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Electrodynamics with Toroid Polarization

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Abstract

A modified system of equations of electrodynamics of continuous media has been obtained. Beside the Lagrangian system an alternative gauge-like formalism has been developed to introduce the toroid moment contributions in the obtained equations. The two potential formalism that was worked out by us earlier has been developed further where along with the two vector potentials we introduce two scalar potentials thus taking into account all four basic equations of electromagnetism.

Key words: Toroid moments, two-potential formalism
PACS 03.50.De Maxwell theory: general mathematical aspects

PACS 11.10.Ef Lagrangian and Hamiltonian approach

1. Introduction

The history of electromagnetism is the history of struggle of different rival concepts from the very early days of its existence. After the historical observation by Hertz, all main investigations in electromagnetism were based on the Maxwell equations. Nevertheless, this theory still suffers from some shortcomings inherited by its predecessors. Several attempts were made to remove the internal inconsistencies of the theory. To be short we refer to very few of them. One of the attempts to modify the theory of electromanetism was connected with the introduction of magnetic charge in Maxwell equation by Dirac [1,2], while keeping the usual definition of **E** and **B** in terms of the gauge potentials. Recently D. Singleton [3] developed this theory introducing two four-vector potentials: $A^\mu = (\phi_e, \mathbf{A})$ and $C^\mu = (\phi_m, \mathbf{C})$. Note that, a similar theory (two potential formalism) was developed by us few years ago (we will come back to it in Sec. 3). The main defect of the theory developed by Singleton in our view is that the existence of magnetic charge still lacks experimental support, hence can be considered as a mathematically convenient one only.

Recently Chubykalo a.o. made an effort to modify the electromagnetic theory by invoking both the transverse and longitudinal (explicitly time independent) fields simultaneously, thus giving an equal footing to both the Maxwell-Hertz and Maxwell-Lorentz equations [4]. To remove all ambiguities related to the applications of Maxwell's displacement current they substituted all partial derivatives in Maxwell-Lorenz equations by the *total* derivatives and separated all field quantities into *two independent classes* with explicit $\{\}^{\star}$ and implicit $\{\}_0$ time dependence, respectively.

Another attempt to modify the equations of eletromagnetism is connected with the existence of the third family of multipole moments, namely the *toroid* one. This theory was developed by us during recent years. Recently, we introduced toroid moments in Maxwel equations exploiting Lagrangian formalism [5]. In the Sec. 2 of this paper we give a brief description of this formalism. Moreover, we develop here an alternative method to introduce toroid moments in the equation of electromagnetism. In Sec. 3 we develop two potential formalisms suggested by us earlier.

2. Introduction of Toroid Moments into the Equations of Electromagnetism

Ya. Zel'dovich [6] was the first to introduce *anapole* in connection with the global electromagnetic properties of a toroid coil that are impossible to describe within the charge or magnetic dipole moments in spite of explicit axial symmetry of the toroid coil. Further, in 1974 Dubovik and Cheskov [7] determined the toroid moment in the framework of classical electrodynamics. Recently, a principally new type of magnetism known as *aromagnetism* was observed in a class of organic substances, suspended either in water or in other liquids [8]. Later it was shown that this phenomena of aromagnetism cannot be explained in a standard way, e.g., by ferromegnetism, since the organic molecules do not possess magnetic moments of either orbital or spin origin. It was also shown that the origin of aromagnetism is the interaction of vortex electric field induced by alternative magnetic moments or axial toroid moments in aromatic elements [9]. These experimental results force the introduction of toroid moments in the framework of conventional classical electrodynamics, that in its part inevitably leads to the modification of the equations of electromagnetism. In the following two subsections we give two alternative schemes for the introduction of toroid moments in the electromagnetic equations.

