$$\phi_i(s_{-i}) \equiv \arg\max_{s_i \in S_i} p_i(s_i, s_{-i})$$
$$\equiv \{s_i \in S_i : p_i(s_i', s_{-i}) \geq p_i(s_i, s_{-i}) \forall \in S_i\}$$

For a game with two players, best responses of both players to other can be defined as;

$s_1^* = \phi(s_2^*)$: s_1^* is the strategy profile of player 1

$s_2^* = \phi(s_1^*)$: s_2^* is the strategy profile of player 2

here it should be,

$$\frac{\partial p_i(s_1,\ldots,s_n)}{\partial s_i} = 0,\ \text{i=1,…,n and } \frac{\partial p_i}{\partial s_i^2} \leq 0.$$

The equilibrium obtained by solving the *n* equations is Nash equilibrium. The fact that the s^* strategy profile is the Pure Strategy Nash equilibrium is because the strategy of a player is the best response to the strategies of other players. $s^* = (s_1^*, s_2^*)$ only if $(s^* \in \varnothing(s^*))$.As a result, the pure strategy Nash equilibrium is as described in the following equation;

$$p_i(s_i^*, s_{-i}^*) \geq p_i(s_i, s_{-i}^*),\ \forall_i \in N,\ \forall s_i \in S$$

2.2.3. Iterated elimination of strictly dominated strategies

Strictly dominated strategy is a strategy that a rational player will not play [11, 12]. Since each player knows that the other player will not play the strictly dominant strategy, these strategies are deleted from the game. This process is named as "Iterated Elimination of Strictly Dominated Strategies" and defined with an algorithm that has an iteratively shrinking strategy set S_i^k (k=0,1,2...) for each player $i \in N$. In every step of the deletion of strictly dominated strategies, a new game is obtained, and this process ends in the fourth step:

Step 1

$S_i^0 = S_i$ for k=0

Step 2

$$S_i^1 = \left\{ s_i \in S_i^0 \setminus \nexists s_i' \in S_i^0\, p_i(s_i', s_{-i}) > p_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}^0 \right\}$$

Step 3 k+1

$$S_i^{k+1} \left\{ s_i \in S_i^k \setminus \nexists s_i' \in S_i^k\, p_i(s_i', s_{-i}) > p_i(s_i, s_{-i}) \forall s_{-i} \in S_{-i}^k \right\}$$

Step 4

$$S_i^\infty = \bigcap_{k=1}^{\infty} S_i^k$$

2.2.4. Mixed Strategy Nash Equilibrium

In some games, there is no pure strategy Nash equilibrium. To find the Nash equilibrium, a mixed strategy must be used, and this game is played by randomizing the strategies.

When $\sum_i$ is mixed strategy space of any player *i*, S_i is a set of probability distributions over $\sum_i$. Thus, if $\sigma_i \in \sum_i$, then $\sigma_i(s_i)$ is the probability that player *i* chosen action s_i.

Probability distribution is $s_i : S_i \rightarrow [0,1]$ over finite not null set of S_i. So,

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1.$$

Let σ is any mixed strategy profile, σ_{-i} is mixed strategy of players other than i, σ_i' is possible strategy of player i, σ_{-i}' is possible strategy of players other than i, σ_i^* is mixed strategy profile of player i, σ_{-i}^* is mixed strategy profile of players other than i, σ^* is mixed strategy profile and $\phi_i(\sigma_{-i})$ is the best mixed strategy set of player i; to explain mixed strategy space, the payoff on mixed strategy can be defined as follows:

$$p_i(\sigma_i, \sigma_{-i}) = \sum_{s_i \in S_i} p_i(s_i, \sigma_{-i}) \sigma_i(s_i)$$
$$= \sum_{s \in S} p_i(s_i, s_{-i}) \left(\prod_{j=1}^{n} \sigma_j s_j \right)$$

This payoff function can be interpreted that the expected payoff should be von-Neumann Morgenstern (VNM) for chosen strategies of players.

$$\phi_i(\sigma_{-i}) \equiv \arg\max_{\sigma_i \in \sum_i} p_i(\sigma_i, \sigma_{-i})$$
$$\equiv \left\{ \sigma_i \in \sum_i : p_i(\sigma_i', \sigma_{-i}) \geq p_i(\sigma_i, \sigma_{-i}) \forall \sigma_i \in \sum_i \right\}$$

