Electronic Energy Spectrum in a DQWS Within a Tilted Magnetic Field

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Abstract

The analytical solution of the Schrödinger equation for a DQWS subjected to an externally applied tilted magnetic field are obtained and the results are discussed. The dependency of energy spectrum of the system on the applied magnetic field direction is also given.

1. Theory

The Hamiltonian of an electron in a DQWS under a externally applied tilted magnetic field (Figure 1 and Figure 2.) can be written as $H = \frac{1}{2\mu}\left(\vec{P} + e\vec{A}\right)^2 + V(z)$, where μ is the effective mass of the electron. Making use of the translational symmetry in the y-direction, the wave function of the system can be written as $\psi(\vec{r}) = e^{ik_y y}\phi(x,z)$, and using the vector potential we can rewrite the Hamiltonian of the system as $H = \left(\frac{P_x^2 + P_z^2}{2\mu}\right) + \frac{1}{2\mu}\left(\hbar k_y - eB(zCos\theta - xSin\theta)\right) + V(z)$. Applying point canonical transformation and some mathematics outlined elsewhere [1], [2] one can obtain a separated and solvable Hamiltonian of which only the z' solutions is not trivial and are of interest. The contribution to the Hamiltonian from z' direction can be written as $H_{z'} = \frac{P_{z'}^2}{2\mu} + \frac{1}{2}\mu\omega^2(z_0' - z')^2 + V(z')$, where potential along z' direction is given by $V(z') = V_0 Cos^2\theta \left\{ S\left(z_{L_1}' - Z'\right) + S\left(z' - z_{R_1}'\right) + S\left(z_{L_2}' - z_{R_1}'\right) + S\left(z' - z_{R_2}'\right) \right\}$. Here, we have used $z_0' = \frac{\hbar k_y}{eB} = a_H^2 k_y$ for the position of the orbit center and $a_H = \sqrt{\frac{\hbar}{\mu\omega}}$ for the magnetic length, $\omega = \frac{eB}{\mu}$ for the cyclotron frequency, and to find the eigenvalues of this system we follow the treatment of Lee et al., Changing variables from z' to $\tilde{u} = \frac{\sqrt{2}}{a_H}(z_0' - z')$, $\tilde{E}_{z'} = \frac{E_{z'}}{\hbar\omega}$, and $\tilde{V}(\tilde{z}') = \frac{V(z')}{\hbar\omega}$ we obtain the Schrödinger equation corresponding to the

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z' motion in terms of dimensionless variables as $\frac{d^2\phi(\tilde{u})}{d\tilde{u}^2} + \left(\tilde{E}_{z'} - \tilde{u} - \frac{1}{4}\tilde{u}^2\right)\phi(\tilde{u}) = 0$. The solution of this equation is given by the well-known Weber functions

$$D_m(\tilde{u}) = 2^{m/2} e^{-\tilde{u}^2/4} \left[\frac{\sqrt{\pi}}{\prod(1/2 - m/2)} F\left(-\frac{m}{2}\left|\frac{1}{2}\right|\frac{1}{2}\tilde{u}^2\right) - \frac{\sqrt{2}\pi\tilde{u}}{\Gamma(-m/2)} F \right.$$
$$\left. \left(-\frac{1}{2} - \frac{m}{2}\left|\frac{3}{2}\right|\frac{1}{2}\tilde{u}^2\right) \right], \text{ where } \Gamma(x)$$

