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Two-level atom in a squeezed vacuum

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Abstract

The master equation for a two-level atom interacting with a strong coherent field and damped into a reservoir formed by a finite bandwidth squeezed vacuum is derived. The master equation extends the Yeoman and Barnett approach to a non-zero detuning of the driving field from the atomic resonance and allows to discuss the role of squeezing bandwidth and the detuning in the level shifts, widths and intensities of spectral lines. The approach is valid for arbitrary values of the Rabi frequency and detuning but for the squeezing bandwidths larger than the natural linewidth in order to satisfy the Markov approximation.

1. Introduction

A squeezed vacuum which can now be produced in laboratory [1-3] is a quantum state of the electromagnetic field with very special properties. If the bandwidth of the squeezed vacuum is large enough it can be treated as a reservoir to the atom subjected to such field. However, the squeezed vacuum, contrary to the ordinary vacuum, carries some phase information and the behavior of the atom in such a reservoir is quite different from its behavior in the ordinary vacuum. Gardiner [4] has shown that in the squeezed vacuum the atomic dipole moment can decay with two vastly different rates, one much longer and the other much shorter than that in the normal vacuum. In consequence, a subnatural linewidth has been predicted in the spontaneous emission spectrum. The addition of a coherent driving field to the problem introduces a strong dependence of the atom dynamics and the fluorescence and absorption spectra on the relative phase between the coherent field and the squeezed field. Carmichael et al. [5] have shown, for example, that the central peak of the Mollow [6], depending on the phase, can either be much narrower or much broader than the natural linewidth of the atom. Apart from

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these quantitative changes, the qualitative changes of the fluorescence spectrum have also been predicted. Courty and Reynaud [7] have found that for a certain detuning of the driving field from the atomic resonance the central line and one of the sidebands can be suppressed due to a population trapping in the dressed state. A number of unusual features in the resonance fluorescence spectra, such as hole burning and dispersive profiles, have been found by Smart and Swain [8-10]. These features, however, appear for Rabi frequencies comparable to the atomic linewidth and are very sensitive to the various parameters involved.

Another spectroscopic feature accessible to experimental verification is the probe absorption spectrum. Mollow [11] has predicted that the absorption spectrum of a weak field probing a system of two-level atoms driven by an off-resonant laser field consists of one absorption and one emission component at the Rabi sidebands and a small dispersion like component at the center of the spectrum. The probe field can be amplified due to the population inversion between the dressed states of the atom despite the fact that there is no population inversion between the bare atomic states.

Most of the studies dealing with the problem of a two-level atom in a squeezed vacuum assume that the squeezed vacuum is broad band, i.e., the bandwidth of the squeezed vacuum is much larger than the atomic linewidth and the Rabi frequency of the driving field. Experimental realizations of squeezed states, however, indicate that the bandwidth of the squeezed light is typically of the order of the atomic linewidth. The most popular schemes for generating squeezed light are those using parametric oscillator operating below threshold, the output of which is a squeezed beam with a bandwidth of the order of the cavity bandwidth [12,13]. There are two types of squeezed field that can be generated by such a parametric oscillator. If the oscillator works in a degenerate regime, the squeezed field has the profile with the maximum squeezing at the central frequency and a small squeezing far from the center. In the non-degenerate regime, the profile has two peaks at frequencies symmetrically displaced from the central frequency. For strong driving fields and finite bandwidth of squeezing this means that the Rabi sidebands can feel quite a different squeezing then the central line. A realistic description of radiative properties of the two-level atom in such a squeezed field must thus take into account the finite bandwidth of the squeezed field.

First studies of the finite-bandwidth effects have been performed by Gardiner et al. [12], Parkins and Gardiner [13] and Ritsch and Zoller [14]. The approaches were based on stochastic methods and numerical calculations, and were applied to analyze the narrowing of the spontaneous emission and absorption lines. The fundamental effect of narrowing has been confirmed, but the effect of finite bandwidth was to degrade the narrowing of the spectral lines rather than enhance it. Later, however, numerical simulations done by Parkins [15,16] demonstrated that for strong driving fields a finite bandwidth of squeezing can have positive effect on the narrowing of the Rabi sidebands. He has found that there is a difference between the two types of squeezed light generated in either degenerate or non-degenerate regime of the parametric oscillator. In the former case it is possible to narrow either both of the Rabi sidebands or the central peak of the fluorescent spectrum, while in the latter case simultaneous narrowing of all three spectral peaks is possible.

