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Flavour Violating τ-Decays in Magnetic Interaction and E_6 Models

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Abstract

In the view of future $c-\tau$-factories, we report an analysis of the rare decays of the τ lepton via $\tau \to \mu e\overline{e}$, $\tau \to e\mu\overline{\mu}$, $\tau \to \mu\mu\overline{\mu}$ and $\tau \to ee\overline{e}$ in the two models, i.e., in Barut's magnetic interaction model (MIM) and in an E_6 model proposed by Gürsey and his collaborators. The energy distributions of one of the final leptons are given for both models. Numerical estimates reveal that these decays can be as large as 10^{-6}, which may be observed in LEP experiments or at other $c-\tau$ factories.

1. Introduction

There are a number of proposals to construct charm-tau factories with center of mass energies of $\sqrt{s} = 3-5$ GeV in order to investigate the properties of the τ-lepton and charmed hadrons in details. Standard (ring-ring) type $c-\tau$-factories [1] allow luminosities up to $L = 10^{33}$ cm^{-2}s^{-1} and this value rises up to $L = 2 \cdot 10^{34}$ cm^{-2}s^{-1} for linac-ring type machines [2]. It is expected that standard type factories will produce $3.5 \cdot 10^7$ and linac-ring type machines $7 \cdot 10^8 \tau^+\tau^-$ pairs per year (10^7 s) at $\sqrt{s} = 4.25$ GeV where $e^+e^- \to \tau^+\tau^-$ cross section can achieve a maximal value of $\sigma = 3.5$ nb. With such a high statistics, it will be possible to investigate the rare decays of the τ-lepton. Charm-tau factories will be sensitive to the branching ratios of lepton flavour violating decay modes ($\tau \to l\gamma$, $\tau \to \mu l^+ l^-$, etc.) down to $BR \simeq 10^{-7} - 10^{-8}$, whereas the present upper limits lie in the range $10^{-4} - 10^{-5}$.

It is well known that the lepton flavour violating decay modes of the τ-lepton are predicted to be extremely small in the framework of the standard model (SM). But some

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non-standard models can give branching ratios large enough for these rare decays. In the case of observation of lepton number violating τ decay modes, the problem of finding out the true underlying model will arise. For this reason, we study rare τ-decays in two different models, i.e., in the magnetic model proposed by Barut [3] and in the E_6 model, originally proposed by Gürsey and his collaborators [4].

2. Models

2.1. Magnetic Interaction Model

The dynamics of magnetic interactions in particle physics is an important problem, especially at short distances. At atomic distances, the magnetic forces are about α^2 times smaller than the electric forces. However, if one extrapolates them down to the nuclear distances they become $1/\alpha^2$ times larger than the electric forces. Considering the anomalous magnetic moment as an intrinsic property, in addition to the minimal Dirac coupling, there would also be a Pauli coupling of the form $a\overline{\psi}\sigma_{\mu v}\psi F^{\mu v}$, where the coupling constant a for this extra term is not connected with the particle's electric charge, but defines the anomalous magnetic moment of a fermion, and it is a gauge invariant quantity [5]. The original version of the model contains lepton flavour-violating vertices like $\gamma - l - l'$. In principle, similar magnetic moment type interactions can be introduced also for the Z boson by an appropriate modification of the model. This possibility will be considered elsewhere.

The corresponding Feynman diagrams for the decays $\tau \to l'll(l, l' = e, \mu)$ are shown in Figures 1-a, b. The contribution from the Figure 1-b comes only in the decays $\tau \to ll\overline{l}$ i.e. when $l' = l$. The transition amplitude is written as

$$M_a = \frac{-e\kappa}{2(p_1 - p_3)^2}\overline{u}(p_3)(\gamma^\mu\gamma \cdot k - \gamma \cdot k\gamma^\mu)u(p_1)\overline{u}(p_4)\gamma_\mu v(p_2), \tag{1}$$

