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The Orbit Center Dependence of the Energy levels in a Multiple Quantum Wells Under External Tilted Magnetic and Electric Fields

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Abstract

An analytical solution to the Schrödinger equation for a multiple quantum well system subjected to an externally applied electric field the growth direction and an externally applied tilted magnetic field are obtained and the results are discussed. The energy dependence energy spectrum of the system as a function of the external electric field and the orbit center is also discussed.

1. Introduction

In order to study a two-dimensional electron gas in heterosystems, an external magnetic field is often applied parallel, perpendicular or at an angle to the growth direction. For the first case, that is when a constant homogeneous magnetic field is applied parallel to the growth direction of a two-dimensional heterostructure, the Landau levels of the confined electrons are formed and the energy spectrum becomes discreate giving rise to an extensively studied yet interesting effect such as the Quantum Hall Effect (QHE), and Shubnikov de Haas ossillations (SdH) [1,2]. In this case, since the magnetic field is in the same direction as the confining electric field, the Hamiltonian can be easily seperated into an electric part leading to subbands and a magnetic part leading to Landau levels. Also, Lee at al. have investigated the situation when an externally applied magnetic field is parallel to the layers [3]. Although an in-plane magnetic field usually has little effect on the two-dimensional properties since the z motion in the confining potentional is only slightly perturbed, it can strongly affect the spectrum of intersubband optical transition [4].

In a parabolic potential well eigenenergies of two-dimensional electrons subjected to a tilted magnetic field have been solved analytically by Maan [5]. This solution was utilized as a good approximation in analysing the experimental findings of transport measurements of these structures. In this paper we have studied the energy spectrum of an electron for different orientations ($\theta = 0$ and $\theta = 18$) of the magnetic field while keeping the electric field parallel to the growth direction. The application of successive transformations makes the Hamiltonian separable in terms of the new coordinates [6]. The general solution goes smoothly approaches the results of the two limits where the magnetic field is either parallel or perpendicular to the layers while the electric field is applied to the growth direction as mentioned above. Thus, we can completely solve the Schrödinger equation using multiple (four) square wells potentials as the confining potential and obtain analytical solutions without making any approximations for two-dimensional semiconductor heterostructures under externally applied electric and tilted magnetic fields.

2. Theory

We consider multiple (four) quantum wells of width L and potential height Vo in an externally applied constant uniform magnetic field B in the $(x - z)$ plane, $B = (xB\cos\theta,\ 0,\ zB\sin\theta)$, where θ is the angle between the direction of B and the x-axes (Figure 1). The magnetic field can be described by the vector potential $A = (0, xB\sin\theta - zB\cos\theta, 0)$, where the gauge is considered as $\nabla \cdot A = 0$. The Hamiltonian of an electron in such a system can be written in SI units as

$$H = \frac{1}{2\mu}(p + eA)^2 + V(z), \tag{1}$$

where μ is the effective mass of the electron and V(z) is the potential energy of the electron in the well which includes the electric field term eFz. A schematic representation of V(z) is given in Figure 1.

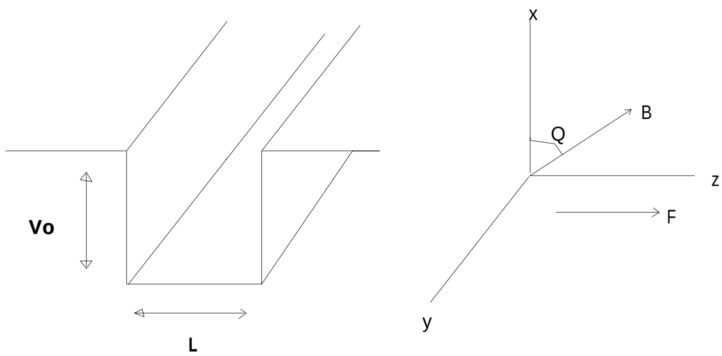


Figure 1. Schematic representation of the potential well and the directions of axes and externally applied magnetic field B.