A. Lagrangian Formalism

As a starting point we consider the interacting system of electromagnetic field and non-relativistic charged particles given by the Lagrangian density [10]

$$\begin{aligned} L &= L_{par} + L_{rad} + L_{int} \qquad (1) \\ L_{par} &= \frac{1}{2}\sum_{\alpha} m_\alpha \dot{q}_\alpha^2 - \frac{1}{2}\sum_{\alpha\neq\beta}\frac{e_\alpha e_\beta}{\mid q_\alpha - q_\alpha} \end{aligned}$$

$$L_{rad} = \frac{1}{8\pi}\int[\frac{\dot{\mathbf{A}}^2}{c^2} - (\text{curl}\mathbf{A}^2)]dr$$
$$L_{int} = \frac{1}{c}\int \mathbf{J}(r).\mathbf{A}(\mathbf{r})dr = \sum_\alpha \frac{e_\alpha}{c}\dot{q}_\alpha.\mathbf{A}(q_\alpha, t).$$

Here L_{par} is the Lagrangian appropriate to a system of charged particles interacting solely through instantaneous Coulomb force; it has the simple form of "kinetic energy minus potential energy". L_{rad} is the Lagrangian for a external radiation field far removed from the charges and currents, and has the form of electric field energy imuns magnetic field energy. The interaction Lagrangian L_{int} couples the particle variables to the field variables. It can be easily verified that variation with respect to the particle coordinates gives the second law of Newton with the Lorentz force:

$$m_\alpha \ddot{q}_\alpha = e_\alpha \mathbf{E}(q_\alpha, t) + \frac{e_\alpha}{c}\dot{\mathbf{q}}_\alpha \times \mathbf{B}(q_\alpha, t). \tag{2}$$

Variation of the Lagragian (2.1) with respect to field variables gives the equation of motion for the vector potential

$$\nabla \times \nabla \times \mathbf{A} + \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{c}J \tag{3}$$

Defining $\mathbf{B}$=$\nabla\times\mathbf{A}$ and $\mathbf{E}$=-$\dot{\mathbf{A}}/c$ one obtains

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}. \tag{4}$$

It should be emphasized that in (2) and (4) $\mathbf{E}$ is the transverse part of the total electric field. The Longitudinal electric field in question is entirely electrostatic.

The Hamiltonian corresponding to the Lagrangian (2.1) reads

$$\begin{aligned} H[\prod, A; p, q] &= \sum_\alpha p_\alpha.\dot{q}_\alpha + \int \prod .\dot{A}dr - L \\ &= \sum_\alpha \frac{1}{2m_\alpha}[p_\alpha - \frac{e_\alpha}{c}\mathbf{A}(q,t)]^2 + \frac{1}{2}\sum_{\alpha\neq\beta}\frac{e_\alpha e_\beta}{\mid q_\alpha - q_\beta \mid} \\ &+ \frac{1}{8\pi}\int[(4\pi c\prod)^2 + (\nabla \times \mathbf{A})^2]dr, \end{aligned} \tag{5}$$

where the corresponding conjugate momenta are

$$P_\alpha = m\dot{q}_\alpha + (e_\alpha/c)\mathbf{A}(q,t), \prod(r) = (4\pi c^2)^{-1}\dot{\mathbf{A}}. \tag{6}$$

It is well known that in classical dynamics the addition of a total time derivative to a Lagrangian leads to a new Lagrangian with the equations of motion unaltered.

Lagrangians obtained in this manner are treated to be equivalent. In general, the Hamiltonians following from the equivalent Lagrangians are different. Even the relationship between the conjugate and the kinetic momenta may be changed [11]. Moreover, let us notice that the basic equations of any new theory cannot be introduced strictly deductively. Usually, either they are postulated in differential form based on the partial integral conservation laws or transformations of basic dynamical variables, whose initial definitions usually have some analog in mechanics. Let us remark that we need to do so not only by inertia of thinking but also because of the fact most of our measurements have their objects individual particles or use them in testing. The situation is the same in electromagnetism and in gravitation. In general, geometrical interpretation of dynamical variables plays the crucial role. An equivalent Lagrangian to that of (2.1) is [5].

$$L^{equiv} = L - \frac{1}{c}\frac{d}{dt}\int [\mathbf{P}(r) + \nabla\times T^{e}(r).]\mathbf{A}(r)dV, \tag{7}$$

where the toroid contribution has been taken into account. Here, T^e is the axial toroid moment (ATM) and is electrical by nature (toroid dipole polarization vector of electric type). Writing it in the explicit form we get the field conjugate to the vector potential $\mathbf{A}$:

$$4\pi c\prod(\mathbf{r}) : -\mathbf{D}(r) = -(\mathbf{E}(r) + 4\pi(\mathbf{P}(r) + \nabla\mathbf{T^e}(r))).$$

Since only the free field $\mathbf{E}$ is generated due to the change of magnetic field $\mathbf{B}$ one writes

$$\nabla\times\mathbf{D}(r) = -\frac{1}{c}\dot{\mathbf{B}}(r) + 4\pi\nabla\times\mathbf{P}(r) + \nabla\times\nabla\times\mathbf{T^e}(r)), \tag{8}$$

under $\nabla\mathbf{E}$(r)=-$\dot{\mathbf{B}}(r)/c$.

