Interference of charged particles in a vector potential with vanishing magnetic field

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Abstract

An interference experiment in a magnetic field free region with non vanishing vector potential created by two perpendicularly intersecting planes carrying uniform currents is discussed. The relation of this configuration to the Aharonov-Bohm potential is examined. An experimental set up which is finite in the direction of electronic motion is studied.

1. Introduction

According to quantum physics charged particles interact with the external electromagnetic potentials even in regions where the field strength is zero. A well known example is the Aharonov-Bohm (AB) effect where the electrons are observed to scatter in the potential of confined magnetic flux [1,2,3]. In another example the roles of the flux line and the electrons are intercharged: Neutrons, which have magnetic dipole moments, are scattered in the Coulomb field of point charge [4]. This effect too, is experimentally verified [5]. The third example one can mention is the electric AB effect in which a homogeneous but time dependent electric potential acts on electrons [2]. The electric AB effect is yet to be experimentally observed. One can think of some other geometrical configurations which may give rise to AB type effects.

Here we present a new example for a vector potential with zero magnetic field. We notice that two perpendicular planes carrying uniform currents in the direction perpendicular to intersection create a region in which we have such a potential. Although it looks quite different, the potential we propose is related to the original AB potential of the magnetic flux line by a simple map: If we represent our vector potential A and the AB

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potential A_{AB} as complex numbers they satisfy the relation $AA_{\mathrm{AB}} = 1$. If the electrons moving in the region of space having this potential are allowed to interfere with the free electrons, hyperbolic pattern of maxima and minima are shown to be created. We hope that it may be possible to realize an experimental setup for this potential and then the scattering of electrons in it can be observed. This hope is the main motivation for writing up the present note. In fact a brief discussion presented in the final section shows that in a less idealized experimental system which is finite in the direction of the electronic motion the effect should still manifest itself.

2. Realization of a new vector potential with vanishing magnetic field

Consider uniform current densities I on the xz and yz-planes which flow in $-x$ and y directions. The corresponding spatial current density $j(x)$ is

$$\vec{j} = -I \left(\delta(y)\hat{x} + \delta(x)\hat{y} \right). \tag{1}$$

The resulting vector potential and the magnetic field (with $c = 1$) are given by

$$\vec{A} = 4\pi I \left(\theta(y)y\hat{x} + \theta(x)x\hat{y} \right) \tag{2}$$

and

$$\vec{H} = 4\pi I \left(\theta(x) - \theta(y) \right) \hat{z}. \tag{3}$$

Four spatial sectors are distinguished: In regions defined by x, $y > 0$ and x, $y < 0$ the magnetic field is zero. In other regions defined by $x > 0$, $y < 0$ and $x < 0$, $y > 0$ the magnetic field values are constant and given by $\vec{H} = 4\pi I\hat{z}$ and $\vec{H} = -4\pi I\hat{z}$ respectively.

The interesting region is the quarter of space defined by the positive x and y-axis where the magnetic field vanishes, while the potential is finite:

$$\vec{H} = 0, \ \vec{A} = 4\pi I(y, x, 0); \ x, y > 0. \tag{4}$$

The above potential is connected to the Aharonov-Bohm (AB) potential by a simple map:

The AB potential produced by the magnetic flux confined to the z–axis is given by

$$\vec{A}_{AB} = \frac{\Phi}{2\pi} \left(\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right). \tag{5}$$

Let us represent our potential of (4) and the AB potential as the complex numbers as

$$A = 4\pi I(x + iy); \ x, y > 0 \tag{6}$$

and

$$A_{AB} = \frac{\Phi}{2\pi} \frac{1}{x^2 + y^2} (x + iy), \tag{7}$$

respectively. If we choose the value of the flux as $\Phi = 8\pi^2 I$, we observe that the above complex numbers are related to each other by

$$A = \frac{1}{\bar{A}_{AB}}, \tag{8}$$

826

where $\bar{A}_{AB}$ is the conjugate of (7). This relation is the same as the conformal map which transforms inside of the unit cylinder to the outside region which, in our case, is valid only for the quarter of space defined by the positive x, y–axis.