For the present case the functional form of V(z) is

$$V(z) = V_0 \sum \left[S(z_{L_i} - z) + S(z - z_{R_i}) - S(z - z_{L_{i+1}} \right] + eFz, \tag{2}$$

where S is step function, and left and right boundaries of the wells are located at $Z = Z_{L_i}$, and $Z = Z_{R_i}$ in the i^{th} well and $Z = Z_{L_{i+1}}$ in the $(i+1)^{th}$ well, respectivelly. Making use of the translational symmetry in the y-direction, the wavefunction of the system can be written as $\Psi(r) = \exp(ik_y y)\varphi(x,z)$, and by using the vector potential we can rewrite the Hamiltonian of the system as

$$H = \frac{1}{2\mu}(p_x^2 + p_z^2) + \frac{1}{2\mu}\left[\hbar k_y - eB(z\cos\theta - x\sin\theta)\right]^2 + V(z). \tag{3}$$

By using the point canonical transformation

$$\begin{pmatrix} z' \\ x' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} z \\ x \end{pmatrix}, \tag{4}$$

where the magnetic field B coincides with the x-axes, the Hamiltonian of the system can be written as;

$$H = \frac{p_{x'}^2 + p_{z'}^2}{2\mu} + \frac{1}{2}\mu\omega^2(z_0' - z')^2 + V(x', z'), \tag{5}$$

where we have used $\omega = eB/\mu$ for the cyclotron frequency, $z_0' = \hbar k_y/eB = a_H^2 k_y$ for the position of the orbit center, and $a_H = (\hbar/\mu\omega)^{1/2}$ for the magnetic length. In order to decompose the potential energy $V(x', z')$ we rewrite the step functions in Equation (2) as follows:

$$\begin{aligned} S(Z_{L_i} - z) &= Cos^2\theta S(z'_{L_i} - z') + Sin^2\theta S(x'_{L_i} - x'); \\ S(Z - Z_{R_i}) &= Cos^2\theta S(z' - z'_{R_i}) + Sin^2\theta S(x' - x'_{R_i}); \\ S(Z - Z_{L_{i+1}}) &= Cos^2\theta S(z' - z'_{L_{i+1}}) + Sin^2\theta S(x' - x'_{L_{i+1}}); \end{aligned} \tag{6}$$

By considering the above equations we can separate the potential as

$$V(x', z') = V(x') + V(z'), \tag{7}$$

where

$$\begin{aligned} V(x') &= V_0 \sin^2\theta \sum_i \left[S(x'_{L_i}) - x' + S(x' - x'_{R_i}) - S(x' - x'_{L_{i+1}}) \right] + eF\sin\theta x'; \\ V(z') &= V_0 \cos^2\theta \sum_i \left[S(z'_{L_i} - z' + S(z' - z'_{R_i}) - S(z' - z'_{L_{i+1}}) \right] + eF\cos\theta z'; \end{aligned} \tag{8}$$

Consequently we can decompose the Hamiltonian of the system as;

$$H = H_{x'} + H_{z'} \tag{9}$$

By using $u = z_0' - z'$ and the dimensionless variables $\tilde{u} = (\sqrt{2}/a_H)u$, $\tilde{E}_{z'} = E_{z'}/\hbar\omega$, and $\tilde{V}_0 = V_0/\hbar\omega$. Hence the Schrödinger equation corresponding to the z'-motion becomes

$$\frac{d^2\phi(\tilde{u})}{d\tilde{u}^2} + (-\frac{1}{4}\tilde{u}^2 - \tilde{V}_0 \cos^2\theta S[\tilde{u}] + \tilde{E}_{z'} - \tilde{\alpha}_0 + \tilde{\beta}\tilde{u})\phi(\tilde{u}) = 0, \tag{10}$$

where

$$S[\tilde{u}] = \left\{S(\tilde{u} - \tilde{u}_{L_i}) + S(\tilde{u}_{R_i} - \tilde{u}) - S(\tilde{u} - \tilde{u}_{L_{i+1}})\right\} \tag{11}$$

