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New Aspects of Vavilov-Cherenkov Radiation

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Abstract

The effects arising from accelerated and decelerated motion of charged point particles inside a medium are studied. It is shown explicitly that, in addition to the bremsstrahlung and Cherenkov shock wave, the electromagnetic shock wave arising from the charge velocity exceeding the medium light velocity should be observed. This shock wave has the same singularity as the Cherenkov one and therefore, it is more singular than the bremsstrahlung shock wave. The space-time regions where these shock waves exist and conditionsunder which they appear are determined.

1. Introduction

Although the Vavilov-Cherenkov effect is a well established phenomenon widely used in physics and technology [1], many of its aspects remain uninvestigated up to now. In particular, it is not clear how a transition takes place from the sub-light velocity regime to the the super-light one. Some time ago [2,3] it was suggested that side by side with the usual Cherenkov and bremsstrahlung shock waves, the shock wave associated with the charged particle overcoming of the light velocity barrier should exist. The consideration had been presented as pure qualitative without any formulae and numerical results. It was grounded on the analogy with the phenomena occurring in acoustics and hydrodynamics. It seems to us that this analogy is not complete. In fact, electromagnetic waves are pure transversal, while acoustic and hydrodynamic waves contain longitudinal components. Further, the analogy itself cannot be considered as a final proof. This fact and experimental ambiguity to distinguish Cherenkov from bremsstrahlung rediation [4] enables us to consider effects arising from the charged particle overcoming of the light barrier in the framework of the completly soluble model.

In Ref. [5] we considered the straight-line motion of the charged particle with a constant acceleration $z = at^2$. This motion law is maintained by the following electric field

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directed along z axis:

$$E_z = \frac{2ma}{e(1-4az/c^2)^{3/2}}. \qquad (1.1)$$

In agreement with Refs. [2,3] we confirmed in [5] the existence of the the shock wave arising at the moment when charged particle exceeds the light velocity inside the medium. This wave has essentially the same singularity as Cherenkov shock wave. It is much stronger then the singularity of the bremsstrahlung shock wave. Previously, the accelerated motion of the point charge in a vacuum was considered by Schott [6]. Yet, his qualitative consideration was pure geometrical, not allowing for numerical investigations.

The electric field tends to ∞ as z approaches $c^2/4a$. This singularity complicates the experimental verification of the shock waves arising from the charge velocity exceeding the light velocity in medium.

Here we consider the straight-line motion of the point charge in a constant uniform electric field and evaluate the arising electromagnetic field (EMF) which is much easier to create than the electric field described by the electric field described by (1.1).

2. Statement of Physical Problem

Let a charged particle move inside a medium with polarizabilities ϵ and μ along the given trajectory $\vec{\xi}(t)$. Then, its EMF at the observation point (ρ, z) is given by the Lienard-Wiechert potentials

$$\Phi(\vec{r},t) = \frac{e}{\epsilon}\sum\frac{1}{|R_i|}, \quad \vec{A}(\vec{r},t) = \frac{e\mu}{c}\sum\frac{\vec{v}_i}{|R_i|}, \quad div\vec{A} + \frac{\epsilon\mu}{c}\dot{\Phi} = 0. \qquad (2.1)$$

Here,

$$\vec{v}_i = (\frac{d\vec{\xi}}{dt})|_{t=t_i}, \quad R_i = |\vec{r} - \vec{\xi}(t_i)| - \vec{v}_i(\vec{r} - \vec{\xi}(t_i))/c_n$$

and c_n is the light velocity inside the medium ($c_n = c/\sqrt{\epsilon\mu}$). The summation in (2.1) is performed over all physical roots of the equation

$$c_n(t - t') = |\vec{r} - \vec{\xi}(t')|. \qquad (2.2)$$

To preserve the causality, the time of radiation t' should be smaller than the observation time t. Obviously, t' depends on the coordinates $\vec{r}, t$ of the point P at which the EMF is observed. Accounting for (2.2) one gets for R_i

