Statistical Description of Lactation Curve of Jersey Bred in Karaköy State Farm

Mehmet N. ORMAN

Ankara University, Faculty of Veterinary Medicine, Departmen of Biostatistics, Ankara-TURKEY

Fetih YILDIRIM

Middle East Technical University, Department of Statistics, Ankara-TURKEY

Abstract: In this study, the parameters of the regression models for the lactation curve were estimated. A total of 45 standard milk yield records obtained from the first, the second and the third lactations of Jersey bred in Karaköy State Farm were used in the present study. Milk production records were corrected for the ages of cows ond the numbers of lactation.

The parameters were estimated in two stages. In the first stage, the data were applied to the five nonlinear models, developed by different authors. The results out of the five models were discussed in detail. The best fitting model is chosen according to the advantages and disadventages of these models.

In the second stage, five different estimation methods were compared. Among the five methods studied, the MRT (Marquardt) method was seen to be more practical and required less computing time than the others.

The best fit model was used with MRT method for the estimation of the parameters. The parameters, which related to the milk yield records, were estimated and the lactation curves were obtained for each of the theree lactation groups. The curves of the three lactation groups were discussed in regard to the animal breeding and statistics.

Key Words: Constrained Nonlinear Astimation, Least Squares, Lactation Curve

Karaköy Tarım İşletmesindeki Jersey Süt Sığırlarının Laktasyon Eğrisinin İstatistiksel Açıklaması

Özet : Eldeki çalışmada, ineklerde laktasyon süt verimi eğrisini en iyi tanımlayacak istatistiksel model ve parametreler araştırılmıştır. Bu amaçla, Karaköy Tarım İşletmesinde yetiştirilen Jersey sürüsüne ait 3 laktasyon grubunda, toplam 45 adet 305 günlük süt verim kayıtları kullanılmıştır.

Parametrelerin tahmini iki aşamada gerçekleştirilmiştir. Birinci aşamada, değişik araştırıcılar tarafından geliştirilen beş farklı doğrusal olmayan model denenmiştir. Madellerin verilere uygulanmasından elde edilen sonuçlardan, modellerin avantaj ve dezavantajları incelenmiş ve bunlardan en uygun model tesbit edilmiştir.

İkinci aşamada ise, uygunluğu saptanan model ele alınarak, beş farklı parametre tahmin metodu karşılaştırımıştır. Bunlar arasında MRT (Marquardt) metodunun daha pratik olduğu ve hesaplamalarda daha kısa zamana ihtiyaç duyulduğu saptanmıştır.

Özetle, en uygun model, MRT metodu ile birlikte kullanılmıştır. Üç laktasyon grubunun herbirisi için laktasyon süt verimi ele ilgili parametreler tahmin edilerek, laktasyon eğrileri çizilmiştir. Daha sonra , sonuçlar istatistik ve hayvan yetiştiriciliği açılarından tartışılmıştır.

Anahtar Sözcükler: Kısıltı Doğrusal Olmayan Tahmin, En Küçük Kareler, Laktasyon Eğrisi

Introduction

The effects of methods selection are important in increasing the production levels of animals farm. The reliabilities of the methods selection depend on mostly the accuracy of the production records of the animals. Since the standard lactation period of cows is 10 months in this long period causes some problems to obtain dependable records of milk production (1).

The accuracy of the production records is instrumental for the present day profit of the dairy enterprise and also for the increased selection response in the following generations. On the other hand, the accuracy of the data, do not competely guaranty the increased production in the next generation. Some other factors may also be involved in the determination.

The lactation curve, would help the researcher, to estimate a missing observation at any stage of the lactation periods and also to estimate the lactation milk yield at different stage of the lactation periods. Some statistical methods have been reported to be used for drawing the lactation curves (2, 3, 4, 5, 6, 7). If regression analysis is considered, linear or nonlinear, two approaches may be used in the estimation (4). Since the estimation procedures with the linear model required less time and the application of this model is easier than the monlinear one, the linear one is usually the choice (4). However, the uses of computers make the nonlinear model easier today to apply the milk production data.