The new Lagrangian is a function of the variables $q_\alpha, \dot{q}_\alpha$ and a functional of the field variables $\mathbf{A}$, $\dot{\mathbf{A}}$, and the equations of motion follow from the variational principle. Applying the Euler-Lagrange equations of motion one gets [5]

$$\nabla\times\mathbf{B}(r) = \frac{1}{c}D(r) + \frac{4\pi}{c}J_{free} + 4\pi\nabla\times(M(r) + \nabla\times\nabla\times T^m(r)). \tag{9}$$

Here, the currents were divided into free and bound state (due to electric polarization and magnetization) as [12]

$$\mathbf{J}(r) = j_{free} + c\mathbf{M}(r) + \dot{\mathbf{P}}(r), \tag{10}$$

with the additional condition imposed on T^e being.

$$\nabla\times\mathbf{T^{m,e}} = \pm\frac{1}{c}\dot{\mathbf{T}}^{e,\mu}, \tag{11}$$

where $\mathbf{T}^m$ is the toroid dipole polarization vector of magnetic type. Relation (2.10) demands some comments. Both $\mathbf{T}^e$ and $\mathbf{T}^m$ represent the closed isolated lines of electric and magnetic fields. So they have to obey the usual differential relations similar to the free Maxwell equations [13,14]). However, we remark that the signs here are opposite to the corresponding signs in Maxwell equations because the direction of the electric dipole is accepted to be chosen opposite to its inner electric field [15].

If we define the auxiliary field **H** to be

$$\mathbf{H}(r) = \mathbf{B}(r) - 4\pi(M(r) + \nabla \times \mathbf{T^m}(r)) \tag{12}$$

then it deduces to

$$\nabla \times \mathbf{H} = \frac{1}{c}\dot{\mathbf{D}} + \frac{4\pi}{c}\mathbf{j}_{free}.$$

But the latter formula is unsatisfactory from the physical point of view. It is easy to image the situation when **B** and M are absent, because the medium may be composed from isolated aligned dipoles $\mathbf{T}^m$ [16-18] and each $\mathbf{T}^m$ is the source of free-field (transverse-longitudinal) potential but not **B** [19]. So the transition to the description by means of potentials is inevitable.

The Hamiltonian, corresponding to the equivalent Lagrangian, in this case reads

$$\begin{aligned} H^{\equiv}[\prod, \mathbf{A}; p, q] = & \sum_{\alpha} \frac{1}{2m_\alpha [p_\alpha} - \frac{e_\alpha]}{c}\mathbf{A}[(\mathbf{q},t)]^2 + \frac{1}{2}\sum_{\alpha\neq\beta}\frac{e_\alpha e_\beta}{\mid q_\alpha - q_\beta} \\ & + \frac{1}{8\pi}\int [4\pi(\mathbf{P} + \nabla \times \mathbf{T}^e) - \mathbf{D}]^2 + (\nabla \times \mathbf{A})^2 dr \\ & + \frac{1}{c}\int \mathbf{J}.\mathbf{A}dr - \int \mathbf{M}.\mathbf{B}dr - \int \mathbf{B}.\nabla \times \mathbf{T}^m dr. \end{aligned} \tag{13}$$

B. Gauge-like Transformation

The Maxwell equations for electromagnetic fields in media can be written as

$$\nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c} j_{free} \tag{14a}$$

$$\nabla.\mathbf{D} = 4\pi\rho \tag{14b}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{14c}$$

$$\nabla.\mathbf{B} = 0 \tag{14d}$$

where

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}, \tag{14e}$$

$$\mathbf{H} = \mathbf{B} - 4\pi M \tag{14f}$$

In the previous subsection we introduced toroid moments into Maxwell equations through Lagrangian formalism. In doing so we first constructed an equivalent Lagrangian. Here we do the same using in an alternative way, which instead employs a gauge transformation. To this end we introduce two vectors $\mathbf{T}^m$ and $\mathbf{T}^e$ (toroid dipole polarization vector of magnetic type and toroid dipole polarization of electric type, respectively) such that

$$\mathbf{P} \Rightarrow \mathbf{P} + \nabla \times \mathbf{T}^e, \tag{15a}$$
$$\mathbf{M} \Rightarrow \mathbf{M} + \nabla \times \mathbf{T}^m. \tag{15b}$$

