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Angular dependence and mode distribution of acoustic phonon emission by hot 2D electrons in GaAs/AlGaAs heterojunctions and quantum wells

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Abstract

We report a detailed theoretical study of the angular dependence and mode distribution of the acoustic phonon emission by hot two-dimensional electron gases in GaAs/AlGaAs heterojunctions and quantum wells and compare the results with some recent heat pulse measurements for carrier temperatures below 50 K. Common to all the experimental results was the strong dependence of the ratio of emitted longitudinal acoustic (LA) phonons to transverse acoustic (TA) phonons from the width of the quantum well and the absence of LA phonons propagating in a direction close to the 2DEG normal for GaAs/AlGaAs heterojunctions. To explain these phenomena and to understand the process of electron–phonon coupling and its dependence on electron confinement we use a model which includes the dynamical screening of the electron–phonon interaction in the 2D electron gas (2DEG), the confinement of the electrons in the direction normal to the 2DEG plane and the strong acoustic anisotropy of the phonon emission and propagation process. Our numerical results show clearly the importance of including all of these factors.

1. Introduction

Detailed information regarding the phonon emission by a hot (quasi) 2DEG can be obtained by heat pulse experiments (for a review see [1]). The electrons in the 2DEG are heated by application of short electrical pulses above the lattice temperature and relax by the emission of phonons. At low temperatures the dominant process of energy relaxation in GaAs devices is the emission of deformation potential (DP) and piezoelectric (PE) coupled acoustic phonons. The emitted phonons propagate ballistically in the substrate and are detected by bolometers. If the heating pulse is very short compared with the time taken for the emitted phonons to reach the bolometer, resolving separately the phonon modes which travel at different speeds is possible. Additionally the angular dependence of the emission can be studied by placing bolometers at different positions relative to the 2DEG. Such temporal and angular resolved phonon emission studies give, in contrast to other methods like transport measurements, more direct information concerning the wavevector and mode dependence of the emission process.

A surprising feature of the heat pulse experiments was the strong dependence of the ratio of emitted longitudinal acoustic (LA) phonons to transverse acoustic (TA) phonons from the width of the quantum well and the absence of LA phonons propagating in a direction close to the 2DEG normal for GaAs/AlGaAs heterojunctions. These results are in contradiction to earlier theoretical estimations which suggested that deformation potential (DP) coupled LA modes should be totally dominant over the TA in the temperature range close to the onset of optic phonon emission. Also the inclusion of phonon focusing in the substrate could not eliminate this discrepancy.

In this paper we describe a theoretical model, which can explain these experimental findings and which allows a detailed theoretical analysis of the angular and mode distribution of the acoustic phonon emission by two-dimensional electron gases and its dependence from electron confinement for electron temperatures below 50 K. The model considers the strong acoustic anisotropy of the electron–phonon matrix elements and the propagation process, includes the dynamical screening of the electron–phonon interaction and uses a realistic model for the confinement of the electrons normal to the 2DEG plane.

2. Phonon emission

We consider a quasi 2DEG with parabolic dispersion relation and N electrons, embedded in a 3D substrate of volume V and density ρ. The electrons have an effective temperature $T_{\mathrm{e}} < 50$ K and the lattice temperature is $T < T_{\mathrm{e}}$.
The general expression of the phonon emission rate per electron is

$$P(t) = \frac{i}{\hbar}\frac{1}{N}\sum_{\mathbf{q},\lambda} \hbar\omega_{\mathbf{q},\lambda} \left\langle [H, \hat{N}_{\mathbf{q},\lambda}] \right\rangle (t) \quad , \tag{1}$$

where $\omega_{\mathbf{q},\lambda}$ is the frequency of a phonon in the substrate with (3D) wave vector $\mathbf{q}$, polarization λ (λ =LA, fast TA, slow TA) and polarization vector $\mathbf{e}_{\mathbf{q},\lambda}$. H is the Hamiltonian for the electron and phonon system including electron–phonon and electron–electron interaction and $\hat{N}_{\mathbf{q},\lambda}$ is the phonon number operator. Within linear response theory and neglecting all higher order phonon processes we obtain [2]