The purpose of this study, is to find the best mathematical model for explaining the lactation curve by nonlinear approach.

Materials and Methods

Materials

The data sets of this study were obtained from the dairy herd at Karaköy State Farm. A of 45 lactation records were used in the study. Each data set is composed of the control days and the total amount of milk at the morning and evening milking of the control days.

Methods

I) Sampling

A total of 300 cows' data set were grouped, according their numbers of lactation. The cows with the first lactation were placed in the first group, with the second lactation in the second group an with the third or more lactation in the third group. Each group containes about 100 data points. After that, 15% of data were selected randomly (8).

II) Mathematical Models

Five models were used in this study. The first three model, given by Morant and Gnanasakty (7):

$$y_1 = \theta_0 - \theta_1 t + \theta_2 EXP(-\theta_3 t)/\theta_3 + e \qquad 2.1$$

$$y_2 = \theta_0 - \theta_1 t + \theta_2 t^2 + \theta_3 EXP(-\theta_4 t)/\theta_4 + e \qquad 2.2$$

$$y_3 = \theta_0 - \theta_1 t + \theta_2 t^2 + (\theta_3(\theta_4 t)) + e \qquad 2.3$$

the fourth model, derived by Bates and Watts(9):

$$y_4 = (\theta_0 + \theta_1)EXP(-\theta_3 t) + \theta_0 EXP(-\theta_4 t) + \theta_1 EXP(-\theta_4 t) + e \qquad 2.4$$

and the last model, derived by Schaeffer et al.(3):

$$y_5 = \theta_0 - \theta_1 t + \ln((1 - EXP(-\theta_2 t))/\theta_2) + e \qquad 2.5$$

where;

Y_t: İs the logarithm of the milk yield at the control day,

t: is the control day,

θ_i: Are the parameters I=0, 1, 2, 3, 4.

In all models each parameter represents a different part (peak, increasing and decreasing) of lactation curve.

Five different estimation methods, namely Conjugate Gradient (CG), Marquardt (MRT), Nelder Mead (NM), Hooke-Jeeves (HJ) and quasi Newton (QN) were used to were used to find the best fit lactation curve and estimation method (9,10,11,12,13,14,15,16)

Results

In the first stage of the study, the conditions of regression analysis were checked (12). The Kolmogorow-Simirnow (K-S) test was used to check, whether the data were normally distirbuted or not (17). Each data set was fitted to two distributions of normal and log-normal.The average significance levels of K-S test values are given in Table 1.

Table 1.   Average significant levels of K-S test values

Groups	Normal Distribution $\overline{X} \pm S\overline{x}$	Log-normal distribution $\overline{x} \pm S\overline{x}$
1	0.84050 ± 715E-2	0.99997 ± 1.25E-4
2	0.99998 ± 1.09E-4	0.99998 ± 6.97E-6
3+	0.81296 ± 7.14E-2	0.99972 ± 1.04E-4

It was found that data sets were log-normally distributed (Table 1.).Therefore, logarithmic transformation is necessary to convert the data to the normal distribution. After transformation, the data sets were again checked and it was decided that the conditions were satisfied.

Stage 1-Comparisons of the models:

One method (QN) and one lactation group (Second) were randomly selected for the determination of the best model (8). The results of the estimations for the cow number 252.84 are given in Table 2. The results may be summarized as follows;

Model no		PARAMETERS					Loss value
		p1	p2	p3	p4	p5	
1	Estimated	2.562	0.002	-1.844	18.572	–	0.218
	S.E of Est	0.049	0.000	0.159	0.106	–	
2	Estimated	2.637	0.004	0.001	-0.147	8.064	0.191
	S.E of Est	0.68	0.001	0.000	0.153	0.082	
3	Estimated	2.756	0.005	0.001	2.505	-3.192	0.157
	S.E of Est	0.080	0.001	0.000	0.588	0.621	
4	Estimated	1.000	1.737	0.816	-0.001	0.004	0.15
	S.E of Est	3.212	3.157	0.197	0.006	0.007	
5	Estimated	2.394	0.002	0.832	–	–	0.212
	S.E of Est	0.324	0.000	0.281	–	–	