and $\tilde{\alpha}_0 = \frac{eFa_H}{\sqrt{2}\hbar\omega_c}\cos\theta \tilde{z}_0'$ and $\tilde{\beta} = \frac{eFa_H}{\sqrt{2}\hbar\omega_c}\cos\theta$. If we shift to normalized coordinate $\tilde{u}$ by an amount of $-2\tilde{\beta}$, $\tilde{\zeta} = \tilde{u} - 2\tilde{\beta}$ the Schrödinger equation is written in the new coordinate

$$\frac{d^2\phi(\tilde{\zeta})}{d\tilde{\zeta}^2} + \left[(m + \frac{1}{2}) - \frac{1}{4}\tilde{\zeta}^2\right]\phi(\tilde{\zeta}) = 0. \tag{12}$$

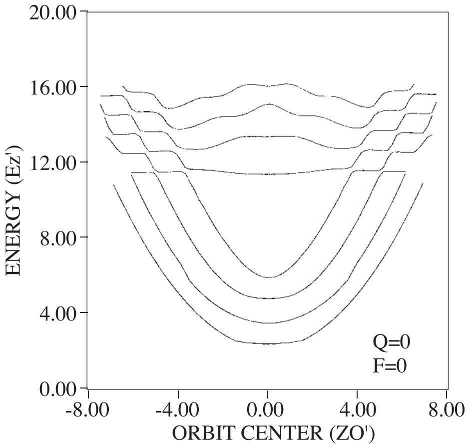
The solution corresponding to the z'-motion is given by the well-known Weber functions $D_m(\tilde{\zeta})$ [7]. The quantum numbers m and m' are related to each other by $(m - m') = \tilde{V}_0 \cos^2\theta$.

3. Results and Discussion

We use the following parameters in this paper:

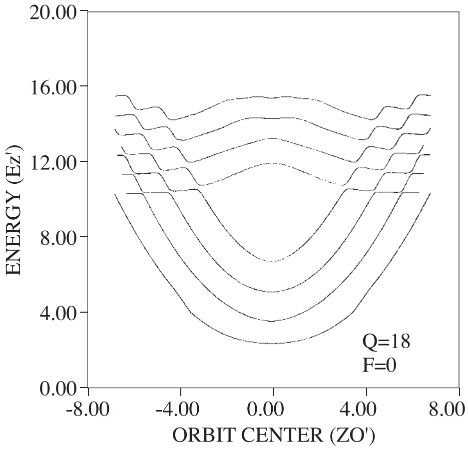
$V_0 = 410$ meV, $\tilde{V}_0 \cong 11$; $L_w = 39.2\AA$, $\tilde{L}_w \cong 1$; $L_b = 11.2\AA$, $\tilde{L}_b \cong 0.286$; $a_H \cong 57\AA$, $\hbar\omega = 37.29$ meV and B=20 Tesla and $F = 5 \times 10$^4 V/cm.

The well width is comparable to the magnetic lenght and the energy levels are quantized by the combined effects of spatial confinement that are barriers of the quantum wells and Landau levels. Consequently, the orbital motion of electrons is reflected by both potential barriers of each quantum well and energy levels with energy smaller than the barrier height are spatially localized. In Figure 2 the numerical solutions of Energy $(E_{z'})$ are plotted versus the position $\tilde{z}_0'$ of the orbit center for different values of θ which defines the angle between the direction of the magnetic field B and x-axes. These results clearly show two different types of energy states: the states confined in the quantum well (the lowest level in Figure 2 (a),(b),(c),(d)), and extended states (the higher levels in Figure 2. (a(,(b),(c) and (d)).When the energy of the lower states are less than the height of the potential the particles are considered to be mostly localized in well regions. For higher states we begin to see the domination of bulk Landau levels at larger $\tilde{z}_0'$ values due to the higher energy and larger cycloid radius.