$$P = \frac{2}{NV}\sum_{\mathbf{q},\lambda} \omega_{\mathbf{q},\lambda}(N^{T}_{\mathbf{q},\lambda} - N^{T_{\mathrm{e}}}_{\mathbf{q},\lambda})|h_{\mathbf{q},\lambda}|^2\, |G(q_\perp)|^2 \mathrm{Im}\left\{ \frac{\chi_{T_{\mathrm{e}}}(\omega_{\mathbf{q},\lambda}, \mathbf{q}_\parallel)}{1 - v(q_\parallel) g(q_\parallel) \chi_{T_{\mathrm{e}}}(\omega_{\mathbf{q},\lambda}, \mathbf{q}_\parallel)} \right\} . \tag{2}$$

$N^T_{\mathbf{q},\lambda} = (e^{\hbar\omega_{\mathbf{q},\lambda}/k_BT} - 1)^{-1}$ is the phonon equilibrium distribution function at temperature T and $h_{\mathbf{q},\lambda}$ is the familiar electron–phonon matrix element

$$h_{\mathbf{q},\lambda} = \sqrt{\frac{\hbar}{2\rho\omega_{\mathbf{q},\lambda}}}\left\{ i\,\Xi_{\mathrm{D}}\mathbf{q}\,\mathbf{e}_{\mathbf{q},\lambda} + 2eh_{14}\frac{e_{\mathbf{q},\lambda_x}q_yq_z + e_{\mathbf{q},\lambda_z}q_xq_y + e_{\mathbf{q},\lambda_y}q_zq_x}{q^2}\right\}, \tag{3}$$

which includes both DP and PE coupling. Ξ_{D}, h_{14} are the relevant coupling constants. The functions

$$G(q_\perp) = \int dr_\perp \varphi^*(r_\perp)e^{-iq_\perp r_\perp}\varphi(r_\perp) \tag{4}$$

and

$$g(q_\parallel) = \int dr_\perp \varphi^*(r_\perp)\varphi(r_\perp)\int dr'_\perp \varphi^*(r'_\perp)\varphi(r_\perp{}')e^{-q_\parallel|r_\perp - r'_\perp|} \tag{5}$$

arise from the finite extension of the component of the electron wave function along the axis normal to the 2DEG plane $\varphi(r_\perp)$ and depend strongly on the chosen confinement potential. To describe the quasi two-dimensionality in a proper way we use both for the heterostructure and the quantum well a confinement potential with finite conduction band offset, allowing a penetration of the electron wave function into the barrier material. We assume, that only the lowest electron subband is occupied.

$\mathrm{Im}\left\{\chi_{T_e}(\omega_{\mathbf{q},\lambda},\mathbf{q}_\parallel)/(1-v(q_\parallel)g(q_\parallel)\chi_{T_e}(\omega_{\mathbf{q},\lambda},\mathbf{q}_\parallel))\right\}$ is the imaginary part of the electron gas response function including a dynamical screening of the electron–phonon interaction. The electron-electron interaction is considered in RPA, thus $\chi_{T_e}(\omega_{\mathbf{q},\lambda},\mathbf{q}_\parallel)$ is the polarizability function for a non-interacting 2DEG at temperature T_e . $v(q_\parallel)$ is the 2D Fourier transform of the Coulomb potential.

Changing in (2) the summation over $\mathbf{q}$ into integration in the spherical variables and integrating over all phonon frequencies we obtain the emitted power per unit angle in $\mathbf{q}$-space (in direction $\hat{\mathbf{q}} = \mathbf{q}/|\mathbf{q}|$) for a given mode λ [3]

$$P_{\hat{\vec{q}}\lambda} = \frac{2}{N(2\pi)^3}\frac{1}{c^3_{\hat{\mathbf{q}},\lambda}}\int d\omega\omega^3|h_{\mathbf{q},\lambda}|^2|G(q_\perp)|^2(N^T_\omega - N^{T_e}_\omega)\mathrm{Im}\left\{\frac{\chi_{T_e}(\omega,\mathbf{q}_\parallel)}{1-v(q_\parallel)g(q_\parallel)\chi_{T_e}(\omega,\mathbf{q}_\parallel)}\right\} \quad . \tag{6}$$