where $k = p_1 - p_3$ is the momentum transfer through the intermediate photon propagator and $\kappa = a_{\tau l}\frac{e}{2m_e}$ denotes the transition magnetic moment. Using the experimental upper bounds for the branching ratios [6]

$$BR(\tau \to e\gamma) < 1.1 \times 10^{-4} \tag{2}$$

$$BR(\tau \to \mu\gamma) < 4.2 \times 10^{-6}, \tag{3}$$

we obtain the upper bounds for the transition magnetic moments $a_{\tau l}$:

$$a_{\tau e} < 0.8 \times 10^{-10} \tag{4}$$

$$a_{\tau\mu} < 1.5 \times 10^{-11}. \tag{5}$$

In the case of two identical leptons in final states ($l' = l$), the transition amplitude will be $M = M_a - M_b$, where $M_b = M_a(p_3 \leftrightarrow p_4)$. The subscripts a and b refer to Figures 1-a and 1-b, respectively. The calculation of the matrix element squared is straightforward, but since it is too long we do not write down it here.

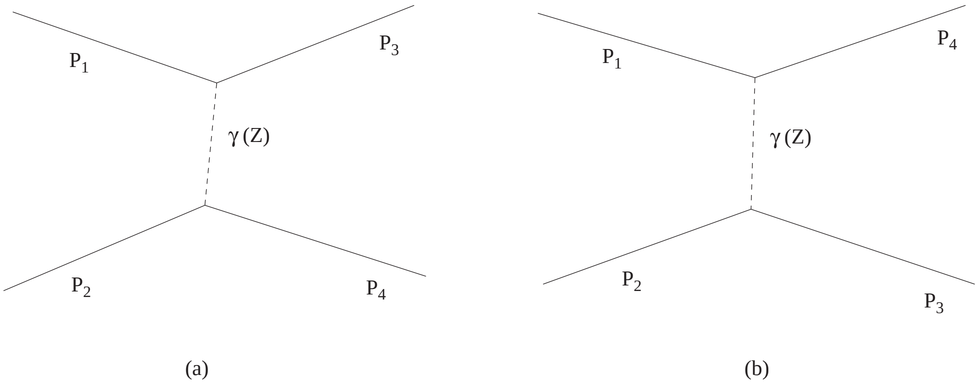

Figure 1. Feynman diagrams for the decays $\tau \to l'l\bar{l}$.

2.2. E_6 Model

This model, originally proposed by Gürsey and his collaborators [4], is now favorable according to the superstring approach (see Ref.[7]). In the model $l-\tau-Z$ vertex arises due to the mixings between right handed components of the ordinary and new heavy charged leptons [8]. The matrix element for the decay $\tau \to l'l\bar{l}$ reads as

$$M_a = \frac{\overline{g}^2 b_{\tau l'}}{8M_Z^2}\overline{u}(p_3)\gamma^\mu(1+\gamma_5)u(p_1)\overline{u}(p_4)\gamma_\mu\left[2x_W - \frac{1}{2}(1-\gamma_5)\right]v(p_2), \tag{6}$$

where $x_W = \sin^2\theta_W = 0.21$ and $b_{\tau l'}$ denotes some combination of the leptonic mixing angles. We use the experimental upper bounds for the lepton flavour-violating Z decays [6],

$$BR(Z \to \tau e) < 9.8\times10^{-6} \tag{7}$$

$$BR(Z \to \tau\mu) < 1.7\times10^{-5}, \tag{8}$$

to obtain the upper bound values:

$$b_{\tau e} < 1.2\times10^{-2} \tag{9}$$

$$b_{\tau\mu} < 1.6\times10^{-2}. \tag{10}$$

The expression for the $|M_a|^2$ is

$$|M_a|^2 = \frac{\overline{g}^4 b_{\tau l'}^2}{M_Z^4}[8x_W^2 p_1\cdot p_2 p_3\cdot p_4 + (8x_W^2 - 8x_W + 2)p_1\cdot p_4 p_2\cdot p_3 - 4x_W(1-2x_W)m^2 p_1\cdot p_3]. \tag{11}$$