$$R_i = c_n(t - t_i) - \vec{v}_i(\vec{r} - \vec{\xi}(t_i)). \qquad (2.3)$$

Consider the motion of a charged point-like particle with rest mass m inside the medium in a constant electric field E along the Z axis. The motion law is given by (see, e.g.,[7])

$$z(t) = \sqrt{z_0^2 + c^2t^2} - z_0, \quad z_0 = mc^2/E > 0. \qquad (2.4)$$

The charge velocity is given by

$$v = \dot{z} = c^2t(z^2 + c^2t^2)^{-1/2}.$$

Clearly, it tends to the light velocity in vacuum for $t \to \infty$. The retarded times t' satisfy the following equation:

$$c_n(t - t') = [\rho^2 + (z + z_0 - \sqrt{z_0^2 + c^2 t'^2})^2]^{1/2}. \tag{2.5}$$

It is convenient to introduce the dimensionless variables

$$\tilde{t} = ct/z_0, \quad \tilde{z} = z/z_0, \quad \tilde{\rho} = \rho/z_0. \tag{2.6}$$

Then,

$$\alpha(\tilde{t} - \tilde{t}') = [\tilde{\rho}^2 + (\tilde{z} + 1 - \sqrt{1 + \tilde{t}'^2})^2]^{1/2}, \quad \alpha = c_n/c. \tag{2.7}$$

In order not to overload exposition we drop the tilda signs:

$$\alpha(t - t') = [\rho^2 + (z + 1 - \sqrt{1 + t'^2})^2]^{1/2}. \tag{2.8}$$

For the treated case of one-dimensional motion the denominators R_i are given by:

$$R_i = \frac{z_0}{\alpha\sqrt{1 + t'^2}}[\alpha^2(t - t_i)\sqrt{1 + t'^2} - t_i(z + 1 - \sqrt{1 + t_i^2})^2]. \tag{2.9}$$

We consider the following two problems:
I. A charged particle rests at the origin up to a moment $t' = 0$. After that it is accelerated in uniform electric field in the positive direction of the Z axis. In this case only positive retarded times t' are nontrivial.
II. A charged particle decelerates in the uniform electric field moving from $z = \infty$ to the origin. After the moment $t' = 0$ it rests there. Only negative retarded times are nontrivial in this case.
It is easy to check that the moving charge acquires the light velocity c_n at the moments $t_l = \pm\alpha/\sqrt{1 - \alpha^2}$ for the accelerated and decelerated motion, resp. The position of a charge at those moments is $z_l = 1/\sqrt{1 - \alpha^2} - 1$.
It is our aim to investigate space-time distribution of the EMF's arising from such particle motions.

3. Numerical Results

We consider at first the typical case corresponding to $|t| = 2$.

3.1. Accelerated Motion

For the first of the treated problems (acceleration of the charge initially resting at the origin) in the uniform electric field the resulted configuration of the shock waves are shown in Fig. 1 ($\alpha = 1/2$).

We see on them the Cherenkov shock wave C_M having the form of the Mach cone, the surface C_L closing the Mach cone and the spherical wave C_0 representing the spherical shock wave arising from the beginning of the charge motion. It turns out that the surface

C_L with a good accuracy is approximated by the part of the sphere $\rho^2+(z-z_l)^2 = (t-t_l)^2$ (shown by the short-dash curve) which corresponds to the shock wave emitted from the point $z_l = (1-\alpha^2)^{-1/2} - 1$ at the moment $t_l = \alpha(1-\alpha^2)^{-1/2}$ when the velocity of the charged particle coincides with the velocity of light in the medium. On the internal sides of the surfaces C_L and C_M electromagnetic potentials acquire infinite values. On the external side of C_M lying outside of C_0 the electromagnetic potentials are zero (as there are no solutions there). On the external sides of C_L and on the part of the C_M surface lying inside C_0 the electromagnetic potentials have finite values.

