ data the upper bound on $m_H$ becomes larger than 600 $GeV/c^2$ [3], which is relatively close to the $TeV$ region of energies. One may ask what would the physical consequences be of a very heavy Higgs boson, much heavier than vector bosons $W, Z$? And why we are so interested in the higher order corrections in the case of heavy Higgs scenario? And this is just the subject we would like do discuss in this report.

It was known many years ago that when Higgs mass is well above $m_W, m_Z$ the perturbation theory comes to the strong coupling regime, since the self coupling in the Higgs sector of $SM$ is proportional to $m_H^2$. In this case calculation of the higher order corrections to the $SM$ parameters becomes very important. About one decade ago it was

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pointed out that $\rho$-parameter

$$\rho = \frac{m_W^2}{m_Z^2 \, \cos^2\theta_W} = \frac{g^2}{8 m_W^2 G_F} \tag{1}$$

grows like $m_H^2$ in the two-loop approximation [4]. Together with the one-loop result it looks numerically as follows

$$\rho \simeq 1 - 5.66 \times 10^{-4} \, \log \frac{m_H^2}{m_W^2} + 2.85 \times 10^{-7} \, \frac{m_H^2}{m_W^2}. \tag{2}$$

There is a strong cancellation among the various terms giving contribution in the second-order expression. It concerns mostly terms of order $m_H^4$, which drop out of the final result in complete agreement with the screening theorem [5]. It is this reason why the magnitude of the two–loop correction becomes sizeable only at $m_H = 3 \sim 4\, TeV/c^2$. So, the Higgs boson must be very heavy before the second-order correction is as large as the first order, it happens only at $m_H \sim 10\, TeV/c^2$. But this estimate is quite meaningless, since then the next order is expected to dominate. Extrapolating our result on the next order correction, one may expect that it will be the same as the two-order correction if $m_H \geq 3.3\, TeV/c^2$. Then the perturbation theory breaks down.

Then it is worth to note derivation of the two-loop radiative correction to the $\rho$-parameter and the $GIM$-violating $Zb\bar{b}$-vertex $\tau$ [6]. Calculations were performed in the limit $m_H >> m_W, m_t >> m_W$ up to the forth power of the Yukawa coupling and for arbitrary values of the ratio $m_H/m_t$. A rather interesting aspect of these corrections is that they have little to do with the gauge structure of the $SM$, since they only arise from the symmetry-breaking sector of the theory restricted to the scalar self-interactions and to the top Yukawa coupling. In fact they survive even in the limit $g \to 0$. It should be noted that one-loop corrections are independent on the powers of $m_H$, this dependence appears only at two-loop level. In the case $m_H >> m_t$ these corrections have the form

$$\begin{aligned} \rho &\simeq 1 + N_c \, x + 5 \times 10^{-5} \, \rho^{(2)}, \\ \tau &\simeq -2\, x - 3.2 \times 10^{-5} \, \tau^{(2)}, \\ x &= \frac{G_F \, m_t^2}{8 \, \pi^2}. \end{aligned} \tag{3}$$

Since the exact formulae are rather lengthy, we shall discuss only numerical values of contributions coming from the two-loop corrections. On the one hand, the $\rho^{(2)}$ magnitude is larger than the one of $\tau^{(2)}$. On the other hand, $\rho^{(2)}$ varies slowly as a function of $m_H/m_t$ in a wide range of values of $m_H$ (Tabl. 1). It means that precise measurement of $\tau^{(2)}$ from decay width $Z \to b\bar{b}$ may be of more relevance for determination of $m_H$. The knowledge of the second-order corrections gives also a way to judge the speed of convergence of the perturbative expansion. It occurs that the speed of convergence is good for any reasonable value of the Higgs mass up to $2 \sim 3 TeV/c^2$. Unfortunately these results are slightly invalidated by the fact, that only the leading order corrections in expansion over $m_W/m_t \sim 0.5$ were calculated.

**Table 1.** Values of $\rho^{(2)}$ and $\tau^{(2)}$ as functions of $m_H/m_t$

| $m_H/m_t$ | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|
| $\rho^{(2)}$ | -8.19 | -8.68 | - 9.11 | - 9.48 | - 9.81 | - 10.1 | - 10.4 | - 10.6 | - 10.8 |
| $\tau^{(2)}$ | 1.24 | 1.23 | 1.26 | 1.33 | 1.42 | 1.53 | 1.66 | 1.80 | 1.95 |

