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Statistical Physics, Neural Networks, Brain Studies

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Abstract

An overview of some aspects of a vast domain, located at the crossroads of physics, biology and computer science is presented: 1) During the last fifteen years, physicists advancing along various pathways have come into contact with biology (computational neurosciences) and engineering (formal neural nets). 2) This move may actually be viewed as one component in a larger picture. A prominent trend of recent years, observable over many countries, has been the establishment of interdisciplinary centers devoted to the study of: cognitive sciences; natural and artificial intelligence; brain, mind and behaviour; perception and action; learning and memory; robotics; man-machine communication, etc. What are the promising lines of development? What opportunities for physicists? An attempt will be made to address such questions, and related issues.

1. Introduction

According to rough estimates, the present world annual output of scientific publications is around one million, among which half are in biology, and ten per cent in physics. The dynamism of modern life sciences is perhaps made even more apparent by the share of coverage they receive, in the two main interdisciplinary journals: 'Nature' and 'Science'.

Actually during the last fifty years, many branches of physics have matured and generated successful technologies (electronics, computers, energy, communication, space). Physics has been a pioneer for Big Science, but the trends toward gigantism are meeting financial bounds, especially after the post Cold War turn, that occurred around 1990 (collapse of Soviet physics, abandon of the SuperCollider, phasing out of fast-breeder and fusion reactors, mitigated prospects for human space flights).

No wonder then if, in this context, a growing proportion of young physicists are paying attention to the new opportunities offered by the physics-biology interface.

This is a vast interface, and the modest purpose of this review is to draw lessons from some acquired experience.

In the eighties, a group of theoretical physicists followed the trajectory described in my title. As one element in this group, I enjoyed the thrill of adventure and the rush to explore wide domains ranging from neurobiology to psychology. But simultaneously I strongly felt the dangers and pitfalls of 'scientific tourism', namely the difficulty to perceive deep issues and to discern promising latent trends. It took time to acquire some intuition for the dynamics of change, in an unfamiliar discipline. Indeed, during the last ten years, I considered as one of my duties to provide that type of assessment for younger physicists, and to help guide them toward adequate openings at the interface between physics and biology.

In such spirit, the focus of this contribution will be on trends and perspectives.

2. Outline

The heyday for the statistical physics of neural networks occurred circa 1984–1988. Of course, in the study of 'formal' or 'real' neural networks, much had been done earlier, and a lot of activity would follow afterwards. But this period was one of momentous excitement when a group of statistical physicists, emboldered by previous breakthroughs in the physics of disordered systems, and inspired by the 1982 article of John Hopfield, applied a kind of 'right of pursuit', that was to project them inadvertently into the fields of neurobiology and brain studies.

Accordingly, a chronological thread runs through the topics listed below, which correspond roughly to successive periods: before, during, after.

1. A trajectory through statistical physics
    - the Kondo problem (magnetic impurity in a metal)
    - phase transitions and critical phenomena - classification of defects
    - disordered systems, with emphasis on spin glasses
2. Neural networks
    - generalities: structures, learning and memory, mental representations
    - short term memory, with emphasis on palimpsests models (models able to learn and forget)
    - neurocomputation: some issues
3. Recent advances in brain studies
    - experimental observations, with emphasis on brain imaging and the advent of fMRI (functional magnetic resonance imaging, born around 1992)
    - memory: subdivision of tasks and areas (brain locations), and the transition from STM (short term memory) to LTM (long term memory)
    - working memory (and attention, planning)
    - perception (and awareness)
4. Other topics in biophysics with emphasis on noise, Brownian motors, and manipulation of single biomolecules

In this article, topic 1. will be detailed and all topics (1–4) will be extensively referenced.