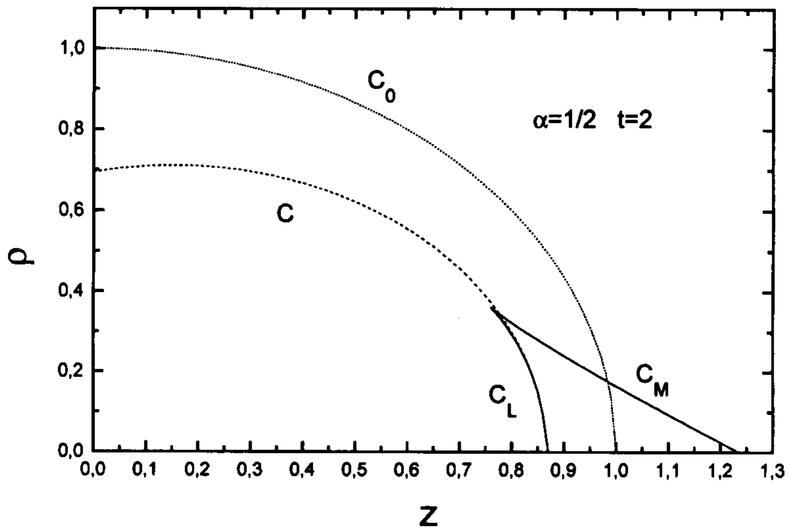

Figure 1. The distribution of the shock waves for an the accelerated charge ($\alpha = 1/2t = 2$). C_M is the Cherenkov shock wave, C_L is the shock wave emitted from the point $z_l = (1-\alpha^2)^{-1/2}$ at the moment $t_l = \alpha(1-\alpha^2)^{-1/2}$ when the charge velocity coincides with the medium light velocity. Part of it with a good accuracy is described by the spherical surface $\rho^2 + (z-z_l)^2 = (t-t_l)^2$ (shown by the short-dash curve). C_0 is the bremsstrahlung shock wave originating from the beginning of the charge motion.

The positions of the shock waves for different observation times are shown in Fig. 2 ($\alpha = 1/2$). The dimension of the Mach cone is zero for $t < t_l$ and continuously rises with time $t > t_l$. The physical reason for this is that the C_L shock wave closing the Mach cone propagates with the lightvelocity c_n, while the head part of the Mach cone C_M attached to the charged particle propagates with the velocity $v > c_n$.