So one can conclude that as far as low energy observable consequences are concerned, a screening theorem seems to be in operation. It makes the Higgs boson practically invisible unless it becomes extremely heavy. But screening disappears when we study either Higgs boson characteristics [7-9], or scattering processes at energies $\sqrt{s} >> m_W$, particularly $WW$ scattering [10-11]. First of all it is worth to mention calculation of the self-energy of the Higgs boson to the order $O(g^4\, m_H^4/m_W^4)$ and as a result, derivation of the leading correction to the Higgs resonance position [7]. The momentum dependence of $Im\Sigma_{HH}(k^2)$ describes corrections to the Breit-Wigner shape. By expanding the self-energy around $k^2 = m_H^2$ and keeping only the first derivative, one may obtain the following correction to the Higgs propagator:

$$\frac{1}{k^2 - m_H^2 + i\, m_H\, \Gamma_H} \to \frac{1}{k^2 - m_H^2 + i\, m_H\, \Gamma_H + i\,(k^2 - m_H^2)\, \Gamma_H'}\ , \tag{4}$$

where $\Gamma_H'$ is a first derivative of $\Gamma$. Then scattering amplitude reaches its maximum at

$$k^2 = m_H^2 - m_H\, \frac{\Gamma\Gamma'}{1 + \Gamma'^{\,2}}, \tag{5}$$

where the two-loop value of $\Gamma'$ looks numerically as

$$\Gamma' \simeq 1.0\, \big(\frac{g^2 m_H^2}{16\, \pi^2\, m_W^2}\big)^2. \tag{6}$$

The magnitude of these corrections becomes large if the Higgs boson is heavier than $1.2 \sim 1.3\, TeV/c^2$, signalling strong coupling in the symmetry breaking sector. As it was expected, this bound is sufficiently lower than the value of $\sim 3.3\ TeV/c^2$, coming from two-loop correction to the $\rho$-parameter. Knowledge of the Higgs self-energy permitted also to derive $O(m_H^4)$- corrections to the Higgs decay width to pairs of fermions [7-8] and vector bosons [9]. The resulting correction factors $K_f,\ K_V$ numerically reads

$$K_f \simeq 1 + 0.111 \times \big(\frac{m_H}{TeV}\big)^2 - 0.089 \times \big(\frac{m_H}{TeV}\big)^4, \tag{7}$$

$$K_V \simeq 1 + 0.146 \times \big(\frac{m_H}{TeV}\big)^2 + 0.169 \times \big(\frac{m_H}{TeV}\big)^4. \tag{8}$$

One may easily find the values of $m_H$ at which one-loop correction is equal to the two-loop one. These are $1.11\, TeV/c^2$ and $930\, GeV/c^2$ correspondingly.

All these calculations (excepting [8]) resort at least partly to the numerical methods. The main obstacle in obtaining an analytical expressions for Higgs self-energy, for example, consisted in calculation of the all-massive master integral

$$J(k^2, m_H^2) = -\frac{1}{\pi^4} \times \int D\,P\ \ D\,Q$$
$$N(P, m_H)\, N(P+k, m_H)\, N(Q, m_H)\, N(Q+k, m_H)\, N(P-Q, m_H),$$

where $N(P, m_H)$ is the Higgs propagator. This integral has a discontinuity that is an elliptic integral and is not expressible generally in terms of polylogarithms. But on the mass shell it is not the case. We have used the dispersive method to find this integral, as well as its derivative, and the final result is very simple

$$m_H^2 \times J(k^2 = m_H^2, m_H^2) = \zeta(3) - \frac{2}{3}\pi\, C, \tag{9}$$

$$m_H^2 \times J'(k^2 = m_H^2, m_H^2) = -\zeta(3) + \frac{2}{3}\pi\, C - \frac{\pi^2}{9}, \tag{10}$$

where $C$- is the maximal value of the Clausen function $Cl(\pi/3)$. As a result all the two-loop renormalization constants in the Higgs scalar sector of $SM$ were evaluated analytically to the order $m_H^4/m_W^4$. There is a possibility to obtain these constants at least in the next-to-leading order which is of utmost necessity for correct description of the $WW$ scattering at comparably moderate energies about $\sim 1\,TeV$.

At the tree level the scattering amplitude for longitudinally polarized intermediate vector bosons grows like $s, t, u$ and this growth continues until the Higgs mass is reached, then the amplitude stays constant $\sim m_H^2$:

$$A_{WW}^0 \simeq i\, g^2\ (2\,\pi)^4 \Big( \delta_{ab}\delta_{cd} \big( \frac{s}{4\,m_W^2} + \frac{s^2}{4\,m_W^2}\ \frac{1}{-s+m_H^2} \big) + \big(s \to t\big) + \big(s \to u\big) \Big),$$

in the approximation in which the weak angle vanishes, and so all $W$ are of the equal mass. At the one-loop level the amplitude $A_{WW}^1$ was already derived in the limit of large $s$, $m_H^2 >> s >> m_W^2$ [10]. The ratio of the $A_{WW}^1$ to the tree-level amplitude at $t = -s$ looks like this:

$$R = \frac{A_{WW}^1}{A_{WW}^0} = -\frac{\alpha_W\ s}{24\,\pi\ m_W^2} \log \frac{s}{m_H^2}. \tag{11}$$

Substituting $m_W, \sqrt{s} \sim 0.5\,TeV$ and taking $m_H \sim 1 - 2\,TeV/c^2$, one may conclude that $R$ remains in the region $3 - 5\%$. Though it is not very much, one may hope to detect signal from heavy Higgs at future colliders with effective energy about $0.5\,TeV$.