It can be easily shown that system (2.13) is invariant under transformation (2.15) if we impose the additional condition of (2.10), i.e.,

$$\nabla \times \mathbf{T}^{e,m} = \pm\frac{1}{c}\frac{\partial \mathbf{T}^{m,e}}{\partial t}. \tag{16}$$

In account of (2.15) and (2.16) we rewrite system (2.13) as

$$\nabla \times \mathbf{B} = \frac{1}{c}\partial \mathbf{D}\partial t + \frac{4\pi}{c} j_{free} + 4\pi\{\nabla \times M + \nabla \times \nabla \times \mathbf{T}^m\} \tag{17a}$$
$$\nabla.\mathbf{D} = 4\pi\rho \tag{17b}$$
$$\nabla \times \mathbf{D} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + 4\pi\{\nabla \times \mathbf{P} + \nabla \times \nabla \times \mathbf{T}^e\} \tag{17c}$$
$$\nabla.\mathbf{B} = 0. \tag{17d}$$

As is seen equations (2.17a) and (2.17c) of the system (2.17) completely coincides with (2.8) and (2.7) of the previous subsection. Thus we introduced toroid moments in Maxwell equations using two different formalisms.

3. Two Potential Formalism

It is commonly believed that the divergence equations of the Maxwell system are" redundant". Recently Krivsky a.o. [20] claimed that to describe the free electromagnetic field it is sufficient to consider the curl-subsystem of Maxwell equations since the equalities $\nabla{\cdot}\mathbf{E}$=0 and $\nabla{\cdot}\mathbf{B}$=0 are fulfilled identically. Contrary to this statement, Jiang and Co [21] proved that the divergence equations are not redundant and that neglecting these equations is at the origin of spurious solutions in computational electromagnetics. Here we construct generalized formulation of Maxwell equations including both curl and divergence subsystems. In this section we develop two potential formalism (a similar formalism was developed by us earlier with the curl-subsytem taken into account only). Note that in the ordinary one potential formalism ($\mathbf{A}$, φ) of the second set of Maxwell equations are fulfilled identically. So that all four Maxwell equations bring their contribution individually, in our view, one has to rewrite the Maxwell equation in terms of two vector and scalar potentials.

Because of introduction of toroid moments (see Sec. 2) now **B** and **D** have lost their initial meaning, hence should be reinterpreted. It means the deduction of the equation of evolution by inserting $\mathbf{B}=\nabla\cdot\mathbf{A}$ and $\mathbf{E}=-\dot{\mathbf{A}}/\mathrm{c}$ is valid no longer and we have to introduce some new potential that could explain the new **B** and **D**. To this end we introduce a so-called double potential [22,23,5]. As was mentioned, due to the introduction of toroid moments the vectors **B** and **D** should be redefined. We denote these new quantities as β and δ, respectively. In account of it, system (2.17) should be rewritten as

$$\nabla \times \beta = \frac{1}{c}\frac{\partial \delta}{\partial t} + \frac{4\pi}{c} j_{free} \tag{18a}$$
$$\nabla . \delta = 4\pi\rho \tag{18b}$$
$$\nabla \times \delta = -\frac{1}{c}\frac{\partial \beta}{\partial t} \tag{18c}$$
$$\nabla . \beta = \tag{18d}$$