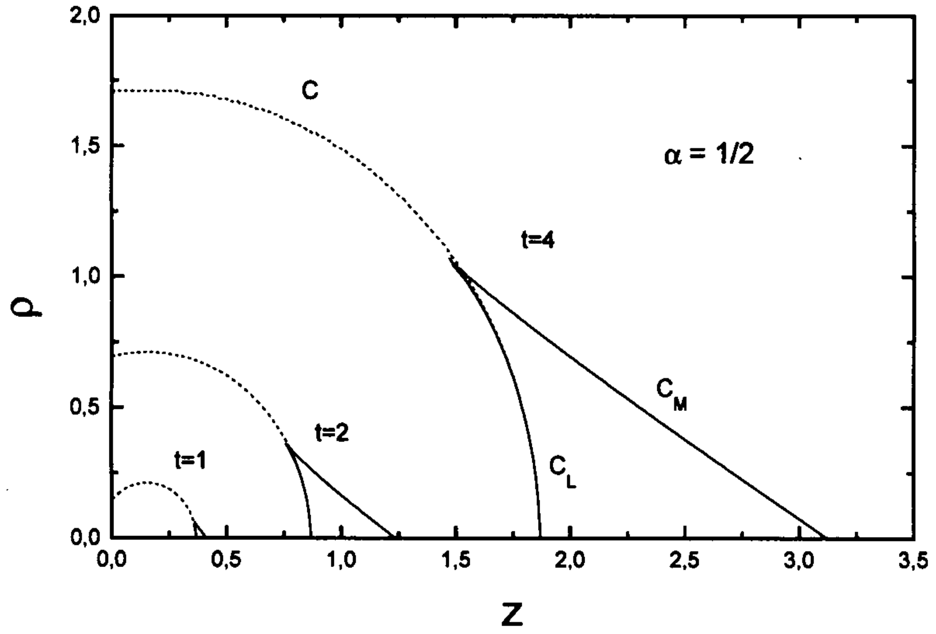

Figure 2. The distribution of shock waves for an accelerated charge and $\alpha = 1/2$. C_L are shock waves emitted from the points in which the charge velocity coincides with the medium light velocity. Short dash curves are the same as in Fig. 1. The distribution of the magnetic vector potential on the surface of cylinder C_ρ. The number of a particular curve means $\beta = v/c$; z and A_z are in units c/ω_0 and $e\omega_0/c$, resp.

3.2. Decelerated Motion

Now we turn to the second problem (deceleration of the charged particle in the uniform electric field along the positive z semi-axis up to a moment $t = 0$ after which it rests at the origin). In this case only negative retarded times t_i have a physical meaning.
For the observation time $t > 0$ the resulting configuration of the shock waves is shown in Fig. 3 for $\alpha = 1/2$. On them we see the bremsstrahlung shock wave C_0 arising from the termination of the charge motion and the blunt shock wave C_L. Its head part with great accuracy is described by the sphere $\rho^2 + (z - z_l)^2 = (t + t_l)^2$ (shown by the short-dash curve) corresponding to the shock wave emitted from the point $z_l = (1 - \alpha^2)^{-1/2} - 1$ at the moment $t_l = -\alpha(1 - \alpha^2)^{-1/2}$ when the velocity of the decelerated charged paricle coincides with the velocity of light in the medium. The electromagnetic potentials vanish outside of C_L (as no solutions exist there) and acquire infinite values on the internal part of C_L. Therefore, the surface C_L represents the shock wave. As a result, for the decelerated motion after termination of the particle motion $t > 0$ one has the shock wave

C_L and the bremsstrahlung shock wave C_0 arising from the termination of the particle motion.

For the decelerated motion and $t < 0$ (i.e., before termination of the charge motion) the physical solutions exist only inside the Mach cone C_M (Figs. 4). On its internal boundary the electromagnetic potentials acquire infinite values. On the external boundary the electromagnetic potentials are zero (as no solutions exist there). In gas dynamics the existence of at least two shock waves attached to a finite body moving with a supersonic velocity was proved on the very general grounds by Landau and Lifshitz ([8] , Chapter 13). In the present context we associate them with the $C_L^{(1)}$ and $C_M^{(1)}$ shock waves.