Once again we see that amplitude at the one-loop level weakly depends on $m_H$. Bearing in mind that even the tree - level amplitude heavily depends on $m_H$ when $\sqrt{s} >> m_H >> m_W$, a group of the authors has recently attempted to calculate $WW$-scattering amplitude in the same limit, but at the two-loop approximation [11]. Though only a few number of two-loop diagrams survives in this limit, the total amount of rather hard work remains enormous. May be it was this reason that led to mistakes and final surprising conclusion that unitarity restricts the $m_H$ to be less than 380 $GeV/c^2$. Reevaluation of these results brought the upper bound on $m_H$ back to the $TeV$ scale [7].

And now there are a few final remarks. In our opinion, amongst the problems which still remains to be solved, there is a very interesting task to obtain the $WW \to WW, ZZ, ..$ - amplitudes in the limit $m_H >> \sqrt{s} >> m_W$ at the two-loop approximation. One can hardly expect a large two-loop contribution, in fact it will be $\sim 0.1 - 0.3\%$ compared with the tree-level amplitude. Nevertheless this problem becomes rather actual now and will be a burning issue when the new generation of $e^+e^-$-colliders will come into operation. We have already made a major part of work, concerned with calculation of the irreducible four-boson vertices to the order $O(s^3/m_W^6), O(t^3/m_W^6), O(u^3/m_W^6)$. The main tool we have exploited in handling with these diagrams was the so called $As$ - operation (method of asymptotic expansion) [12]. This technique reduces calculation of the two-loop diagrams, for example, to derivation of either two-loop master integrals with zero external momenta or one-loop integrals of general type. It is significantly easier to obtain two - and three – point vertices, so we hope to complete derivation of the $WW$ - scattering amplitudes in the coming months.

To conclude, we can state that higher order corrections are very important in the case of heavy Higgs scenario. If the Higgs boson is heavy, they will be numerically large and will play a role in the Higgs search. Knowledge of the higher order corrections is the only way to find out, to which point the perturbative theory predictions can be trusted. In our case the ultimate Higgs mass is about 1 $TeV/c^2$. And finally, knowledge of corrections to the processes with heavy particles or large momenta on the external legs is of considerable importance, because it permits to see the details of the symmetry breaking mechanism, otherwise hidden by the screening theorem.

We would like to thank the Ankara University (Fen Fakultesi) and Z. Aydin for hospitality. This work was supported in part by the Alexander von Humboldt Foundation and the Russian Foundation for Basic Research grant 96-02-19-464.

## References

[1] D. Buskulic et. al., *Phys. Lett.* **B384** (1996) 427.

[2] A. Blondel, Plenary talk at the Int. Conf. on High Energy Physics, Warsaw, 1996.

[3] P. Langacker, Talk at Int. Workshop on Supersymmetry and Unification of Fund. Interactions (SUSY 95), Palaiseau, France, 15-19 May 1995.

[4] J. van der Bij and M. Veltman, *Nucl. Phys.* **B231** (1984) 205.

[5] M. Veltman, *Acta Phys. Polon.* **B8** (1977) 475.

[6] R. Barbieri et. al., *Nucl. Phys.* **B409** (1993) 105.

[7] A. Ghinkulov and J. van der Bij, *Nucl. Phys.* **B436** (1995) 31.
A. Ghinkulov, *Phys. Lett.* **B337** (1994) 137.

[8] V. Borodulin and G. Jikia, *Phys. Lett.* **B391** (1997) 434.

[9] A. Ghinkulov, *Nucl. Phys.* **B455** (1995) 21.
A. Frienk, B. A. Kniehl, D. Kreimer and K. Riesselman,
*Phys. Rev.* **D54** (1996) 4548.

[10] M. Veltman and F. Yndurain, *Nucl. Phys.* **B325** (1989) 1.

[11] P. Maher et al., *Phys. Rev.* **D48** (1993) 1061,
*Phys. Rev.* **D48** (1993) 1084,
*Phys. Rev.* **D52** (1995) 553 (Err).

[12] F. Tkachov, *Int. J. Mod. Phys.* **A8** (1993) 2047.