But to compare with heat pulse experiments we need the power emitted in selected directions $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$ of real space. In real space the phonons are propagating in the direction of the group velocity $\mathbf{v}_{\mathbf{q},\lambda}$. In the long-wave limit the group velocity depends only on the direction $\hat{\mathbf{q}}$ of the phonon wavevector $\mathbf{q}$ but, in general, is not parallel to $\mathbf{q}$. This feature strongly influences the angular distribution of the emitted power in the substrate. So the emitted power of mode λ in the unit solid angle around $\hat{\mathbf{r}}$ has to be expressed by the product of $P_{\hat{\vec{q}}\lambda}$ with the corresponding focusing factor $\mathcal{A}_{\hat{\mathbf{q}}\lambda}$

$$P_{\hat{\mathbf{r}}} = \sum_{i=1}^{n_{\hat{\mathbf{q}}\lambda}} \mathcal{A}_{\hat{\mathbf{q}}_i\lambda}\hat{P}_{\hat{\mathbf{q}}_i\lambda} \quad , \tag{7}$$

where $\hat{\mathbf{q}}_i$ $(i = 1, ..., n_{\hat{\mathbf{q}}\lambda})$ are the solutions of the equation $\hat{\mathbf{v}}_{\hat{\mathbf{q}}\lambda} = \hat{\mathbf{r}}$ and the focusing factor is defined by $\mathcal{A}_{\hat{\mathbf{q}}\lambda} = d\Omega_{\hat{\mathbf{q}}}/d\Omega_{\hat{\mathbf{v}}_{\hat{\mathbf{q}}\lambda}}$, where $d\Omega_{\hat{\mathbf{v}}_{\hat{\mathbf{q}}\lambda}}$ is the body angle subtending all vectors

$\mathbf{v}$ corresponding to all vectors $\mathbf{q}$ subtended by $d\Omega_{\hat{\mathbf{q}}}$ (for details of calculation see [4]). As input for equation (7) we have to calculate the group velocity $\mathbf{v}_{\mathbf{q},\lambda}$, the phase velocity $c_{\mathbf{q},\lambda}$ and the polarization vector $\mathbf{e}_{\mathbf{q},\lambda}$ for all phonons $(\mathbf{q}, \lambda)$.

2.1. Results and discussion

In Figure 1 the angular dependence of the phonon emission by hot 2D electrons is compared for two quantum wells with the width of 3 nm (a,b) and 15 nm (c,d), respectively. For both cases the calculated phonon energy flux on the surface of a (100) wafer is presented separately for LA phonons (a,c) and TA phonons (b,d) as a function of detector position. The images cover an area of $2.5t \times 2.5t$, where t is the thickness of the wafer. The device is located at the centre of the image on the opposite face of the wafer.