3. An ideal interference experiment

The Schrödinger equation for a particle with mass μ and charge e in the region $x, y > 0$ is (with $\hbar = 1$)

$$-\frac{1}{2\mu}\left[(\partial_x - 4i\pi eIy)^2 + (\partial_y - 4i\pi eIx)^2 + \partial_z^2\right]\Psi_{++} = i\frac{\partial}{\partial t}\Psi_{++}. \tag{9}$$

The solution differs from the free wave function Ψ_0 by a pure phase:

$$\Psi_{++}(\vec{x}, t) = e^{4i\pi Iex_+y_+}\Psi_0(\vec{x}, t) \tag{10}$$

where x_+, y_+ stand for the coordinates in the region $x, \ y > 0$.

Suppose a coherent electron beam, which is prepared to move in the positive z–direction, is split into two parts and then are allowed to enter the $x, \ y > 0$ and $x, \ y < 0$ regions [Fig.1]. The wave function of the beam moving in $x, \ y > 0$ region is given by

$$\Psi_{++} = e^{4i\pi Iex_+y_+}\Psi_0^k \tag{11}$$

where

$$\Psi_0^k = \frac{1}{\sqrt{2\pi}}e^{-i\frac{k^2}{2\mu}t}e^{ikz} \tag{12}$$

is the free wave function for the motion parallel to the z–axis. Note that the phase picked up by the electron beam remains constant, i.e., depends only on the transverse position in the xy–plane, independent of the distance traveled in z–direction. The second beam moving in $x, \ y < 0$ region will simply be described by the free wave function:

$$\Psi_{--} = \Psi_0^k. \tag{13}$$

If we recombine the above beams to interfere somewhere at the asymptotic values of z–axis, the resulting wave function will be

$$\Psi = \frac{1}{\sqrt{2}}(1 + e^{4i\pi Iex_+y_+})\Psi_0^k. \tag{14}$$

The corresponding probability density is

$$|\Psi|^2 = (1 + \cos(4\pi Iex_+y_+))\left|\Psi_0^k\right|^2 \tag{15}$$

which is dependent on the position of the first beam in the $x, \ y > 0$ region.

If we let the second beam coming from the free region $x, \ y < 0$ interfere with the first beam which traveled in the $x, \ y > 0$ region at a position with coordinates x_+ and y_+ given by

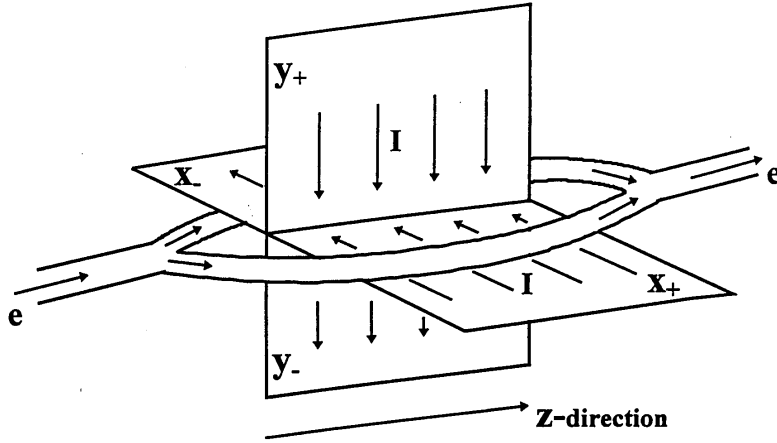$$x_+y_+ = \frac{2n}{4eI}; \ n = 1, 2, 3, \ldots \tag{16}$$

Figure 1. The Intereference Experiment.

we should observe a maximum.

On the other hand if the free beam interferes with a beam from the x, $y > 0$ region which traveled at a position

$$x_+ y_+ = \frac{2n+1}{4eI}; \ n = 1, 2, 3, \ldots \tag{17}$$

we must have a minimum.

Repeating the experiment with different transverse positions of the beam coming from the x, $y > 0$ region we should be able to observe the hyperbolic curves of the interference pattern [Fig.2]

$$y_+ = \frac{n/2eI}{x_+}. \tag{18}$$

4. Discussion

So far we have discussed an ideal arrangement involving infinite planes. Of course, in practice"infinite" means the employment of distances which are sufficiently large in comparison to the wave lengths of the particles. Since the phase in the wave function (11) is independent of the distance traveled in the z–direction, we should still expect to observe an interference effect for a setup which is of finite size along the $z-$ direction.