Recently, Yeoman and Barnett [17] have proposed an analytical technique for investigating the behavior of a coherently driven atom damped by a squeezed vacuum with finite bandwidth. In the approach, they derived a master equation and analytic expressions for the fluorescent spectrum for the simple case of a two-level atom exactly resonant with the frequencies of both the squeezed field and the driving field. Their analytical results agree with that of Parkins [15,16] and show explicitly that the width of the central peak of the fluorescent spectrum depends solely on the squeezing present at the Rabi sideband frequencies. They have assumed that the atom is classically driven by a resonant laser field for which the Rabi frequency is much larger than the bandwidth of the squeezed vacuum, though this is still large compared to the natural linewidth. Unlike conventional theory, based on uncoupled states, it is possible to obtain a master equation consistent with the Born-Markov approximation by first including the interaction of the atom with the driving field exactly, and then considering the coupling of this combined dressed atom system with the finite-bandwidth squeezed vacuum. The advantage of this dressed atom method over the more complex treatments based on adjoint equation or stochastic methods [15,16,18] is that simple analytical expressions for the spectra can be obtained, thus displaying explicitly the factors that determine the intensities of the spectral features and their widths. Recently, the idea of Yeoman and Barnett has been extended by Ficek et al. [19] to the case of a fully quantized dressed-atom model coupled to a finite bandwidth squeezed field inside an optical cavity. They have studied the fluorescence spectrum under the secular approximation and have found that in the presence of a single-mode cavity the effect of squeezing on the fluorescence spectrum is more evident in the linewidths of the Rabi sidebands rather than in the linewidth of the central component. In the presence of a two-mode cavity and a two-mode squeezed vacuum the signature of squeezing is evident in the linewidths of all spectral lines. They have also established that the narrowing of the spectral lines is very sensitive to the detuning of the driving field from the atomic resonance. The dressed atom method, including a detuning of the driving field from the atomic resonance has also been applied to calculate the probe absorption spectra of a driven two-level atom in a narrow bandwidth squeezed vacuum [20].

In this paper I present the derivation of the master equation as well as the atomic Bloch equations that we have obtained recently [21].

2. Master equation and atomic Bloch equations

We consider a two-level atom driven by a detuned monochromatic laser field and damped by a squeezed vacuum with finite bandwidth. Applying the approach of Yeoman and Barnett [17] we derive a master equation describing the time evolution of the system which includes squeezing bandwidth effects. In this approach, we first perform the dressing transformation to include the interaction of the atom with the driving field and next we couple the resulting dressed atom to the narrow bandwidth squeezed vacuum field. We derive the master equation under the Markov approximation which requires the squeezing bandwidth to be much greater than the atomic linewidth, but not necessarily greater than the Rabi frequency of the driving field and the detuning. For simplicity, we assume that the squeezing properties are symmetric about the central frequency of the squeezed field which, in turn, is exactly equal to the laser frequency. Our model differs from that of Yeoman and Barnett in performing Markov approximation in the time domain instead of pole approximation in the Laplace transform domain and in adding a nonzero detuning.

We start from the Hamiltonian of the system which in the rotating-wave and electric-dipole approximations is given by

$$H = H_A + H_R + H_L + H_I \,, \tag{1}$$

where

$$H_A = \frac{1}{2}\,\hbar\omega_A\,\sigma_z = -\frac{1}{2}\hbar\Delta\,\sigma_z + \frac{1}{2}\hbar\,\omega_L\,\sigma_z \tag{2}$$

is the Hamiltonian of the atom,

$$H_R = \int_0^\infty \hbar\omega\, b^+(\omega)\, b(\omega)\, \mathrm{d}\omega \tag{3}$$

is the Hamiltonian of the vacuum field,

$$H_L = \frac{1}{2}\hbar\Omega\, [\, \sigma_+ \exp(-i\omega_L t) + \sigma_- \exp(i\omega_L t)\,] \tag{4}$$