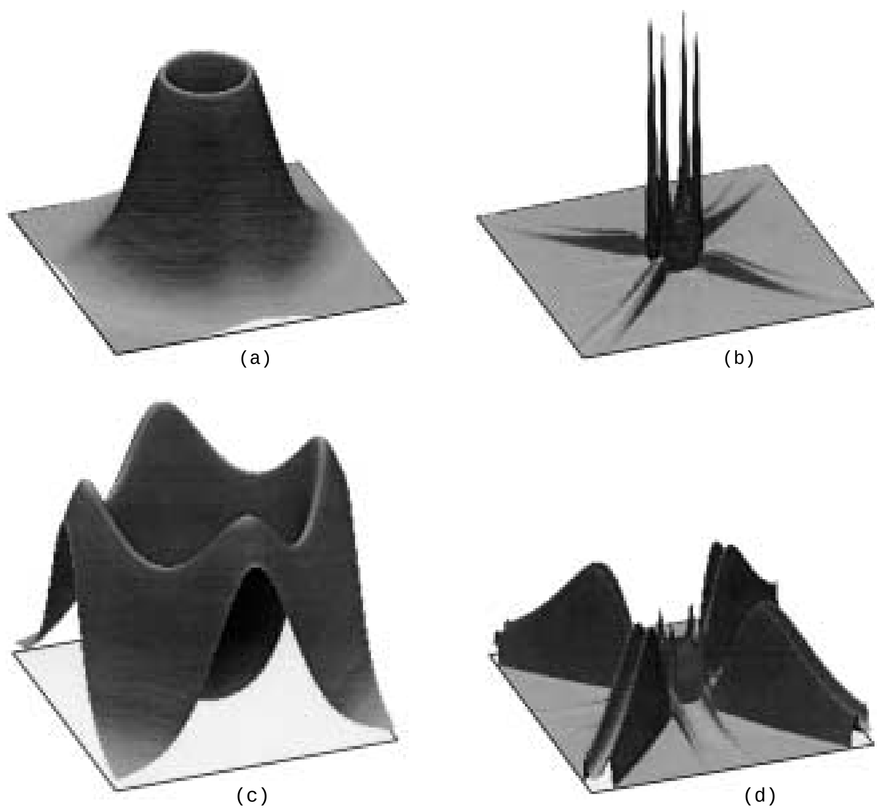

Figure 1. Theoretical images of the phonon energy flux emitted by a hot 2DEG ($T_e = 25K$) as a function of the detector position for two quantum wells with different width: (a,b) 3 nm, (c,d) 15 nm. For the 15 nm well the flux is multiplied by a factor 30. In (a,c) only the LA phonons emitted by DP and PE coupling are considered, in (b,d) only the TA phonons (slow TA+ fast TA). The density of the 2D electrons is 1.8×10^{15} m^{-2} for both quantum wells.

Figure 1 demonstrates that the emission is highly anisotropic and depends strongly on the electron confinement (quantum well width). To show the results for the two different quantum wells the flux for the 15 nm well (Figs. 1c, 1d) has to be multiplied by a factor 30. This confirms the strong suppression of acoustic phonon emission with increasing well width, particularly for emission close to the 2DEG normal.

The respective influence of screening, acoustic anisotropy in the electron–phonon matrix elements and phonon focusing in the substrate on the calculated patterns of phonon energy flux is shown in the Figs. (2-5) for the case of a (001) heterostructure. Each point of the patterns corresponds to a detector position, the device is again at the centre on the opposite side of the wafer. The patterns are separately displayed for DP coupled LA phonons (Fig. 2), PE coupled LA phonons (Fig. 3), DP coupled TA phonons (Fig. 4) and PE coupled TA phonons (Fig. 5). In all cases the *exact* theoretical results (a) for the emission as function of detector position are compared with a model without screening (b), with a calculation using the isotropic approximation for the electron–phonon matrix elements (c) and with the situation, where the phonon focusing in the substrate is ignored (d). The large overestimation of phonon emission close to the 2DEG normal in the case without screening for DP coupled LA phonons (Fig. 2b) is remarkably. But interesting is also the fact, that a significant contribution from the DP coupled TA phonons (Fig. 4a) exists, but this term is completely zero in the common used isotropic approximation for the electron-phonon matrix elements (Fig. 4c). Altogether, the results for the different models clearly demonstrate the fact, that only a complete theory, that takes proper account of the acoustic anisotropy and the screening of the electron–phonon interaction, allows a satisfactory description of the angular and mode dependence of phonon emission by 2D electrons.

To compare with the heat pulse experiments, we integrated the phonon flux in a window corresponding to the detector size. The detector (100×10 μm) is centrally opposite the 2DEG (120×50 μm). Table 1 summarizes the numerical results of the ratio LA/TA for quantum wells with different width and in addition for a GaAs/AlGaAs heterostructure and compares them with the corresponding experimental results [5]. We use a finite well depth of 350 meV for the confining potential of the quantum wells and a conduction band offset of 225 meV for the heterostructure. To compare with our model the ratio is also calculated using a conventional model.