Let us now consider such a system in greater detail. If the length of the system Z in this direction is large compared to the wave length $\lambda = k_z^{-1}$ of the electron, for example if $Z > 10^2 \lambda$ we, expect that in the central sections around $z \sim Z/2$ the potentials to be roughly the same as those in the ideal case. Then, the electron moving in the transverse
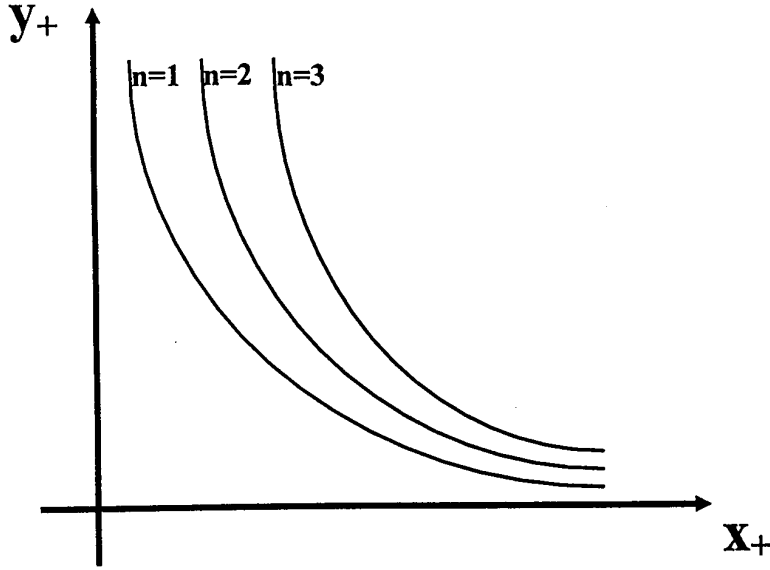
Figure 2. Interference pattern to be obtained after repeated experiments with different transverse positions of the beam coming from the $x, y > 0$.

position x, $y > 0$ picks up the phase

$$e^{4i\pi I e x_+ y_+}, \tag{19}$$

whereas the other branch of the electron beam travelling along the z–axis in any part of the x, $y < 0$ region acquires no phase in this central section.

If coherence between two paths of the electron beam is disturbed at the entrance and the exit sections corresponding to $z \sim 0$ and $z \sim Z$, respectively, the ideal wave functions (11) and (13) are replaced by

$$\Psi_{++} = e^{4i\pi I e x_+ y_+} \Psi_0^+ \tag{20}$$

and

$$\Psi_{--} = \Psi_0^-. \tag{21}$$

Here, Ψ_0^+ and Ψ_0^- are some functions different from Ψ_0^k of (12) due to the disturbances caused by entrance and exit effects. If the proportion of Ψ_0^+ and Ψ_0^- is given by a function $\beta(\vec{x})$ in the form

$$\Psi_0^+ = \beta \Psi_0^- \tag{22}$$

the probability density (15) must then be replaced by

$$|\Psi|^2 = \frac{1}{2}\left(1 + |\beta|^2 + \beta e^{4i\pi I e x_+ y_+} + \beta^* e^{-4i\pi I e x_+ y_+}\right)\left|\Psi_0^-\right|^2.$$

(23)

We now consider two cases:
(i) For real β it is obvious that we have

$$|\Psi|^2 = \left(\frac{1 + \beta^2}{2} + \beta\cos(4\pi I e x_+ y_+)\right)\left|\Psi_0^-\right|^2\,;\ \beta = \text{real},$$

(24)

thus, the hyperbolic interference pattern of Fig.2 is still valid provided that the sign of β does not depend on the transverse coordinates x_+, y_+.
(ii) If β is a complex function given by

$$\beta = |\beta|e^{i\alpha}$$

(25)

the probability density becomes

$$|\Psi|^2 = \left(\frac{1 + |\beta|^2}{2} + |\beta|\cos(\alpha + 4\pi I e x_+ y_+)\right)\left|\Psi_0^-\right|^2,$$

(26)

which for $|\beta|$ and α being slowly varying functions of the transverse coordinates x_+, y_+ we again have the same hyperbolic interference pattern modulated by α.

Acknowledgment

References

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