is the interaction between the atom and the classical laser field, and

$$H_I = i\hbar \int_0^\infty K(\omega)\, \left[\, \sigma_+\, b(\omega) - b^+(\omega)\, \sigma_- \,\right]\, \mathrm{d}\omega \tag{5}$$

is the interaction of the atom with the vacuum field. In (2)-(5), $K(\omega)$ is the coupling of the atom to the vacuum modes, $\Delta = \omega_L - \omega_A$ is the detuning of the driving laser field frequency ω_L from the atomic resonance ω_A, and σ_+, σ_-, and σ_z are the Pauli pseudo-spin operators describing the two-level atom. The laser driving field strength is given by the Rabi frequency Ω, while the operators $b(\omega)$ and $b^+(\omega)$ are the annihilation and creation operators for the vacuum modes satisfying the commutation relation

$$[b(\omega), b^+(\omega')] = \delta(\omega - \omega')\,. \tag{6}$$

For simplicity, we assume that the laser field phase is equal to zero ($\phi_L = 0$).

In order to derive the master equation we perform the two-step unitary transformation. In the first step we use the second part of the atomic Hamiltonian (2) and the free field Hamiltonian (3) to transform to the frame rotating with the laser frequency ω_L and to the interaction picture with respect to the vacuum modes. After this transformation our system is described by the Hamiltonian

$$H_0 + H_I^r(t)\,, \tag{7}$$

where

$$H_0 = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\Omega(\sigma_+ + \sigma_-)\,, \tag{8}$$

and

$$H_I^r(t) = i\hbar\int_0^\infty K(\omega)\left[\sigma_+\, b(\omega)\,\exp[i(\omega_L-\omega)\,t] - b^+(\omega)\,\sigma_-\,\exp[-i(\omega_L-\omega)\,t]\right]\mathrm{d}\omega\,. \tag{9}$$

The second step is the unitary dressing transformation performed with the Hamiltonian H_0, given by (8). The transformation

$$\sigma_\pm(t) = \exp[-\frac{i}{\hbar}H_0t]\sigma_\pm\exp[\frac{i}{\hbar}H_0t] \tag{10}$$

leads to the following time-dependent atomic raising and lowering operators:

$$\sigma_\pm(t) = \frac{1}{2}\left[\sigma_a \pm (1\mp\tilde{\Delta})\sigma_b\exp(i\Omega' t) \pm (1\pm\tilde{\Delta})\sigma_c\exp(-i\Omega' t)\right]\,, \tag{11}$$

where

$$\begin{aligned}
\sigma_a &= \tilde{\Omega}\left[\tilde{\Omega}(\sigma_+ + \sigma_-) - \tilde{\Delta}\sigma_z\right]\,,\\
\sigma_b &= \frac{1}{2}\left[(1-\tilde{\Delta})\sigma_+ - (1+\tilde{\Delta})\sigma_- - \tilde{\Omega}\sigma_z\right]\,,\\
\sigma_c &= \frac{1}{2}\left[(1+\tilde{\Delta})\sigma_+ - (1-\tilde{\Delta})\sigma_- + \tilde{\Omega}\sigma_z\right]\,,
\end{aligned} \tag{12}$$

are the 'dressed' operators oscillating at frequencies 0, Ω' and $-\Omega'$, respectively, and

$$\tilde{\Omega} = \frac{\Omega}{\Omega'}\,,\quad \tilde{\Delta} = \frac{\Delta}{\Omega'}\,,\quad \Omega' = \sqrt{\Omega^2+\Delta^2}\,. \tag{13}$$

For $\Delta = 0$, transformation (11) reduces to that of Yeoman and Barnett [17]. Under transformation (11) the interaction Hamiltonian takes the form

$$\begin{aligned}
H_I(t) = i\hbar\int_0^\infty K(\omega)\left[\sigma_+(t)b(\omega)\exp[i(\omega_L-\omega)\,t] - b^+(\omega)\sigma_-(t)\right.\\
\left.\exp[-i(\omega_L-\omega)t]\right]\mathrm{d}\omega\,.
\end{aligned} \tag{14}$$