Before developing the two potential formalism we first rewrite system (2.13) in terms of vector and scalar potentials **A**, ϕ such that $\mathbf{B}=\nabla\times\mathbf{A}$ and $\mathbf{E}=-\nabla\varphi-(1/c)(\partial\mathbf{A}/\partial t)$. Following any text book we can write system (2.13) as

$$\Box\mathbf{A} = -\frac{4\pi}{c}\mathbf{j}_{tot} = -\frac{4\pi}{c}[j_{free} + \frac{\partial \mathbf{P}}{\partial t} + c\nabla \times M] \tag{19a}$$
$$\Delta\phi = -4\pi[\rho - \nabla.\mathbf{P}] \tag{19b}$$

under Lorentz gauge, i.e., $\nabla\mathbf{A}+(1/\mathrm{c})(\partial\phi/\partial t)=0$ and

$$\Box\mathbf{A} = -\frac{4\pi}{c}[j_{tot} = -\frac{4\pi}{c}\nabla\frac{\partial \phi}{\partial t}] \tag{20a}$$
$$\nabla^2\phi = -4\pi[\rho - \nabla.\mathbf{P}] \tag{20b}$$

under Coulomb gauge, i.e., $\nabla.\mathbf{A}=0$. Here, $\Box = \nabla^2 - (1/c^2)(\partial^2/\partial t^2)$. Note that to obtain (3.2) or (3.3) it is sufficient to consider (2.13a) and (2.13b) only since the two others are fulfilled identically.

Let us now develop a two potential formalism. Two potential formalism was first introduced in [22] and further developed in [23,5]. In both papers we introduce only two vector potentials α^m, α^e and use only the curl-subsystem of the Maxwell equations with the additional condition $\mathrm{div}\alpha^{m,e}=0$. Thus, in our view our previous version of two potential formalism lack of completeness. In the present paper together with the vector potentials φ^m and φ^e such that

$$\bar{\beta} = \nabla \times \alpha^m + \frac{1}{c}\frac{\partial \alpha^e}{\partial t} - \nabla\varphi^m. \tag{21a}$$
$$\bar{\delta} = \nabla \times \alpha^e - \frac{1}{c}\frac{\partial \alpha^m}{\partial t} - \nabla\varphi^e. \tag{21b}$$

It can be easily verified that system of equations (3.1) are invariant under this transformation and take the form

$$\Box\alpha^{\mathbf{m}} = -\frac{4\pi}{c}[\mathbf{j} + c\nabla\nabla\mathbf{T}^m], \tag{22a}$$
$$\nabla^2\varphi^m = 0 \tag{22b}$$
$$\Box\alpha^{\mathbf{e}} = -\frac{4\pi}{c}[\nabla\mathbf{P} + \nabla\nabla\mathbf{T}^e], \tag{22c}$$
$$\nabla^2\varphi^e = -4\pi\rho \tag{22d}$$

under $\nabla\alpha^{m,e} + (1/c)(\partial\varphi^{e,m}/\partial t)$=0 and

$$\Box\alpha^{\mathbf{m}} = -\frac{4\pi}{c}[\mathbf{j} + c\nabla\times M + \nabla\times\nabla\times\mathbf{T}^m - \frac{1}{4\pi}\nabla\frac{\partial\varphi^e}{\partial t}], \tag{23a}$$
$$\nabla^2\varphi^m = 0 \tag{23b}$$
$$\Box\alpha^{\mathbf{e}} = -\frac{4\pi}{c}[\nabla\times\mathbf{P} + \nabla\times\nabla\times\mathbf{T}^e - \frac{1}{4\pi}\frac{\partial\varphi^m}{\partial t}], \tag{23c}$$
$$\nabla^2\varphi^e = -4\pi\rho \tag{23d}$$

under $\nabla\alpha^{m,e}$=0. The solutions to systems (3.5) and (3.6) can be written as follows (see for example [5,25]): The solutions to the d'Alembert equation

$$\Box F(r,t) = f(r,t) \tag{24}$$

look

$$F(\mathbf{r},t) = -\frac{1}{4\pi}\int_{\text{all space}}\frac{f(r',t')dr'}{r-r'}\,|_{t'=t-|r-r'|/c}, \tag{25}$$

whereas the solutions to the Poisson equation

$$\nabla^2F(r) = f(r) \tag{26}$$

read

$$F(r) = -\frac{1}{4\pi}\int\frac{f(r')dr'}{|r-r'|}. \tag{27}$$

It is necessary to emphasize that the potential descriptions electrotoroidic and magnetotoroidic media are completely separated. The properties of the magnetic and electric potentials α^m and α^e under the temporal and spatial inversions are opposite [13]. The potential $\alpha^e(\alpha^m)$ is related to the toroidness of the medium $\mathbf{T}^e$ ($\mathbf{T}^m$) as $\mathbf{B}$ ($\mathbf{D}$) to M ($\mathbf{P}$).