For example, for a two-player game, the Nash equilibrium of a σ^* mixed strategy is due to the fact that a player's mixed strategy is the best response to the mixed strategies of other players.

$$\sigma^* = (\sigma_1^*, \sigma_2^*)$$

Mixed strategy Nash equilibrium can be defined as (https://warwick.ac.uk/fac/soc/economics/staff/dsgroi/ec202/w06_dominant_strategies_and_iesds.pdf; http://www.sam.sdu.dk/~psu/teaching/phd/draft.pdf);

$$p_i(\sigma_i^*, \sigma_{-i}^*) \geq p_i(\sigma_i, \sigma_{-i}^*), \quad \forall_i \in N, \quad \forall \sigma_i \in \sum_i$$

Here, we suppose that the row players are males, and the column players are females. The payoff matrix is constructed by adding the breeding values of individual male and female animals. If we want to optimize the milk yield and fat percentage of *i*th male and *j*th female animals, for instance, then *aij* (*bij*) is the addition of *i*th male and *j*th female animals milk (fat) characteristic values.

Then, one finds the Nash equilibrium of the game to match each male with a desired number of female animals. When *j*th female is matched with a male the *j*th row of the bimatrix is deleted. When ith male is matched with a desired number of females, the *i*th column is deleted from the bimatrix. This process is continued until all animals are matched.

To compare the expected benefits (EB) and coefficient of variation (CV) between methods Mann–Whitney U test was used [13]. The variability of CV was calculated among sires.

3. Results

The selection index method applied to more than one character in classical breeding is calculated as a linear combination of individual breeding values and maximization is aimed at selection [14, 15]. The benefits obtained in the negative genetic correlation scenarios were shown in Table 2 for the selection index and in Table 3 for the game theory methods. The benefits obtained in the positive genetic correlation scenarios were shown in Table 4 for the selection index and in Table 5 for the game theory methods.

When the mating design generated by the selection index and game theory is examined in terms of the traits with negative genetic correlations between them, it is seen that the animals selected for breeding are the same animals in both methods. In other words, the mating method didn't affect the selection of the animals for breeding. But different mating couples of animals were observed for methods. Results showed that the total

Table 2. The expected benefits of the mating program generated by the selection index in terms of traits have negative genetic correlations for Jersey cattle.

Bull / Cow	101	106	110	111	122
1	–104.2	137.4	–0.8	179.3	254.9
2	–106.2	135.3	–1.1	175.5	249.5
3	–121.6	120.8	–1.9	169.5	247.8
4	–128.8	119.2	–3.9	168.8	230.1
5	–132.5	116.7	–9.8	167.3	218.8
6	–133.1	113.5	–11.4	164.56	215.7
7	–137.4	111.4	–24.9	162.1	200.1
∑EBi	–863.8	854.3	–53.9	1187.2	1616.9
CV	10.8	8.4	113.2	3.6	8.9
∑EB	2740.6				
CV	166.3				

The numbers at the first row and column shows the animal ID.

Table 3. The expected benefits of the mating program generated by the game theory in terms of traits have negative genetic correlations for Jersey cattle.

Bull / Cow	101	106	110	111	122
1	–19.3	106.6	114.1	155.5	109.8
2	–23.1	106.2	108.7	153.4	107.8
3	–29.1	105.5	106.9	138.9	92.4
4	–29.9	103.4	89.2	137.3	85.2
5	–31.4	97.6	77.9	134.8	81.6
6	–34.1	95.9	74.8	131.7	80.9
7	–36.5	82.4	59.3	129.5	76.6
∑EBi	–203.4	697.7	630.9	981.1	634.3
CV	20.7	8.8	22.8	7.3	14.7
∑EB	2740.6				
CV	75.1				

The numbers at the first row and column shows the animal ID.

Table 4. The expected benefits of the mating program generated by the selection index in terms of traits have positive genetic correlations for Saanen goat.