The master equation for the reduced density operator ρ of the system can be derived using standard methods [22]. In the Born approximation the equation of motion for the reduced density operator is given by [22]

$$\frac{\partial\rho^D}{\partial t} = -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\left\{[H_I(t),[H_I(t-\tau),\rho_R(0)\rho^D(t-\tau)]]\right\}\,\mathrm{d}\tau\,, \tag{15}$$

where the superscript D stands for the dressed picture, $\rho_R(0)$ is the density operator for the field reservoir, Tr_R is the trace over the reservoir states and the Hamiltonian $H_I(t)$ is given by (14). We next make the Markov approximation [22] by replacing $\rho^D(t-\tau)$ in (15) by $\rho^D(t)$, substitute the Hamiltonian (14) and take the trace over the reservoir variables. We assume that the reservoir is in a squeezed vacuum state in which the operators $b(\omega)$ and $b^+(\omega)$ satisfy the relations [4]

$$\begin{aligned} \langle b(\omega)b^+(\omega')\rangle &= [N(\omega)+1]\,\delta(\omega-\omega')\,, \\ \langle b^+(\omega)b(\omega')\rangle &= N(\omega)\,\delta(\omega-\omega')\,, \\ \langle b(\omega)\,b(\omega')\rangle &= M(\omega)\,\delta(2\,\omega_L-\omega-\omega')\,, \end{aligned} \tag{16}$$

where $N(\omega)$ and $M(\omega)$ are the parameters describing the squeezing and that the carrier frequency of the squeezed field is equal to the laser frequency ω_L. In the Markov approximation we can extend the upper limit of the integration over τ to infinity and next perform necessary integrations using the formula

$$\int_0^\infty \exp(\pm i\,\epsilon\,\tau)\,\mathrm{d}\tau = \pi\,\delta(\epsilon) \pm i\,\mathcal{P}\frac{1}{\epsilon}\,, \tag{17}$$

where $\mathcal{P}$ means the Cauchy principal value. After lengthy calculations we obtain the master equation which, in the frame rotating with the laser frequency ω_L, can be written as

$$\begin{aligned} \dot\rho = {} & \frac{1}{2}\,i\,\gamma\,\delta\,[\sigma_z,\rho] \\ & + \frac{1}{2}\,\gamma\,\tilde N\;(2\,\sigma_+\rho\,\sigma_- - \sigma_-\sigma_+\rho - \rho\,\sigma_-\sigma_+) \\ & + \frac{1}{2}\,\gamma\,(\tilde N+1)\;(2\,\sigma_-\rho\,\sigma_+ - \sigma_+\sigma_-\rho - \rho\,\sigma_+\sigma_-) \\ & - \gamma\,\tilde M\,\sigma_+\rho\,\sigma_+ - \gamma\,\tilde M^*\,\sigma_-\rho\,\sigma_- \\ & - \frac{1}{2}\,i\,\Omega\,[\sigma_+ + \sigma_-,\rho] + \frac{1}{4}\,i\;(\beta\,[\sigma_+,[\sigma_z,\rho]] - \beta^*[\sigma_-,[\sigma_z,\rho]])\,, \end{aligned} \tag{18}$$

where γ is the natural atomic linewidth,

$$\tilde N = N(\omega_L+\Omega') + \frac{1}{2}\,(1-\tilde\Delta^2)\,\gamma_n\,, \tag{19}$$

$$\tilde M = \Big(\,|M(\omega_L+\Omega')| + i\,\tilde\Delta\,\delta_M\Big)\;\mathrm{e}^{i\phi} - \frac{1}{2}\,(1-\tilde\Delta^2)\,(\gamma_n - i\,\delta_n)\,, \tag{20}$$

$$\delta = \frac{\Delta}{\gamma} + \tilde\Delta\,\delta_N + \frac{1}{2}(1-\tilde\Delta^2)\,\delta_n\,, \tag{21}$$

$$\beta = \gamma\tilde\Omega\,\Big[\,\delta_N + \delta_M\,\mathrm{e}^{i\phi} - i\,\tilde\Delta\;(\,\gamma_n - i\,\delta_n\,)\Big]\,, \tag{22}$$