3. A Trajectory through Statistical Physics

3.1. Kondo Problem

The study of dilute magnetic alloys (e.g., a small concentration of manganese atoms, substituted at random inside a copper matrix) was marked by two events:

- in the fifties, the experimental discovery of a low temperature anomaly in the resistivity,
- in the sixties, the theoretical discovery of a divergence in a second-order perturbation term,

within the simplest conceivable model of one magnetic impurity in a metal (exchange coupling between the impurity magnetic moment $\vec{S}$ and the spin density $\vec{s}(0)$ of conduction electrons at the impurity site).

$$\mathcal{H} = -J\vec{S} \cdot \vec{s}(0)$$

Anomalous behaviour appears when the coupling constant J is negative, i.e., antiferromagnetic.

In the mid-sixties, it seemed that the Bardeen-Cooper-Schrieffer theory (1957), followed by the Josephson effect, had finally solved the long-standing problem of superconductivity. A group of condensed matter theorists was looking for a new challenge. The Kondo problem turned out to be a fortunate choice in this regard (much more so indeed than anyone could have guessed at the time), because it has proved fertile over many years, and still remains active nowadays.

After a few attempts at approximate solutions, which remained inconclusive (for good reasons: the problem is too hard to be treated perturbatively), some of the productive developments were:

- early use of novel renormalization ideas, providing hints of an infinite coupling fixed point, attractive for all negative values of J,
- original formulation of the renormalization problem, with a numerical solution which produced novel quantitative predictions (K.Wilson),
- elegant and powerful Fermi liquid theory (P.Nozières),
- analytical solution based on the Bethe Ansatz (N.Andrei).

In subsequent decades, the magnetic impurity problem was to be revived in various contexts: valence fluctuations, heavy fermions, correlated electrons. Current work is touching on: impurity pairs, Kondo lattice, coexistence between magnetism and superconductivity.

Morality:

- The story of the Kondo problem may be seen as an example of a scientific process, which has occurred repeatedly; at first only a faint anomaly is observed (in an experiment, or in a theoretical model), and this hint is disregarded as an artefact by most people; a few scientists insist however and, step by step, bring to light a large and novel structure, which so far laid hidden; such a discovery may be dubbed as originating from a 'tip of the tail of the cat' process.
- The Kondo story serves also as an illustration of the thesis that deep, original, fundamental problems may arise at all hierarchical levels of matter, and even in odd or seemingly obscure corners; this thesis was elaborated by P.W.Anderson in his prophetic article 'More is different' (1972).

3.2. Phase Transitions, Critical Phenomena, Renormalization Group

This domain has seen one of the two great success stories of physics in the seventies (the other one being the unification of fundamental interactions within the 'standard model'). It did much for the fame of statistical physics.

In the thirties, the Landau theory had established a universal framework for continuous phase transitions (essentially, a general formulation of mean-field theory). But its predictions for the critical exponents were too universal: they did not fit experiments, nor exact results obtained on low-dimensional models. Subsequently the scaling theory led to four scaling laws relating critical exponents; one among the four, the Josephson scaling law, included an explicit dependence on space dimension d.

What struck me most, early in the game (I was an outside observer, at the time), was the revelation about the special role of $d = 4$. It appeared so plainly obvious retrospectively. How could it have been overlooked for so long? Mean-field theory was now finding its realm in highest dimensions, down to $d_c = 4$ (the only dimension for which mean-field exponents obey Josephson's scaling law). Below the critical dimension, the exponents had a restricted universality: they were dependent on d (and also on the nature of the symmetry breaking associated with the transition). As a consequence, the results obtained on bulk materials, in $d = 3$, could be neatly re-appraised as interpolating between the mean-field values (valid for $d = 4$) and those derived from exactly soluble one-dimensional or two-dimensional models. Everything was falling in its place (like the chemical elements in the Mendeleiev table, to use a bold analogy).

At the time, percolation (which qualifies as the simplest collective phenomenon in disordered systems) was not commonly treated as a phase transition. But the existence of a threshold, and of critical exponents, was well recognized.

Putting together the various pieces of existing data, it struck me that they were incompatible with $d_c = 4$, but that everything would fit beautifully, if the critical dimension for percolation was 6, instead of 4. Although the conjecture met initial skepticism, it was shortly afterwards confirmed by Scott Kirkpatrick, via numerical simulations made in high dimensions.