In this case focusing is included, but a isotropic approximation for the electron–phonon matrix elements is used, screening of the DP is ignored and only a simple static screening is applied to PE interaction. In addition, to show the influence of the penetration of the electron wave function into the barrier material on the emission, the ratio LA/TA is calculated for a model with infinitely high conduction band offset. In all cases only our complete theory is in reasonable agreement with the experimental results, validating this approach. No free parameters were used in our theory, a further improvement could be reached by fitting the ratio of the coupling constants. Other reasons for the remaining small discrepancies could be the uncertainty in the temperature of the hot 2DEG and/or an additional contribution of TA phonons from the decay of renormalized (plasmonlike) optical modes [6].

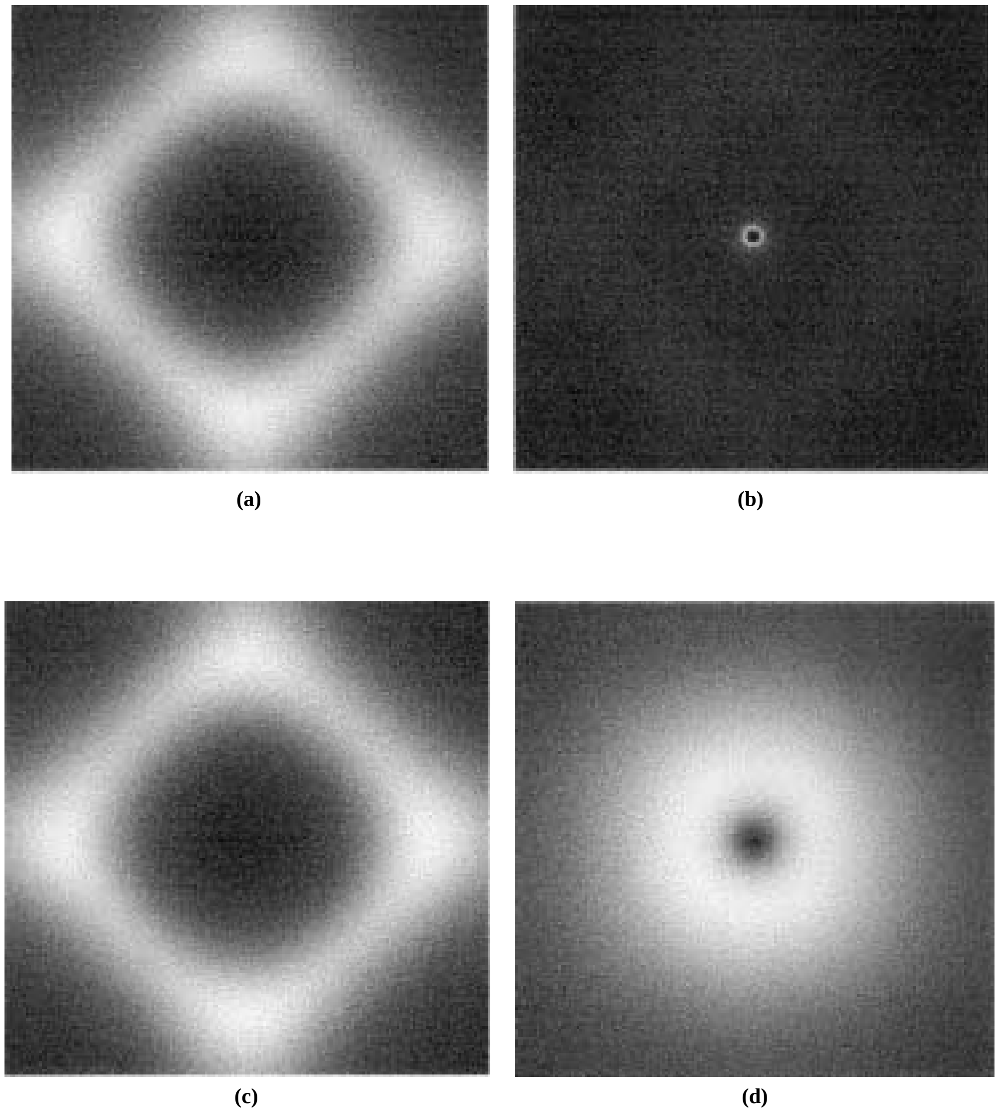

Figure 2. Patterns of the angular dependence of LA phonon emission for DP coupling. Pattern (a) is the result of the exact calculations, in (b) the screening of electron-phonon interaction is ignored, (c) is the result of the isotropic approximation for the electron-phonon matrix elements and (d) is without phonon focusing in the substrate. It is used a gray tones scale, where black means zero phonon flux in this direction and white corresponds to the maximum value of phonon signal. The electron temperature is $T_e = 20$ K, the electron density 2.8×10^{15} m^{-2}.