$$\gamma_n = N(\omega_L) - N(\omega_L + \Omega') - (|M(\omega_L)| - |M(\omega_L + \Omega')|) \cos\phi \,, \tag{23}$$

$$\delta_n = (\,|M(\omega_L)| - |M(\omega_L + \Omega')|\,) \sin\phi \,, \tag{24}$$

$$\delta_N = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{N(x)}{x+\Omega'}\,\mathrm{d}x \,, \qquad \delta_M = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{|M(x)|}{x+\Omega'}\,\mathrm{d}x \,, \tag{25}$$

and ϕ is the phase of squeezing ($M(\omega) = |M(\omega)| \exp(i\phi)$). In the derivation of equation (18) we have included the divergent frequency shifts (the Lamb shift) to the redefinition of the atomic transition frequency. Moreover, we have assumed that the squeezed vacuum is symmetric about the central frequency ω_L, so that $N(\omega_L - \Omega') = N(\omega_L + \Omega')$ and a similar relation holds for $M(\omega)$.

The master equation (18) has the standard form known from the broadband squeezing approaches with the new effective squeezing parameters $\tilde{N}$ and $\tilde{M}$ given by (19) and (20). There are also new terms, proportional to β which are essentially narrow bandwidth modifications to the master equation. All the narrow bandwidth modifications are determined by the parameters γ_n, δ_n and the shifts δ_N and δ_M (see [17,21]). These parameters become zero when the squeezing bandwidth goes to infinity.

From the master equation (18) we easily derive the optical Bloch equations for the mean values of the atomic operators

$$\begin{aligned} \langle\dot{\sigma}_-\rangle &= -\gamma\,(\frac{1}{2} + \tilde{N} - i\,\delta)\langle\sigma_-\rangle - \gamma\,\tilde{M}\,\langle\sigma_+\rangle + \frac{i}{2}\Omega\,\langle\sigma_z\rangle \,, \\ \langle\dot{\sigma}_z\rangle &= i\,(\Omega + \beta^*)\langle\sigma_-\rangle - i\,(\Omega + \beta)\,\langle\sigma_+\rangle - \gamma\,(1 + 2\,\tilde{N})\,\langle\sigma_z\rangle - \gamma \,. \end{aligned} \tag{26}$$

The equation for $\langle\sigma_+\rangle$ is obtained as a Hermitian conjugate of equation for $\langle\sigma_-\rangle$. Defining the Hermitian operators σ_x and σ_y as

$$\sigma_x = \frac{1}{2}(\sigma_- + \sigma_+) \,, \qquad \sigma_y = \frac{1}{2i}(\sigma_- - \sigma_+) \tag{27}$$

we get from (26) the following equations of motion for the atomic polarization quadratures:

$$\begin{aligned} \langle\dot{\sigma}_x\rangle &= -\gamma\,\left(\frac{1}{2} + \tilde{N} + \mathrm{Re}\,\tilde{M}\right)\langle\sigma_x\rangle - \gamma\,\left(\mathrm{Im}\,\tilde{M} + \delta\right)\,\langle\sigma_y\rangle \,, \\ \langle\dot{\sigma}_y\rangle &= -\gamma\,\left(\mathrm{Im}\,\tilde{M} - \delta\right)\langle\sigma_x\rangle - \gamma\,\left(\frac{1}{2} + \tilde{N} - \mathrm{Re}\,\tilde{M}\right)\langle\sigma_y\rangle + \frac{1}{2}\,\Omega\,\langle\sigma_z\rangle \,, \\ \langle\dot{\sigma}_z\rangle &= 2\,\mathrm{Im}\,\beta\,\langle\sigma_x\rangle - 2\,(\Omega + \mathrm{Re}\,\beta)\,\langle\sigma_y\rangle - \gamma\,(1 + 2\,\tilde{N})\,\langle\sigma_z\rangle - \gamma \,. \end{aligned} \tag{28}$$

The modified Bloch equations (28) describe the evolution of the atom and together with the quantum regression theorem can be used for calculations of the resonance fluorescence as well as absorption spectra.

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