As $M_b = M_a(p_3 \leftrightarrow p_4)$

$$|M|^2 = |M_a|^2 + |M_b|^2 - 2M_a^* M_b, \tag{12}$$

where the cross term is

$$M_a^* M_b = \frac{\overline{g}^4 b_{\tau l'}^2}{32M_Z^4}[4x_W^2 m m_1(p_2\cdot p_3 + p_2\cdot p_4 - p_3\cdot p_4) + (1-4x_W)m m_1 p_2\cdot p_3 + 8x_W^2 m^3 m_1]. \tag{13}$$

3. Calculation

To proceed, we first write the usual form of the differential decay rate

$$d\Gamma = \frac{|M|^2}{2m_\tau}\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1 - p_2 - p_3 - p_4), \tag{14}$$

where M is the total transition amplitude, $E_i = \sqrt{\vec{p}_i^{\,2} + m_i^2},\ i = 2,3,4$. The total rate can be calculated from Eq. (14) integrating over $\vec{p}_2, \vec{p}_3$ and $\vec{p}_4$. In the rest frame of the initial τ lepton, the decay rate is reduced to the following form:

$$\Gamma(\tau \to l'l\bar{l}) = \frac{1}{64\pi^3 m_\tau}\int_m^{E_2^+} dE_2 \int_{E_4^-}^{E_4^+} dE_4 |M|^2. \tag{15}$$

The phase-space boundaries to be used in the evaluation of (15) are found to be

$$E_4^\pm = \frac{1}{2(m_\tau - E_2 - |\vec{p}_2|)}\{0.5(m_\tau^2 - m_3^2 + 2m^2 - 2m_\tau E_2) \\ +[[0.5(m_\tau^2 - m_3^2 + 2m^2 - 2m_\tau E_2)]^2 \mp 2m^2|\vec{p}_2|(m_\tau - E_2 - |\vec{p}_2|)]^{1/2}\} \tag{16}$$

and

$$E_2^+ = \left[\frac{1}{4m_\tau^2}(m_\tau^2 - m_3^2)(m_\tau^2 - m_3^2 - 4mm_3 - 4m^2) + m^2\right]^{1/2}. \tag{17}$$

Here, m refers to $m_2 = m_4$.

The energy distributions of the final (anti) leptons, $\frac{1}{\Gamma}\frac{d\Gamma}{dE_2}$ versus E_2, are shown in the Figures 2, 3 in MIM and in the Figures 4, 5 in E_6. It is seen that the models under considerations lead to drastically different energy distributions of the final leptons. This fact will help to separate the models.

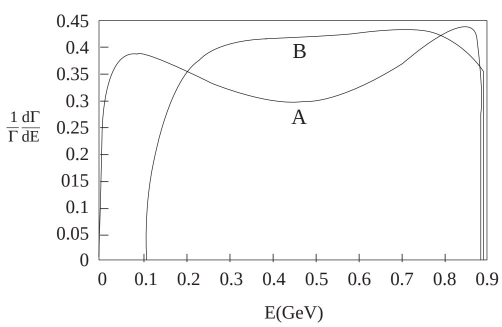

Figure 2. Energy distributions of the final antilepton in MIM. Lines A and B correspond to the decays $\tau \to \mu e\bar{e}$ and $\tau \to e\mu\bar{\mu}$, respectively.

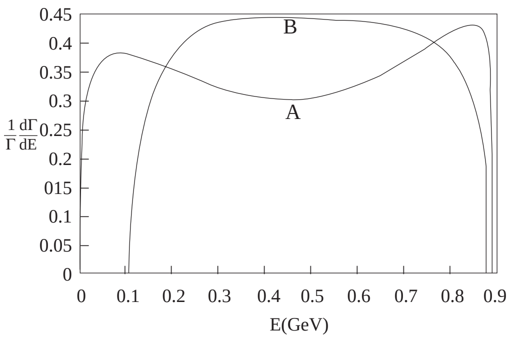

Figure 3. Energy distributions for the final antilepton in MIM for the decays $\tau \to \mu\mu\bar{\mu}$ (line A) and $\tau \to ee\bar{e}$ (line B).