Note that if $\nabla\delta \neq$0 and there does exist free current in the medium we have to use the direct method for finding all constrains in the theory suggested by Dirac. Dirac applied his method to electrodynamics and found that electromagnetic potentials have only two degrees of freedom described by transverse components of vector potential. This method was developed by Dobovik and Shabanov [24], where classical and quantum dynamics of a system of non-relativistic charged particles were considered.

4. Conclusion

The modified equations of electrodynamics has been obtained in account of toroid moment contributions. The two-potential formalism has been further developed for the equations obtained. Note that introduction of free magnetic current j^m_{free} and magnetic charge p^m in the equations (3.1c) and (3.1d) respectively leads to the equations obtained by Singleton [3].

References

[1] P. A. M. Dirac, Proceedings of Royal Society, A **133**, 60-72 (1931)

[2] P. A. M. Dirac, Physical Review **74**, 817,830 (1948)

[3] D. Singleton, American Journal of Physics **64**(4), 452-458 (1996).

[4] A. E. Chubykalo and R. Smirnov-Rueda, Modern Physics Letters A, **12**(1) 1-24 (1997)

[5] V. M. Dubovik, B. Saha and M. A. Martsenyuk, ICTP Internal Reports IC/IR/96/9/, 23 p. (1996)

[6] Ya. B. Zel'dovich, Zhurnal Eksperimentalnoi i Thereticheskoi Fiziki, **6** 1184 (1958)

[7] V. M. Dubovik and A. A. Cheskhov, Fizika Elementarnikh Chastits i Atomnogo Yadra, **5** 318 (1974)

[8] A. A. Spartakov, and N. A. Tolstoi, Pis'mav Zhurnal Eksperimentalnoi i Thereticheskoi Fiziki, **52**(3) 796 (1990)

[9] M. A. Martsenyuk, and N. M. Martsenyuk, Pis'ma v Zhurnal Eksperimentalnoi i Thereticheskoi Fiziki, **53**(5) 229 (1991)

[10] W. P. Healy, Non-Relativistic Quantum Electrodynamics, (Academic Press, 1982)

[11] E. A. Power, and T. Thirunamachandran, American Journal Physics, **46**(4), 370 (1978)

[12] E. M. Purcell, Electricity and Magnetism, (McGrow-Hill, 1969)

[13] V. M. Dubovik, and A. M. Kurbatov, in Barut, A. O., Feranchuk, I. D., Shnir, Ya. M. and Tomil'chik, L. M. (eds), Quantum Systems: New Trends and Methods, (World Scientific, Singapore), 117 (1994)

[14] M. A. Miller, Uspekhi Fizicheskikh Nauk, **142**(1) 147 (1984)

[15] V. L. Ginzburg, Works of Lebedev Physics Institute, **176**, 3 (1986).

[16] I. S. Zheludev, Izvestia Academi Nauk USSR, Physics, **33** 204 (1969)

[17] V. M. Dubovik, M. A. Martsenyuk, and N. M. Martsenyuk, Phys. Part. Nucl., **24**(4) 453 (1993).

[18] V. M. Dubovik, I. V. Lunegov, and M. A. Martsenyuk, Fizika Elementarnikh Chastits i Atomnogo Yadra, **26** 72 (1995)

[19] V. M. Dubovik, and V. V. Tugushev, Physics Reports, **187**(4), 145 (1990)

[20] I. Yu. Krivsky, V. M. Simulik, and Z. Z. Torich, Preprint: National Academy of Science Ukraine, KINR - 96 - 3, 20p.

[21] Bo-nan Jiang, Jie Wu, and L. A. Povinelli, J. Comp. Phys., **125**, 104 (1996)

[22] V. M. Dubovik and E. N. Magar, J. Mosc. Phys. Soc., **3**, 1 (1994).

[23] V. M. Dubovik, B. Saha and M. A. Martsenyuk, S. Jefferst et al. (eds.) The Present Status of the Quantum Theory of Light, (Kluwer Acad. Publs.), 141-150. (1997)

[24] V. M. Dubovik, and S. V. Shabanov, in Lakhtakia, A. (ed.), Essays on the Formal Aspects of Electromagnetic Theory, (World Scientific, Singapore), 399 (1993).

[25] O. D. Jefimenko, Electricity and Magnetism, (Electret Scientific Company, 1989)