Around the same period, polymer statistics (namely, their geometric properties, in the limit of large size) were brought also within the framework of critical phenomena. This discovery instigated a lot of activity in chemical physics ... and also a sustained quarrel between opposite groups of chemists and physicists. Many polymer chemists claimed that these physical studies were irrelevant for polymer science, and they insisted that the real core of interest was in the effects of chemical diversity, and not in asymptotic properties of homopolymers. This controversy is worth mentioning here, because quite similar attitudes of outright rejection (for good or bad reasons) are spontaneously adopted by many biologists about any models which bear the mark of physics.

Morality: The advances of statistical physics in the early seventies fulfilled many criteria of great success, according to physicists' standards. Unification of many past results, previously scattered in different subfields of physics, or even outside; powerful concepts and tools: scale invariance, renormalization theory, numerical simulation. This success nurtured a conquering mood: lots of newcomers were attracted to statistical physics, while many of the protagonists felt equipped, and confident, for novel adventures and explorations.

3.3. The Topological Classification of Defects in Ordered Media

The study of defects in ordered media was then a collection of results accumulated over a secular history: dislocations in solids, vortices, singular lines in smectic and nematic liquid crystals, magnetic domain walls, etc. Yet, there was no unifying theory, able to predict the stability of defects, and to derive specific predictions from the broken symmetry of the ordered phase.

Mounting pressure, for an unambiguous determination of all topologically stable defects, was coming from the discovery of complex ordered phases, recently found in superfluid Helium 3 and in liquid crystals. Clearly, geometrical intuition had reached its limits, and was no longer sufficient. Cross-fertilization between particle and condensed matter physics proved again useful; diverse hints helped to put the search on the right track.

The theory (1976) involved the use of homotopy groups, a beautiful branch of algebraic topology, and its formulation came almost as a flash. On a minor level, it was an event remindful of some earlier famous steps in physics, when a piece of highly abstract mathematical theory (Hilbert spaces, or non-Euclidian geometries), developed for purely internal reasons, suddenly finds unexpected applications. The theory produced surprising predictions, specially concerning obstructions to the crossing of line defects (as a consequence of noncommutative properties of homotopy groups). But the topic did not become an 'industry', partly because the classification could now be established straightfully. Thus was listed and explained right away almost everything that could be observed, and accounted for within the compass of a topological theory.

Morality: The topological theory was accepted without difficulty, because its visual 'evidence', for simple cases, was just irresistible; and the mathematics brought to bear, on hard cases, was awesome. On the other hand, this story is also an illustration of the self-defeating character of scientific research. The firework was unusually sparkling, and brief.

3.4. Frustration and Spin Glasses

One running theme, in the previous sections, is disorder: impurities, defects, percolation in heterogeneous systems, statistics of polymer configurations.

Spin glasses constitute another class among disordered systems. They came onto the theoretical stage in the mid-seventies, and stayed there prominently for a decade. Subsequently, there was an explosion of activity spreading in many directions (to such extent that the spin glass problem was to be described as a 'cornucopia'), but spin glass physics itself is not exhausted, with important remaining issues like: fluctuation effects (beyond mean-field theory) and the nature of the phase transition in real materials, the slow dynamics of various aging phenomena.

Typically spin-glass materials contain magnetic impurities, with competing interactions. For instance, the dilute alloy CuMn may be described as a Kondo system, at low concentrations, and as a spin glass system, at higher concentrations. Around 1976, it became clear that spin-glass behaviour comes through a conspiracy of two ingredients: disorder and frustration.

Frustration (in physics) means competing interactions. Three Ising spins (binary vectors pointing upwards or downwards), located on the vertices of a triangle, and coupled with antiferromagnetic interactions, cannot find a configuration where all interactions are satisfied. In a metal, the interaction between two magnetic impurities oscillates with distance; thus impurities distributed at random necessarily experience interactions of both sign, and some amount of frustration is thereby induced.

As a consequence of frustration, typically, there is no ideal low-lying ground state, and instead many metastable states of similar energies.