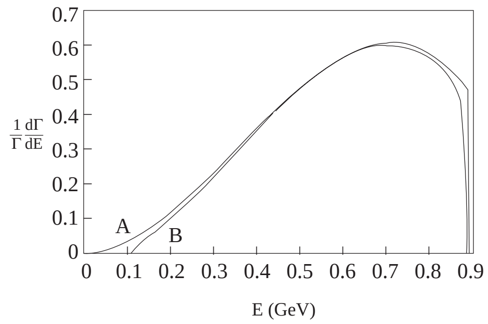

Figure 4. Same distributions as in Figure 2 for the E_6 model.

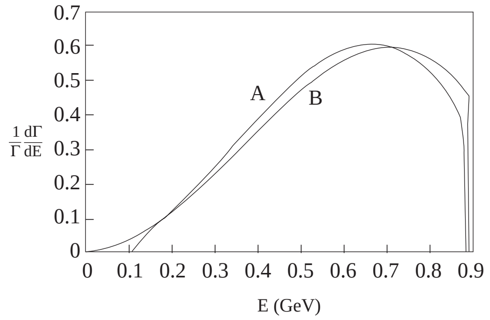

Figure 5. Same distributions as in Figure 3 for the E_6 model.

The experimental value for the total width of the τ lepton is $\Gamma_\tau = 2.1481 \times 10^{-12}$ GeV [6], so we obtain the following values for the branching ratios in the MIM (using the limits in Eqs. (4) and (5)):

$$\begin{aligned} BR(\tau \to \mu e\overline{e}) &< 5.5 \times 10^{-7} \\ BR(\tau \to e\mu\overline{\mu}) &< 3.3 \times 10^{-6} \\ BR(\tau \to \mu\mu\overline{\mu}) &< 1.1 \times 10^{-7} \\ BR(\tau \to ee\overline{e}) &< 1.6 \times 10^{-5}. \end{aligned} \tag{18}$$

Using the limits in the Eqs. (9) and (10) we obtain the corresponding branching ratios in the E_6 model:

$$\begin{aligned} BR(\tau \to \mu e\overline{e}) &< 6.1 \times 10^{-6} \\ BR(\tau \to e\mu\overline{\mu}) &< 3.4 \times 10^{-6} \\ BR(\tau \to \mu\mu\overline{\mu}) &< 5.3 \times 10^{-6} \\ BR(\tau \to ee\overline{e}) &< 3.6 \times 10^{-6}. \end{aligned} \tag{19}$$

Finally, in the MIM the branching ratios for the processes $\tau \to 3l$ and $\tau \to l\gamma$ are not independent and one has the following relations:

$$\begin{aligned} BR(\tau \to ee\overline{e}) &< 0.15 BR(\tau \to e\gamma) \\ BR(\tau \to e\mu\overline{\mu}) &< 0.03 BR(\tau \to e\gamma) \\ BR(\tau \to \mu\mu\overline{\mu}) &< 0.026 BR(\tau \to \mu\gamma) \\ BR(\tau \to \mu e\overline{e}) &< 0.13 BR(\tau \to \mu\gamma). \end{aligned} \tag{20}$$

Similarly, in the E_6 model one can write the following set of relations:

$$BR(\tau \to ee\overline{e}) < 0.37 BR(Z \to e\tau)$$

$$\begin{aligned} BR(\tau \to e\mu\overline{\mu}) &< 0.35 BR(Z \to e\tau) \\ BR(\tau \to \mu\mu\overline{\mu}) &< 0.31 BR(Z \to \mu\tau) \\ BR(\tau \to \mu ee) &< 0.36 BR(Z \to \mu\tau). \end{aligned} \tag{21}$$

A comparison of these two sets of relations can also be used to distinguish the two scenarios if lepton number violation will be observed in the tau sector.

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