Table 2. Sample results of cow number 252.84

Methods	Group1 $\bar{x} \pm S\bar{x}$	Group2 $\bar{x} \pm S\bar{x}$	Group3+ $\bar{x} \pm S\bar{x}$
MRT	0.271846± 34.863E-3	0.2475199± 40.188E-3	0.2910805± 46.578E-3
CG	0.271852± 34.862E-3	0.2475200± 40.188E-3	0.2910805± 46.578E-3
HJ	0.271846± 34.762E-3	0.2474199± 40.188E-3	0.2910863± 46.477E-3
NM	0.271846± 34.863E-3	0.2474976± 40.189E-3	0.2910794± 46.578E-3
QN	0.258386± 36.317E-3	0.2468000± 40.125E-3	0.2870667± 45.847E-3

Table 3. Mean loss function value of each lactation groups.

I-The mean loss function of the first model is 0.29775.This model did not fit the curve, at many initial points for three cases.

II-The mean loss function of the second model is 0.38380. This model gave a negative value for some parameters with the same data sets. Some very large values of standard errors of parameter were also estimated.

III-The third model did not satisfy the conditions. The mean loss function values were rather large.

IV-The fourth model seemed to se more suitable for the data sets. However, one of the sets gave a high loss function value.Even after the high loss function value was eliminated the mean value of the loss function would still be very large.

V-Average loss function value of the fifth model was 0.304.Although this value is slightly larger than the result obtained with the first model. The standard errors of the esmimators of parameters satisfied better the conditions than the first model. Also the values of the parameters were very close to each other.

It was concluded that the fifth model was the best fit for explaining the lactation curve of the cows.

-Stage 2: Comparisons of estimation methods

Two statistical package programs were used in this study, The first program was SYSTAT. The SYSTAT statistical package had only QN method (10).The second packed program was developed by Nash and Walker-Smith (13).This program contains the other four methods which were used in this study,

In considering the loss function, QN method gave smallest values in three lactation groups. The reason of the smallest value for QN method was due to the negative parameters. The results are presented in Table 3.

The values of gradients, total elapsed time and functions obtained are presented in Table 4.

Estimated values of gradients for HJ and NM methods were found as zero in the calculations, while those methods do not use gradient. As seen in Table 4, the CG and MRT methods resulted very small mean values of the time, gradient and iterations. More func-

Groups	Method	Time(sec) $\overline{x} \pm S\overline{x}$	N.of Gradient $\overline{x} \pm S\overline{x}$	N.of Function $\overline{x} \pm S\overline{x}$
	MRT	3.6 ± 0.5327	1.8 ± 0.2430	5.267 ± 1.089
	CG	2.8 ± 0.4276	1.6 ± 0.325	9.200 ± 0.957
1	HJ	11.3 ± 6.6580	0.0 ± 0.0	59.600 ± 4.083
	NM	31.8 ± 7.1582	0.0 ± 0.0	127.800 ± 29.581
	MRT	2.8 ± 0.3550	1.3 ± 0.1594	4.267 ± 0.945
	CG	3.1 ± 0.3160	1.6 ± 0.2726	12.000 ± 1.746
2	HJ	27.6 ± 13.2430	0.0 ± 0.0	148.933 ± 70.026
	NM	26.7 ± 2.0990	0.0 ± 0.0	102.533 ± 8.590
	MRT	4.1 + 0.7029	2.1 ± 0.4415	6.800 ± 1.066
	CG	4.2 ± 0.5273	2.1 ± 0.5336	14.330 ± 1.438
3	HJ	12.6 ± 1.0456	0.0 ± 0.0	119.270 ± 55.217
	NM	24.2 ± 1.0441	0.0 ± 0.0	93.800 ± 3.937

Table 4. Total number of functions, total elapsed time and gradients for th methods

tion evaluation and longer time needed for HJ and NM methods. When all the groups were compared together with respect to thir elapsed times the MRT method was seen to be the faster among the five.