is the Gamma Function, and $F(a\,|b|\,2)$ is the Confluent Hypergeometric Function and the parameter m is related with eigenvalues E_z. Applying continuity conditions of the wave functions and its first derivative at the boundaries we obtain a set of linearly independent equations. Furthermore, to have a unique solution we impose the condition that the determinant of the coefficients C_1 through C_8 must be zero, which gives us the master equation (not given here due to space limitation). From the master equation, one can obtain the quantum number m in terms of k_y and θ. Therefore, treating θ as a parameter we can calculate m values as a function of k_y numerically, if the parameters of quantum structure and the magnitude of B are given.
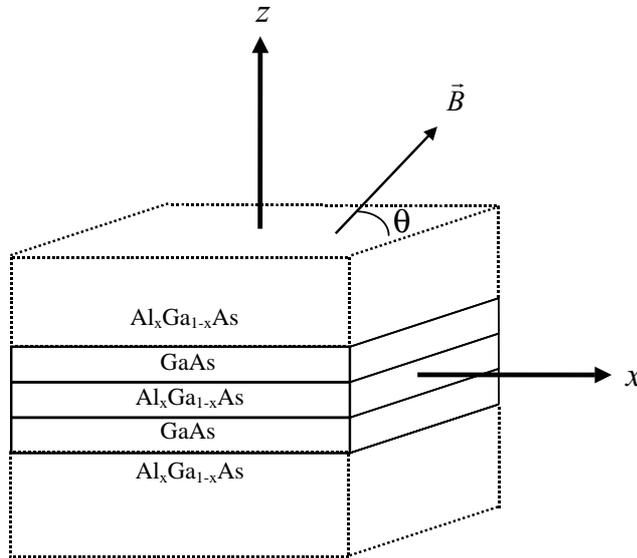
Figure 1. The DQW structer under externally applied tilted magnetic field Growth direction is chosen along z axis. Well width and barier width is 36 Å(the parameter x is taken to be 0.3 for this value V_0=225 meV. and B value is 6
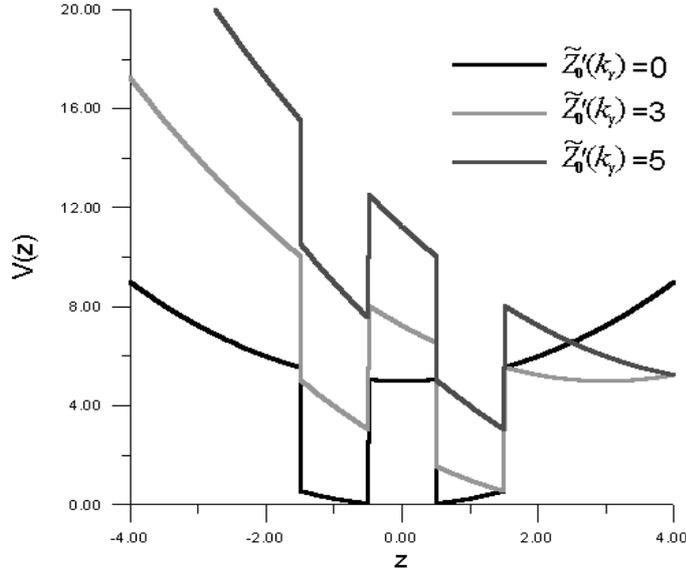
Figure 2. The $\tilde{Z}_0'(k_y)$ dependence of the Potential after the coordinate transformation.

2. Conclusions

In all numeric calculations, the following parameters are used: $Vo = 225\ meV$ (for $x = 0.3$), $B = 26T$, $L = 36\overset{\circ}{A}$ and the results are shown on Figure 3.

- The complete and exact solution of a symmetric DQWS is given;

- Beyond $\left|\tilde{Z}\right| \geq 6$ the energy levels are flat and has the same value on boths ides of the DQWS. These are bulk Landau levels (just shifted by $\tilde{V}_0 Cos^2\theta$) which clearly means that in these regions the spatial confinement has no effect (except by a constant shift of Landau levels);

- In region $\left|\tilde{Z}_0'\right| \leq 2$ strong spatial confinement effects are seen. For smaller energy values the states behaves more like the square well states, as the energy increases the state behavior changes toward Landau level behavior;

- In region $2 \prec \left|\tilde{Z}_0'\right| \succ 6$; we observe the transformation from well-like states to bulk Landau states

- The first two energy plots ($\theta = 0^0, \theta = 30°$) shows two bound states (meaning bound by spatial confinement) whereas the last two plots ($\theta = 45°, \theta = 60°$) shows