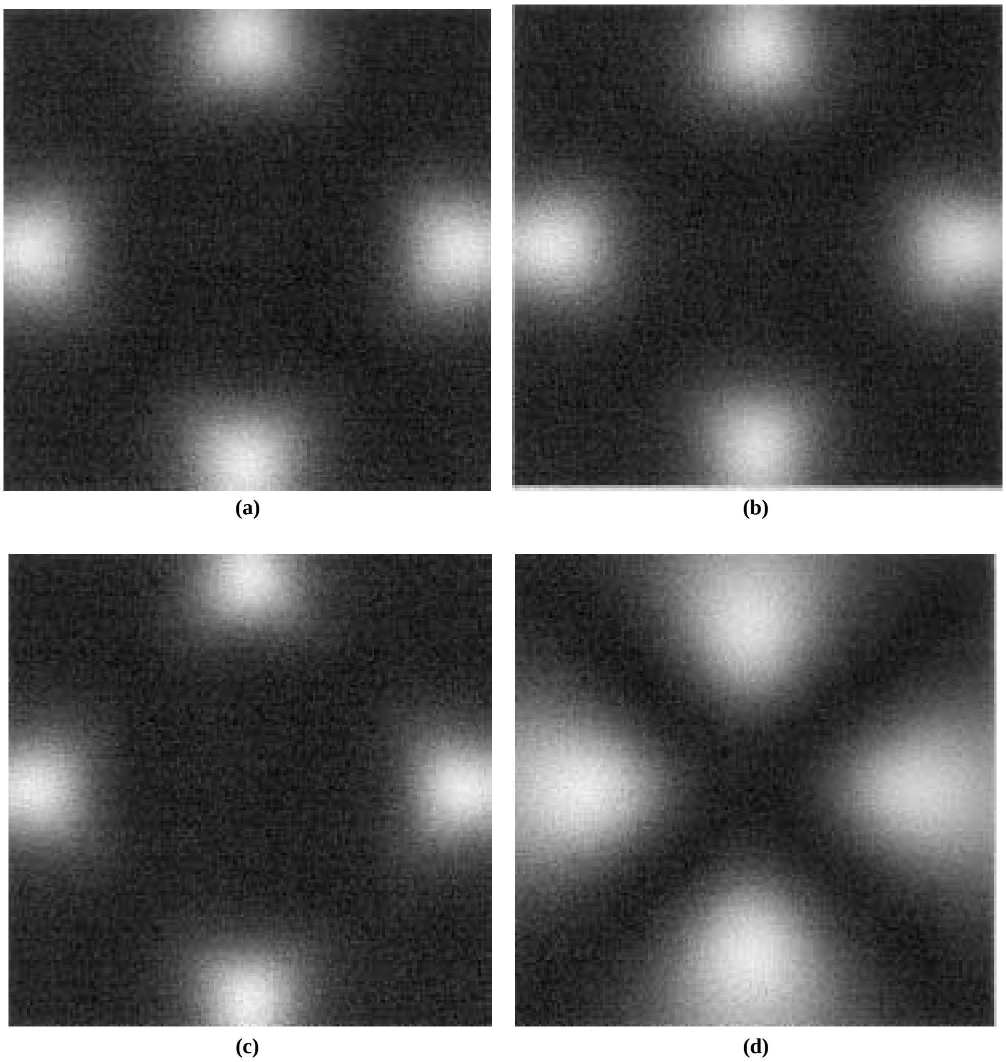

Figure 3. Patterns of the angular dependence of LA phonon emission for PE coupling. For details see Figure 2.