The data sets, all contain auto-correlated error. Most of the authors have not taken to in the consideration this type of error (3,4,10,12,13). However, Glasbey (18) did consider the auto-correlated errors. He found out that the logarithmic transformation was necessary to eliminate the type of error. Schaeffer et al. (3), also considered autocorrelated error, but they used multiplicative method for the elimination. The method proposed by Schaeffer et al. (3) was used in this study.

Conclusions

The parameters of nonlinear regression models for milk production of Jersey cows at Karaköy State Farm were estimated in each of the three lactation groups. The data are shown to foollow log-normall distribution. Therefore, logarithmic transformation was used to convert the data to the normal distribution. Five different nonlinear regression models were used. The lactation group and QN methods were selected randomly for the determination of the most efficient model. Then each model were fitted to the data in all three groups. In testing the models, mean valsue of loss function, standard error of estimators of the parameters and similarity of estimators values of parameters were used as principal criteria. Because of the weaknesses the first four models were eliminated.

The first model did not fit the curve for the there cases and also gave negative valued estimates of the parameters. Negative results are not meaningful and the model was eliminated. The second model gave very large values of standard errors for estimatrs of the parameters and it was also eliminatded. The third

model had very large loss function value than the others have and also in many data sets optimum solutions were not found. Morant and Gnanasakty (7), reporter similar results the fourth model was seen to be suitable, but one of te data sets gave high loss function value. As a result the curve was not acceptable for the lactation. The last model very close to each other, so the fifth model was shown to be the best for explaining the lactation curve in cows.

The selected model and five different estimation methods were used for the lactation groups. Then these five methods were compared with each other. The mean values of loss function, the average elapsed time and the number of iteration and the gradients were considered as the bases for comparisons. The mean loss function valsue were close to each other in all methods. The QN method, on the other hand, yielded with the smallest value. The negative parameter values were restricted and tested. Then the restrictions were applied for all cases. So the restriction was valid for the model used in each of the three lactation groups. When the total elapsed time was considered, the methods MRT and CG, which were completed ealier than the methods of NM and HJ. Because, the package program of QN method did not give out the total elapsed time during the iteration. This method was omitted. The method that requires derivatives needs less iteration, than the others. Because of the NM and HJ methods do not require derivetives, the mean values of number of gradient were zero. The result showed that the methods that require derivatives are more efficient than the others. Furthermore, The MRT was faster than CG. As a result, it may be stated that MRT was the best among the other methods used in this study.

Considering the above explanation the lactation curves were obtained for each of the theree lactation groups using the fifth model and MRT method. The

result was presented in Figure 1 and three models are as follows:

Group 1:

y=EXP(2.0798-0.0023t+Ln((1-EXP(-0.9773t)) /0.9773)

Group 2:

y=EXP(1.9623-0.0032t+Ln((1-EXP(-0.6003t)) /0.6003)

Group 3:

y=EXP(1.6184-0.0030t+Ln((1-EXP(-0.4076t)) /0.4076)

Depending on the results obtained the following conclusions were reached:

i)Using incomplete lactation records, 305-day milk yield of the cows can be predicted.

ii)The estimated parameters and lactation curves may be used determine the portions of the effects of genotype and environment.

iii)The lactation curve of the herd may be used as a standard to determine the possible deviations. So that the necessary measures can be taken to improve the production.