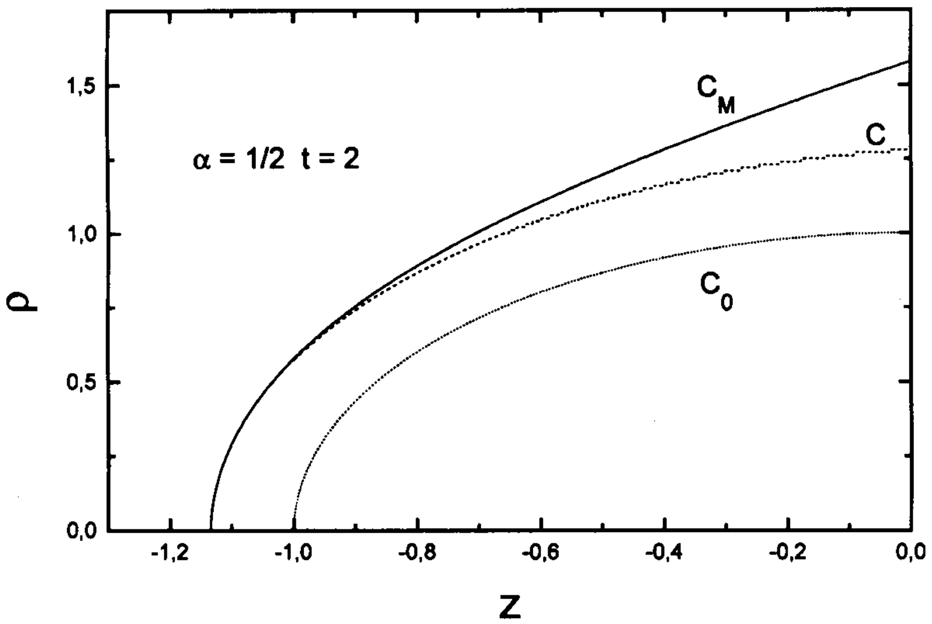

Figure 3. The distribution of the shock waves for the decelerated charge ($\alpha = 1/2$, $t = 2$) in the uniform electric field. C_M is the blunt shock wave. Part of it, with good accuracy, is approximated by the spherical surface $\rho^2 + (z - z_l)^2 = (t + t_l)^2$ (it is shown by short dash curve C). C_0 is the bremsstrahlung shock wave originating from the termination of the charge motion.

In order not to hamper the exposition we did not mention in this section the continuous radiation which reaches the observer between the arrival of two shock waves or after the arrival of the last shock wave. It is easily restored from the complete exposition presented in Ref. [5] for the $z = at^2$ motion law. The results of calculations for other values of charge velocity may be found in Ref. [9].

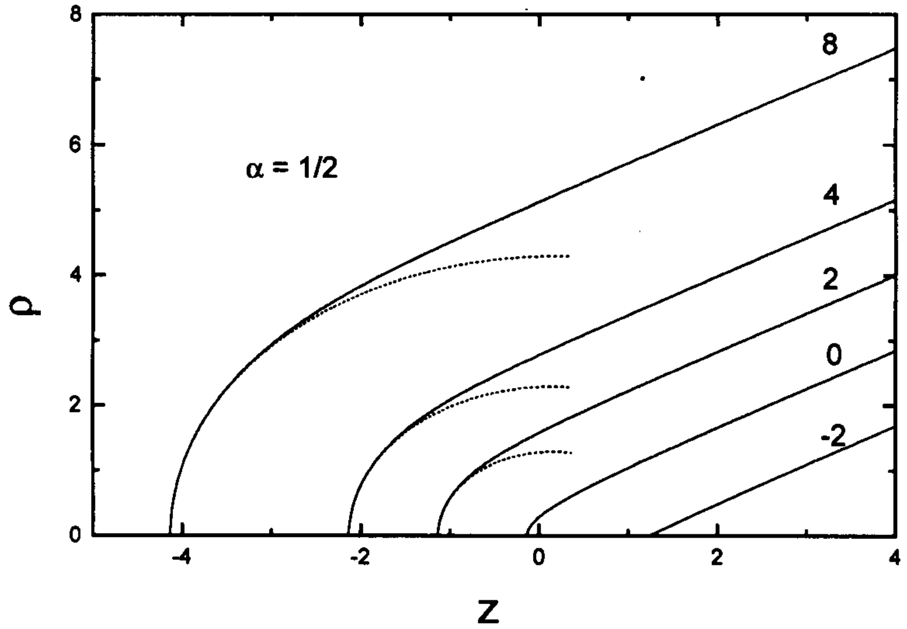

Figure 4. The continuous transformation of the Cherenkov shock wave shown at the right of figure into the blunt shock wave shown at the left for the decelerated motion for $\alpha = 1/2$. The numbers on the curves denote the observation times. Short dash curves are the same as in Fig. 3.

4. Conclusion

Thus we confirm the qualitative predictions of refs. [2,3] concerning the existence of the shock waves arising from the charge overcoming the light velocity barrier (inside the medium). It would be interesting to observe them experimentally.

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