Buck / Goat	101	102	110	115	119
1	–0.7	0.2	–0.5	1.6	2.7
2	–0.7	0.1	–0.5	1.5	2.6
3	–0.7	0.1	–0.5	1.5	2.5
4	–0.8	0.0	–0.6	1.5	2.5
5	–0.9	0.0	–0.6	1.5	2.3
6	–0.9	0.0	–0.7	1.5	2.2
7	–0.9	–0.2	–0.7	1.5	2.2
∑EBi	–5.7	0.3	–4.0	10.4	16.9
CV	12.0	287.6	14.2	2.3	8.1
∑EB	17.9				
CV	247.2				

The numbers at the first row and column shows the animal ID.

Table 5. The expected benefits of the mating program generated by the game theory in terms of traits have positive genetic correlations for Saanen goat.

Buck / Goat	101	102	110	115	119
1	–0.0	–1.1	–1.5	2.0	2.7
2	–0.7	0.3	–1.4	1.7	2.5
3	–0.3	0.3	–0.2	1.3	2.2
4	0.0	–0.5	–0.5	0.9	2.6
5	–0.7	–1.0	–1.0	1.0	2.0
6	–0.5	0.3	–1.1	0.8	2.3
7	–0.3	0.2	–0.9	1.5	1.7
∑EBi	–2.5	–1.6	–6.5	9.2	15.9
CV	81.1	271.1	49.3	32.5	15.3
∑EB	14.4				
CV	310.0				

The numbers at the first row and column shows the animal ID.

Table 6. Comparison of selection index and game theoretic approaches.

	Negative genetic correlation		Positive genetic correlation	
	EB	CV	EB	CV
Selection index	548.1 ± 447.24	28.9 ± 21.08	3.6 ± 4.35	64.8 ± 55.71
Game theory	548.1 ± 198.60	14.9 ± 3.09	2.9 ± 4.17	89.9 ± 46.59
P for Mann Whitney U	0.60	0.60	0.75	0.12

expected benefits were equal (2740.63) for both methods. This may be caused from the fact that selected animals for breeding were same for both methods. When variation coefficient (CV) was examined, it can be seen that the value obtained from the mating design realized by the game theoretic approach was much lower than the value obtained from the mating design according to the selection index. This result can be regarded an indicator of a more homogeneous expected benefit that can be obtained at the new generation from the mating program realized by the game theoretic approach than selection index. Comparison of the selection index and game theoretic approaches were given in Table 6.

4. Discussion

Although the index method is still popular for its various advantages nowadays, it is difficult to calculate the values used in the calculation of the index equation, which contains high sampling error, the contribution of each genotype has different effect on population genotypes and the maximization of the individuals obstructed the calculation of economic contribution to the population. It also brings disadvantages that one of the most prominent problems of the selection index method is that the traits or yields that enter the index while individuals selected can change out of control from positive to negative [9, 15] This problem has been overcome since the expected benefit of the population is optimized in the developed game theoretic approach.

In the comparison by expected benefit and coefficient of variation (CV) for negative genetic correlated data, for the bull with ear number of 110, the expected benefit was higher (–53.94<630) in the game theoretic approach than the index method, and CV was lower (166.28>75.11) in the game theoretic approach than the index method. In the index method, the expected benefit of the bull 122 was decreased nearly 61% and CV was increased nearly two times when comparing with the game theory. This result shows that the game theory method is more likely to provide a number of advantages such as a more homogenous mating design and, thus, the increase of the desired genotypes in the population in the sense of animal breeding and the simplicity of maintenance and feeding conditions and the ease of herd management in terms of raising animals.

For positive genetic correlated data, selected animals for breeding were not same for two methods. Only 82.86% of animals were same for mating selection. It was found that the expected benefit obtained by the index method was a bit higher (17.913>14.438) than the game theoretic approach. When the CV was examined, it was found that the index method was 25% homogeneous than the game theoretic approach. For both sets of data that contain both negative and positive genetic correlations, the common feature of the two methods is that the best optimizing bull / buck is over after the other bull / buck. This leads to the conclusion that the data may be related to cardinal values (numerical quantities).

According to the results obtained, it is understood that the game theoretic approach produces homogenous next generation expectancy especially when it is aimed to perform selection and mating design in terms of features having negative genetic correlation between them. The homogeneity of the expected utility of the next generation in animal breeding is gaining importance, which is why the variance of the response given to the environmental conditions that will arise will be reduced, and, therefore, the environmental conditions can be controlled more easily [16]. The homogeneity of the trait to be breed also increases the success of the statistical methods used in animal breeding [17]. The optimization of both sexes is more important in animal breeding, especially for fattening characters, even if used methods are based on the selection of male individuals and their maximization without any expected benefit loss.