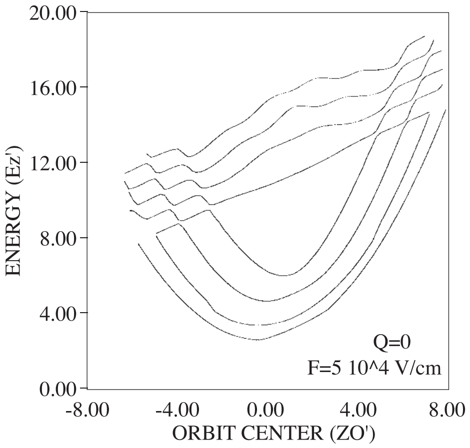
(a)

Figure 2.(a) electronic structures of the potential well under a magnetic field for a tilt angle $\theta = 0$ and electric field strength F=0



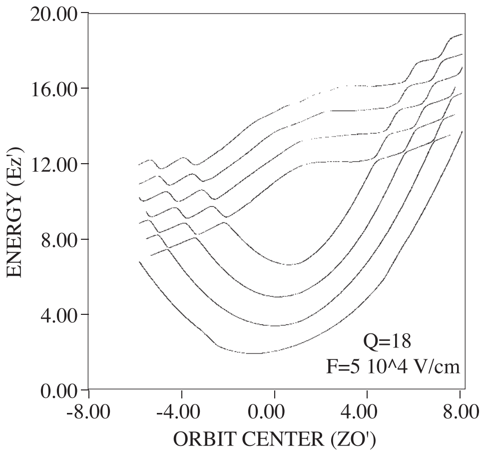
(b)

Figure 2.(b) electronic structures of the potential well under a magnetic field for a tilt angle $\theta = 18^\circ$ and electric field strength F=0.



(c)

Figure 2.(c) electronic structures of the potential well under a magnetic field for a tilt angle $\theta = 0$ and electric field strength $F = 5\times10^4$ V/cm.



(d)

Figure 2.(d)(d) electronic structures of the potential well under a magnetic field for a tilt angle $\theta = 18^\circ$ and electric field strength $F = 5 \times 10^4$ V/cm.

Conclusion

In summary, we present the solution of the Schrödinger equation of a square well potential problem under the influence of an externally applied tilted magnetic field by making the Hamiltonian separable via a substitution followed by a orthogonal transformation. Then, we discuss the energy of the system as a function of tilted parameter θ and the center of the cycloid orbit center of the electron. Where we find that in both limits of θ the results are in complete agreement with that of previous work found in literature. It is also found that if the orbit center is moved far away from the quantum well the usual bulk Landau levels take over with a shift of V_{eff} in energy.

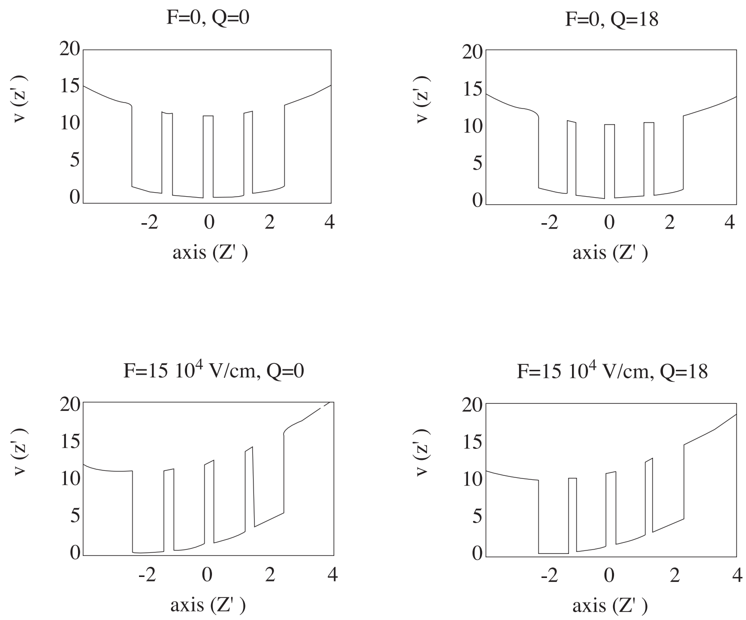


Figure 3. Potential profiles at the $z_0' = 0$

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