This situation happens in quite a variety of different materials. Indeed, when the term 'frustration' was introduced with such meaning, it became clear how general was the need for it, because its use spread swiftly around the planet, and across disciplines.

Several momentous surprises were to occur in spin-glass theory, but the story started in 'tip of the tail' style:

- experimentally, in 1972, an ac susceptibility measurement found a sharp cusp, suggesting that the spin glass transition might be sharp, after all, and not 'glassy', as the term 'spin glass' coined by Bryan Coles a few years earlier, had so far suggested;
- theoretically, in 1975, a model with infinite range couplings (adequate for the sudy of mean-field theory) was found to exhibit, or so it seemed, a negative low-temperature entropy (small indeed, but still radically unacceptable!).

Eventually a cure was found for this anomaly, but it requested the creation of novel abstract concepts. The whole theoretical enterprise, to an unusual degree, had the character of a bold flight of fancy. And two decades later, some genuine features of the spin-glass theory remain very strange indeed; such as:

i) a mean-field theory which is quite odd and complex (making it extremely difficult to introduce fluctuation corrections),

ii) a free energy of the low-temperature 'ordered' phase, which is oddly higher (instead of lower, as usual) than the analytic continuation of the free energy for the paramagnetic high-temperature phase,

iii) an ergodicity breaking into valleys, which are not related by any symmetry; yet, the transition may nevertheless be described as spontaneous symmetry breaking, in terms of a purely formal construct, namely replica symmetry.

All this complexity is coming out of the simple-looking Hamiltonian:

$$\mathcal{H} = -\sum_{(i,j)} J_{ij} S_i S_J$$

where the couplings J_{ij} are randomy positive or negative.

The role of the order parameter, in the spin-glass phase, is taken up by an 'order function', $P(q)$, which is the overlap distribution function of the valleys, properly weighted by Boltzmann factors; the overlap of two valleys, q, is a measure of their closeness in configuration space.

A striking discovery was the ultrametricity property of the valleys. As a consequence, the valley distribution can be described as generated by a branching process (valleys, within valleys, within valleys).

At this stage, the turn toward neural network theory was fostered by two considerations:

i) a system with many metastable states is a good candidate to function like a memory, as was brought to public attention by John Hopfield's seminal paper; furthermore, nervous systems, specially in the mammalian cortex, conspicuously possess the two ingredients of disorder and frustration (largely random anatomical structures and connections, with excitatory and inhibitory synapses);

ii) some exactly soluble neural network models were found to exhibit remarkable behaviour (rich phase diagrams); unexpectedly, these models proved to be ideal testing grounds for replica symmetry breaking, and thus they helped to strengthen spin-glass mean-field theory, and to put it on a wider, and thus more convincing, basis.

Darwin, early on, had the intuition that 'life is an irregularly branched tree'. Indeed evolution trees introduce a hidden simplicity into the confounding diversity of species and biomolecules. It was surprising and intriguing to find a similar kind of simplicity lurking behind spin-glass models. Furthermore, the algorithm of simulated annealing, devised by Scott Kirkpatrick in order to study spin glasses, proved to be a somewhat universal tool for hard optimization problems. All of this seemed to converge, and was felt as encouragement by a number of statistical physicists, who became eager to try and extend their new conceptual and numerical instruments onto complex issues of biological interest.

Morality: In some sense, an instructive duality exists between the topological defects story and the spin-glass story, because, under comparison, they stand as neat examples of opposite courses. For the solution of the defects problem, mathematics was ahead of physics. Whereas in the spin-glass saga, physics was the pioneer, and the abstract constructs and operations that were devised by physicists, in *ad hoc* manner, remain as heresy for the mathematicians.

Some references are presented, on: physics and biology [1–8]; a trajectory through statistical physics (a few steps) [9–14]; neural networks [15–19]; palimpsest models for short-term memory [20,21]; two complimentary books, one from a pioneer biologist and the other exploring the neural code [22,23]; topical easily accessible mini-surveys with recent references [24–33]; et quelques références en français [34–40].

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