While there is individual benefit in the selection index, population utility is the forefront in the game theoretic approach. Use of the game theory may be more beneficial on the populations that desired breeding aims have been nearly reached. It is desired to increase the homogeneity in the obtained progeny population with using the game theoretic approach. In this way, it will be possible to manage the environmental conditions much more easily in practice, and the operating costs can be reduced. In this respect, it is important that the offspring population can be obtained homogeneously [6]. Taking all the analysis results into consideration, it is quite clear that there is a need for further study on the game theory which was a new approach to animal mating designs in order to validate the efficiency of the game theoretic approach on experimental breeding studies to show the methods superiority for optimization and maximization.

Acknowledgments
This study is a short summary of PhD thesis of the FIRST AUTHOR under co-supervision of the SECOND AUTHOR and the THIRD AUTHOR, all contribution are equal for this study.

Funding
We didn't use any grant for this study.

Conflict of interests
Authors declare no competing interests.

Informed consent
There is no animal or human study requires ethics permission.

References

1. Camerer CF. Behavioural studies of strategic thinking in games. Trend in Cognitive Sciences 2003; 7 (5): 225-231. doi:10.1016/S1364-6613(03)00094-9.
2. Mitchell K, Ryan S. Game theory models of animal behavior. Lexington, USA: COMAP; 2003.
3. Dall SRX, Mcnamara JM, Wedell R, Hosken DJ. Debating sexual selection and mating strategies. Science 2006; 312: 689-697.
4. Turocy TL, Stengel VS. Game theory. Encyclopaedia of Information Systems. London, UK: Academic Press; 2002.
5. Schuster P, Sigmund K. Towards a dynamics of social behaviour: strategic and genetic models for the evolution of animal conflicts. Journal of Social and Biological Structures 1985; 8: 255-277.
6. Garrick D, Bullock D, Enns RM, Imumorin I, Mistzal I et al. Beef sire selection manual. Ames, USA: National Beef Cattle Evaluation Consortium; 2010.
7. Kaygısız A, Vanlı Y. Genetic analysis of the Brown Cattle herd of Van agricultural vocational high school. Tarım Bilimleri Dergisi 1997; 3 (2): 23-32.
8. Chatterjee B. Bimatrix Game MATLAB Central File Exchange. Berlin Heidelberg, Germany: Springer-Verlag; 2008.
9. Düzgüneş O, Eliçin A, Akman N (1996). Animal breeding. Ankara, Turkey: Agricultural Faculty Publications no: 1437; 1996.
10. Quint T, Shubik M, Yan D. Dumb bugs and bright noncooperative players: games, context and behavior. Connecticut, USA: Cowles Foundation Discussion Paper No: 1094, 31; 1995.
11. Gibbons R. Game theory for applied economists. New Jersey, USA: Princeton University Press; 1992.
12. Bergemann D. Game theory and information economics. Berlin Heidelberg, Germany: Springer-Verlag; 2006.
13. Onder H. Nonparametric statistical methods used in biological experiments. Black Sea Journal of Engineering and Science 2018; 1 (1): 1-6.
14. Henderdson CR. Statistical Genetics and Plant Breeding. Washington, USA: National Academy of Science National Research Council; 1961.
15. Cerón-Rojas JJ, Crossa J, Sahagún-Castellanos J. Statistical sampling properties of the coefficients of three phenotypic selection indices. Cropp Science 2015; 56: 51-58.
16. Formoso-Rafferty N, Cervantes I, Ibáñez-Escriche N, Gutiérrez JP. Correlated genetic trends for production and welfare traits in a mouse population divergently selected for birth weight environmental variability. Animal 2016; 10 (11): 1770-1777.
17. Langhammer M, Michaelis M, Hoeflich A, Sobczak A, Schoen J et al. High-fertility phenotypes: two outbred mouse models exhibit substantially different molecular and physiological strategies warranting improved fertility. Reproduction 2014; 147: 427-433.