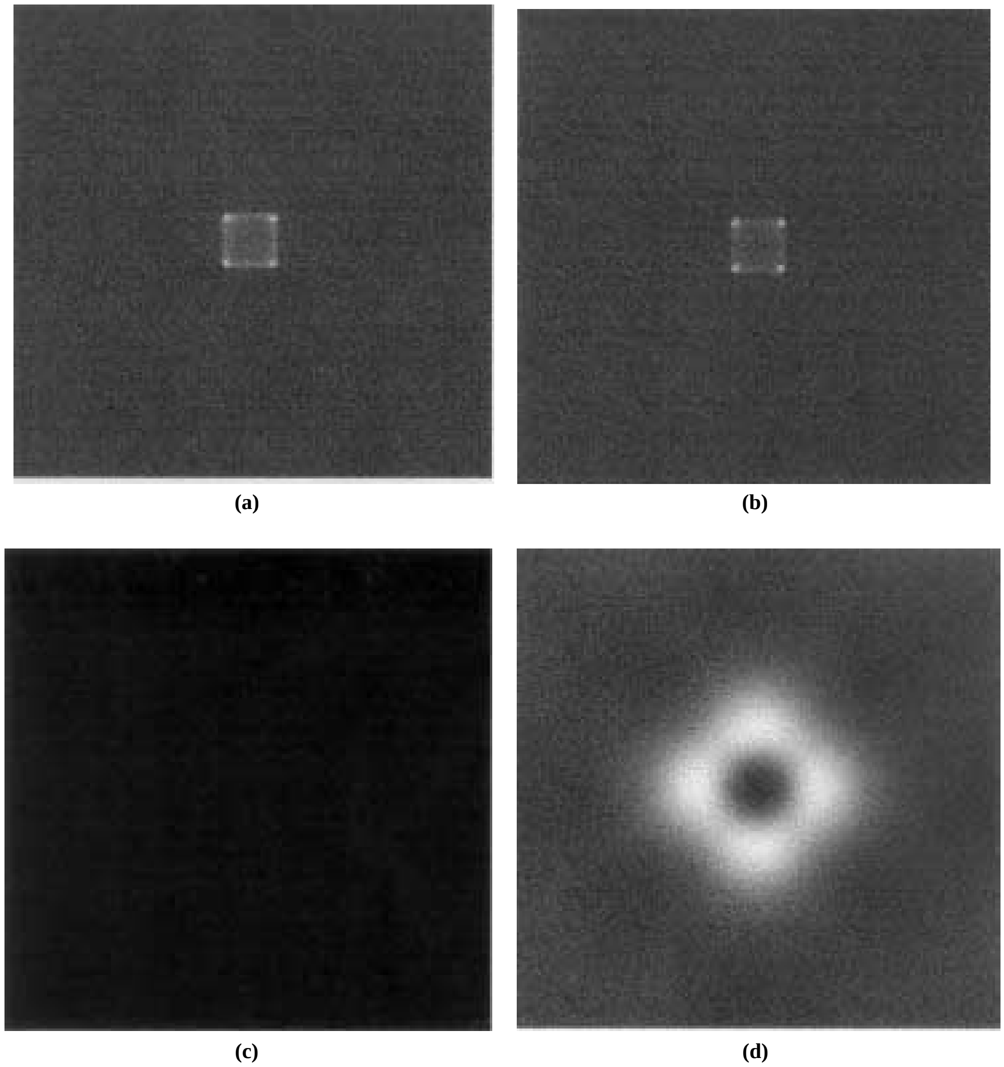

Figure 4. Patterns of the angular dependence of LA phonon emission for DP coupling. For details see Figure 2.