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only one bound state. This is interesting, since this shows that by just changing the tilt angle of magnetic field it is possible to change the characteristics of the particle from bound to extended.
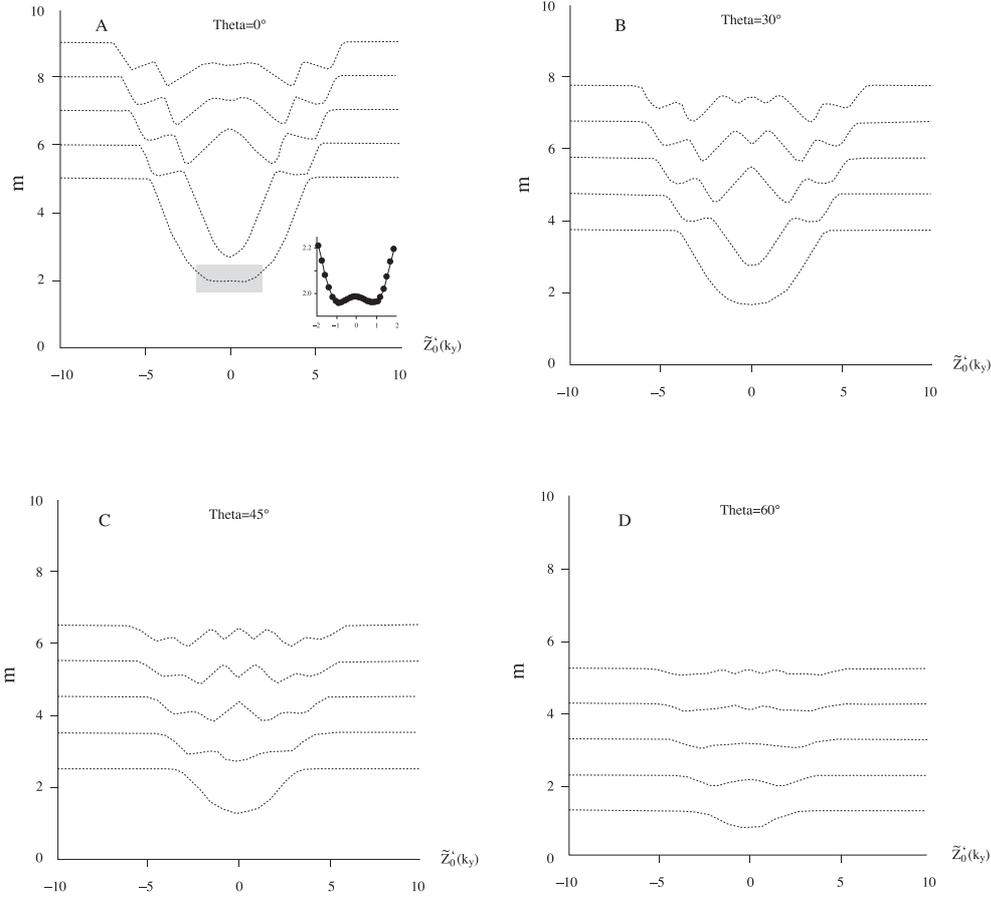
Figure 3. The plot of an electronic energy spectrum m vs. dimensionless orbit center Z_0' for a symmetric DQWS subjected to externally applied tilted magnetic field for tilt angle Theta= $0°$, Theta= $30°$, Theta= $45°$ ve Theta= $60°$

3. Summary

The complete and exact solution of a symmetric DQWS under externally applied, homogenous, tilted magnetic field is given. Eigenvalues and eigenstates of the system are obtained. This is followed by a discussion of the orbit center position dependency of the eigenvalue behavior of the system as a function of θ. Furthermore it is pointed out that, by just changing the tilt angle θ it is possible to change the character of a state from

confined to extended or the other way around an important result promising future field direction sensitive device designs.

References

[1] H. R. Lee, H. G. Oh, T. F. George, and C. I. Um, J. Appl. Phys. 66, 2442 (1989).

[2] I. Sökmen, H. Sari, S. Elagoz, Y. Ergün and S. Erzin, Superlattices Microstructures 17, 3 (1995).