iv) The results obtained in this study may be considered useful for the explanation of the other productive characteristics of animals such as egg production or growth rate.
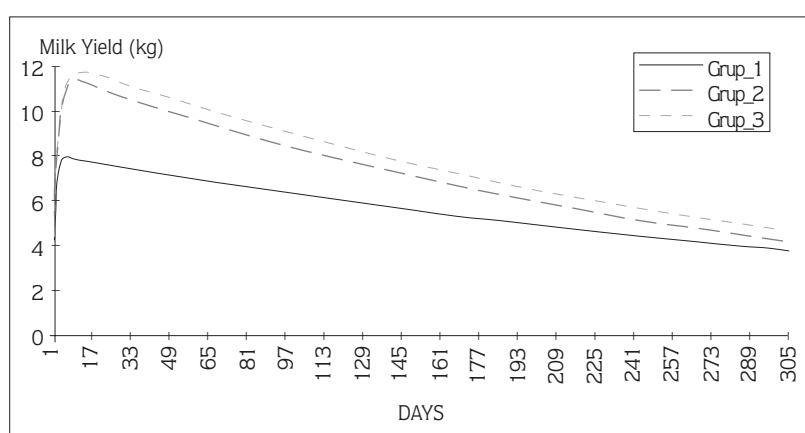

Figure 1. Lactation curve of the three lactation groups

References

1. Alpan,O: Sığır Yetiştiriciliği ve Besiciliği. Ankara, 2.Basım, 1992.

2. Schaeffer, L.R. and Burnside, E.B.: Estimating the Shape of the Lactation Curve. Can.J. Anim. Sci., 1976, 56:157-170.

3. Schaeffer L.R., Minder, C.E., McMilian, I. and Burnside, E.B.:Nonlinear Techniques for Predicting 305-Day Production of Holsteins and Jersey. J. Dairy Sci. 1977,60: 1636-1344.

4. Keown, J.F. and Van-Veleck, L.D.: Extending lactation Records in progress to 305-Day Euivalent. J.Dairy Sci. 1972, 56(8):1070-1079.

5. Khoda, V.K., Trivedi, K.R. and Patel, J.P.: Age Correction Facors for Lactation Records in Jersey Cows. Indian Vet.J. 1989,66:749-755.

6. Roy, T.C. and Katpatal, B.G.: Predicting of 300 Days Milk Yields from Part Lactation Records in Jersey Cattle. Indian Vet.J.1989, 66:749-755.

7. Morant, S.V. and Gnanasakthy A.: A New Approach to the Mathematical Forlulation of Lactation Curves. Anim. Prod. 1998, 49: 151-162.

8. Cohran, W.G.: Sampling Techniques. Third ed. Joph Wiley and Sons, New York, 1977.

9. Bates,D.M. and Watts, D.G.: Non-Linear Regression Analysis and Its Application. John Wiiley and Sons, New York, 1988.

10. Wilkinson, L. SYSTAT The System for Statistics Evolution. II. SYSTAT Inc. 1990.

11. Marquard, D.W.: An Aygorithm for Least Squares Estimation of Nonlinear Parameters. J. Soc. Indust. Appl.Math. 1963,11 (2):431-441.

12. Watts, D.G.:An Introduction to Nonlinear Least Squares. in L Endreyni Ed. Kinetic Data Analysis, Design and Analysis of Enzyme Pharmacokinetic Experiments. Plenium, New York, pp 1-24.1987.

13. Nash, C.J. and Walker-Smith, M.: Nonlinear Parameter Estimetion. Marcel Dekker Inc New York, 1987.

14. Gallant, A.R.: Nonlinear Regression. The American Statistician. 1975,29: 73-81.

15. Gallant, A.R.: Nonlinear Statistical Model. John Wiley and Sons, New York, 1987.

16. Gallant, A.R.: and Goebel, J.J.: Nonlinear Regression with Autocorrelated Errors. American Statist. Asoc.J-Theory and Method. 1976, 71:961-967.

17. Connover, J.: Practical Nonparametric Statistical Methods. II. Ed.John Wiley and Sons, New York, 1980.

18. Glasbey, C.A.: Correlated Residual In Nonlinear Regression Applied to Growth Data. Appl. Statist. 1979, 28(3):251-259.