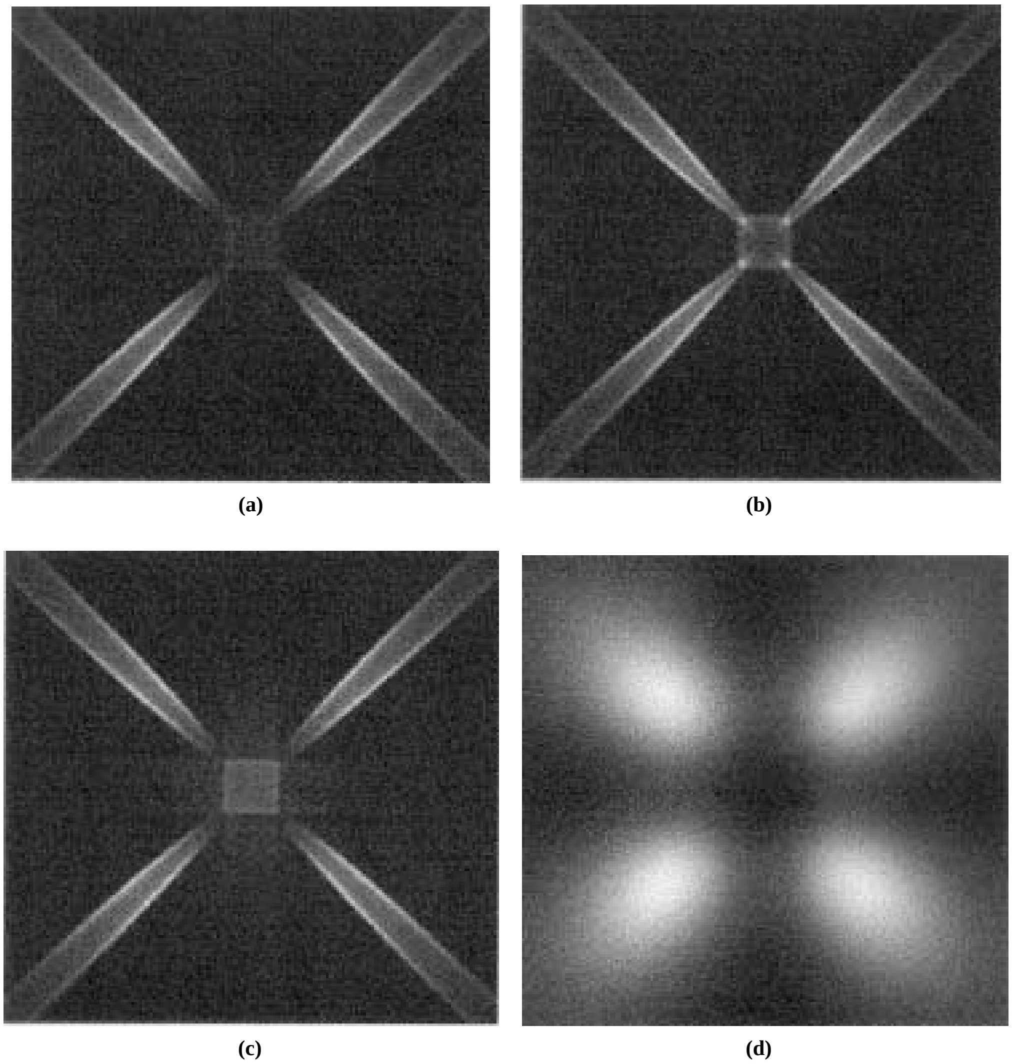

Figure 5. Patterns of the angular dependence of LA phonon emission for PE coupling. For details see Figure 2.

Table 1. Observed and calculated ratio LA/TA for QWs with different width and for a (001) heterostructure (HET), calculations for $T_e = 50$ K

well width [nm]	5.1	6.8	12	15	HET
2DEG density [10^{15} m^{-2}]	1.8	2.0	3.7	3.6	2.8
LA/TA (experiment)	1.3±0.3	1.1±0.2	0.5±0.1	0.16±0.03	≤0.05
LA/TA (conv. theory)	144	94	29	18	3.5
LA/TA (inf. pot. step)	9.8	2.8	0.34	0.22	0.13
LA/TA (complete theory)	1.59	0.92	0.27	0.16	0.10

2.2. Conclusion

Our numerical calculations and their comparison with experimental results show clearly the sensitivity of the angular and mode dependence of acoustic phonon emission to the four factors: acoustic anisotropy of the electron–phonon interaction, screening, confinement of the 2D electrons and focusing in the substrate. Neglecting of only one of these factors can result in large errors in the theoretical description of heat pulse experiments.

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